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BILLET' SPLIT LENS IN A NEARLY UNKNOWN HYPOSTASIS

F. S. Uliu
Department of Physics 1, Faculty of Sciences, University of Craiova,
Str. Al. I. Cuza, 13, 1100 Craiova, Romania.

The classical problem of the Billet split lens as interference device is revisited. We have in view only the situation in which a central portion of a thin convergent lens is cut out symmetrically and both remaining halves are tightly fitted against each other. We calculate the width of an interference fringe and the number of interference bands in the field of interference, for various positions of the light source (on the symmetry axis) and of the observation screen. Thus, the paper presents the theoretical results required to design a Billet interference device, in a "new" configuration. Billet lenses can be tailored in amorphous chalcogenide materials to be used in optical elements at infrared vawelengths.

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1. Introduction

During the nineteenth century, the French physicist Félix Billet (1808-1882), professor at the University of Dijon, imagined and achieved deeply detailed experimental research on "two light-sources" interference device which to date now is called by his name [1]. The Billet split lens consists of two halves of a thin convergent lens, cut along a diameter. The two halves are slightly separated and this space is covered by an opaque screen (black paste). The whole arrangement is described and analysed in many books on Optics [2-11] as one of the classical devices used to observe interference by means of conventional (nonlaser) light sources. Of course, the principles used for studying interference with the Billet split lens are similar to those of Young's two-slit arrangement, of the Fresnel mirrors or biprism, or of the Lloyd mirror: to obtain coherent light waves, light from a single classical point-like source is split into two systems of waves by dividing the incident wavefront. Indeed, the two halves of the Billet split lens produce two noncoincident real images of the light source (placed on the symmetry axis) and the interference of light from these coherent secondary sources is observed on a screen.

This paper aims to illustrate the functioning of a "new" arrangement with the Billet split lens, which is seldom mentioned in the usual literature on wave optics (this is the reason we placed "new" in quotes) - see [12] - and, by this, we think, much more interesting. For various possible positions of the light source and of the observation screen, the details of geometrical optics calculations required to design such a Billet interference device are presented.

2. The "new" arrangement

Let us suppose that a central portion of width d is symmetrically cut out of a thin convergent lens of diameter $D(d << D)$, and focal length f (Fig. 1 a)), and both halves are tightly fitted against each other. The "new lens" receives monochromatic light (wavelength λ) from a point-like source S, at a distance $SC \equiv a$ from it. Interference of light is showing on a screen (E) lying on the opposite side of the lens.

After the cutting operation the two semi-lenses are as large as $(D-d)/2$ each so that, in Fig. 1 b), C_j are the curvature centres of the spherical surfaces denoted by the index j (=1,2,3 and 4). We see that a ray of light emitted from the source S - situated on the symmetry axis (Δ) - would suffer deviation located below and above the principal axis (Δ) as if it was passing through prisms with common basis. If the curvature radii R_i of the spherical surfaces i = (1,2) or (3,4), were considerable and the semi-lenses could be seen as thin prisms with A as refringent angle, the deviations would be accessible by approximated formula $\delta \approx (n-1)A$, where n would be the refractive index of glass (air is assumed on the outside).

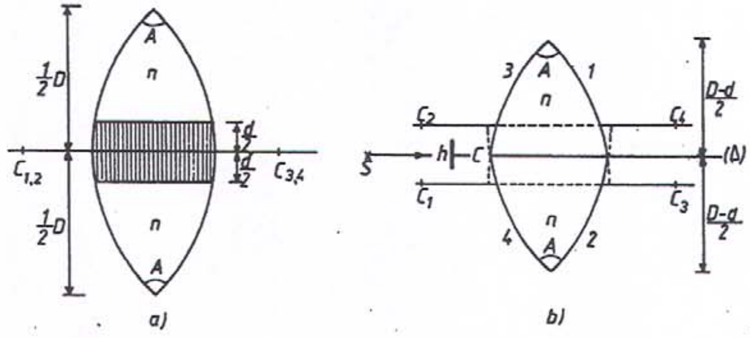

Fig. 1 A thin convergent lens (a) and bilens' manufacturing (b). Essential geometric elements are shown.

To avoid penetration in the bilens of direct light ray SC (and of some other adjacent rays) and in order to achieve a real division of the incident wavefront, this time too it is necessary to implant a tiny opaque screen in front of C (of width $h \approx g(n-1)d/2nR_{in} < d$, where g is the thickness of the lens and R_{in} is the curvature radius of the front entrance of the lens). Otherwise, after refraction in C at the surface 3 or 4, the inner ray would go towards the surface 2 or 1, respectively, and after a new refraction, the corresponding emergent ray would not propagate towards the points I' or I'' (see Fig. 2).

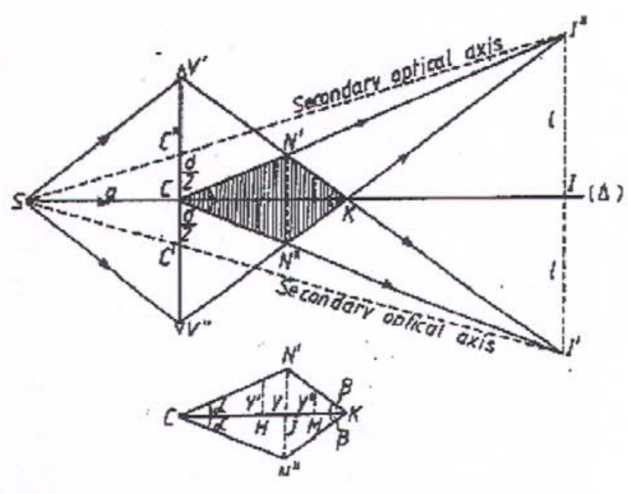

Fig. 2 Interference with Billet's bilens in the case SC = a > f. The coherent sources are the real images (I' and I'') of the point-like source S in the two halves of the lens.

3. Theory and computational details

We intend to study the functioning of this bilens as an interference device in two distinct situations: 1). $a \geq f$ and 2). $a < f$.

3.1. The $a \geq f$ case

To understand the construction of images in Fig. 2 we notice that C' and C'' are the optical centres of the semi-lenses as large as D/2 (if removing not achieved!) and the lines SC' and SC'' are secondary optical axes for the semi-lenses from the upper part and from the downside part of the drawing. Of course, these axes are shown in order to ease locating, in the paraxial approximation, the images I' and I'' of the source S through the respective semi-lenses. It is easy to see that the image of S, given by the downward semi-lens is formed on the upper side (I'') and the image of S, given by the upward semi-lens is formed down-side (I').

The region of interference is situated between the points C and K (harrowed in the drawing). Therefore, we are due to place the screen on which we aim to observe the interference pattern, between C and K. There is a point J on the line CK where the interference field is the largest and, from this point of view, we have a resemblance to Meslin's bilens device [10].

A peculiar property of this arrangement is that the interference region is situated on the left side of the "Young - sources" I' and I'', and not on their right side as in the other classical types of interference arrangements, but this is not bothering for us. We know that the optical path from S to I', or from S to I'', is the same for all rays converging in I' or in I''. By invocating the principle of reversibility for the path of light rays, we could imagine that the light propagates from I' and I'' (as Young-sources) to the left, i.e. to the interference region. By this "trick" the phase difference of the light rays over lapping in the CN'KN''C-zone is altered only in sign (mathematically) and this is irrelevant in interference term.

If we were to denote $b \equiv CI$, by means of the lens equation in the Gaussian form (1/a+1/b=1/f) we should obtain b=af/(a-f). Then, from the similarity of triangles SCC'' and SII'' it should follow that the distance between I' and I'' is

$$2l = 2II' = 2II'' = \frac{d}{a}(a+b) = \frac{ad}{a-f}. \quad (1)$$

On the other hand, from the similarity of triangles CKV'' and IKI'' we could deduce the length of the interference region on the axis

$$CK = \frac{b(D-d)}{D-d+2l} = \frac{af(D-d)}{D(a-f)+fd}. \quad (2)$$

From Fig. 2 we see that

$$tg\alpha = \frac{l}{b} = \frac{d}{2f}, \quad (3)$$

i.e. the angle α is the same for all positions of the source S.

Aiming to locate the inner point J of the interference field, we write $y = CJ \cdot tg\alpha = JK \cdot tg\beta = (CK - CJ) \cdot tg\beta$, where $tg\beta = (D-d)/2CK$, and finally we get to:

$$CJ = \frac{af(D-d)}{D(a-f)+d(a+f)}. \quad (4)$$

Then

$$JI = CI - CJ = b - CJ = \frac{2a^2fd}{(a-f)\left[D(a-f)+d(a+f)\right]}, \quad (5)$$

and the maximal width of the interference field, in the transversal direction, is

$$2y = 2CJ \cdot tg\alpha = \frac{ad(D-d)}{D(a-f)+d(a+f)} . \qquad (6)$$

Now, we remember the theory of Young's device [4], which demonstrates that the width of an interference fringe is $i = \lambda \cdot$[distance between the plane of the slits and the observation screen] /[distance between the two slits].

If we were to suppose that the screen is a plane perpendicular in J to the axis (Δ), we could write

$$i = \lambda \frac{IJ}{2l} = \frac{2a\lambda f}{D(a-f)+d(a+f)} . \qquad (7)$$

Between N' and N'', the number of interference bands on the screen, is

$$N = \frac{2y}{i} = \frac{d(D-d)}{2\lambda f} . \qquad (8)$$

Of course, the number of interference rings (circles of maximum and minimum intensity) is N/2. We notice that the numbers N and N/2 must be considered always as "integer parts", in the mathematical meaning. The fact that the number N is not depending on the source position (when $a \geq f$) is highly remarkable. We also notice that N is inversely proportional to the focal distance f of the lens. When d=0 (absence of cutting), from (8) we have N=0, which means the absence of the interference phenomenon.

In the general case, when $d \neq 0$, we could particularize the above results to the limits $a \to +\infty$ (i.e. the incident beam of light is parallel), and $a \to +f$.

In the first situation, namely $a \to +\infty$ (Fig. 3), we get to $b \to +f$, $2\ell \to d$,
$CK \to f(1-d/D)$, $CJ \to f(D-d)/(D+d)$, $JI \to 2fd/(D+d)$, $2y \to d(D-d)/(D+d)$.

With the screen in J we obtain

$$i = \frac{2\lambda f}{D+d} \quad \text{and} \quad N = \frac{d(D-d)}{2\lambda f} . \qquad (9)$$

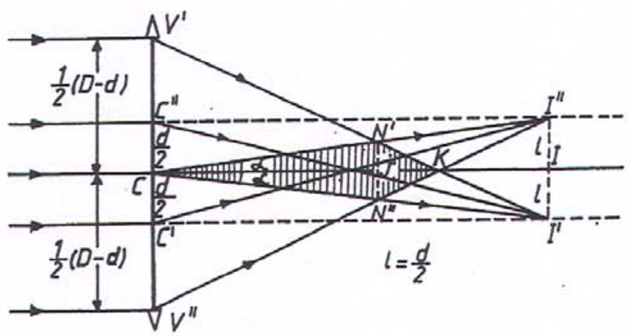

Fig. 3 Interference with Billet split lens in the case SC = a →+∞ (the incident beam of light is parallel).

In the second situation when $a \to +f$ (Fig. 4), we obtain $b \to +\infty$, $2\ell \to +\infty$, $CK \to f(D/d-1)$, $CJ \to (1/2)CK = (f/2)(D/d-1)$, $JI \to +\infty$, $2y \to (D-d)/2$ and, with the screen in J, we have

$$i = \frac{\lambda f}{d} \quad \text{and} \quad N = \frac{d(D-d)}{2\lambda f} . \qquad (10)$$

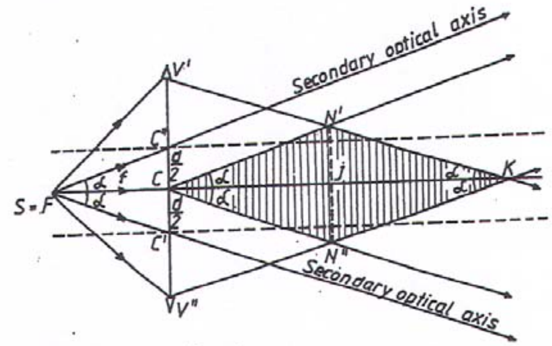

Fig. 4 Interference with Billet's bilens in the case SC = a = +f.
The coherent sources I' and I'' are infinitely moved in the right side of the figure.

3.1.1. The screen arranged between C and J (in the point H)

Let be $CH \equiv L$, so that $y' = L \cdot tg\alpha = Ld/2f$. In this case we have HI=b-L=l /tgα–L=2 ℓ f/d-L. The fringe separation is

$$i' = \lambda \frac{HI}{2l} = \frac{\lambda}{ad}\left[af - L(a-f)\right] . \tag{11}$$

For the number of interference fringes we obtain

$$N' = \frac{2y'}{i'} = \frac{aLd^2}{\lambda f\left[af - L(a-f)\right]} \tag{12}$$

and, this time, the number N' depends on the position of the source.

In the limit $a \to +\infty$ we have

$$i' = \lambda \frac{f-L}{d} \quad \text{and} \quad N' = \frac{Ld^2}{\lambda f(f-L)} . \tag{13}$$

On the other hand, in the limit $a \to + f$ we find

$$i' = \frac{\lambda f}{d} \quad \text{and} \quad N' = \frac{Ld^2}{\lambda f^2} . \tag{14}$$

3.1.2. The screen installed between J and K (in the point M)

Let be CM≡L, so that MK=CK-L, with CK expressed as in eq. (2). We can write y''=MK.tgβ=MK.(V'C/ CK)=$(D-d)$.MK/2. CK, so that finally we find

$$2y'' = \frac{af(D-d) - L\left[D(a-f) + fd\right]}{af} . \tag{15}$$

On the other hand,

$$MI = b - L = \frac{af - L(a-f)}{a-f} . \tag{16}$$

The fringe separation is

$$i'' = \lambda \frac{M}{2l} = \frac{\lambda}{ad}\left[af - L(a-f)\right] \tag{17}$$

and, for the number of interference bands on the screen, we obtain

$$N''=\frac{2y''}{i''}=\frac{d\{af(D-d)-L[D(a-f)+fd]\}}{\lambda f[af-L(a-f)]}. \tag{18}$$

The validity of the relations (12) and (18) could be verified by applying them to the particular value from eq. (4). We immediately obtain

$$N'=N''=\frac{d(D-d)}{2\lambda f}, \tag{19}$$

i.e. the maximal number of interference bands previously found in the relation (8).

When $a \to +f$, from the general relations (17) and (18) we find

$$i''=\frac{\lambda}{d}(f-L) \quad \text{and} \quad N''=\frac{d[f(D-d)-LD]}{\lambda f(f-L)}, \tag{20}$$

and, when $a \to +f$, from the same relations we get

$$i''=\frac{\lambda f}{d} \quad \text{and} \quad N''=\frac{d[f(D-d)-Ld]}{\lambda f^2}. \tag{21}$$

3.2. The a < f case

In this case, the images of the object S given by the two semi-lenses are virtual - as it is shown in Fig 5, and the device is functioning like the Fresnel biprism. The interference zone, which is unlimited along the symmetry axis, has its own axis on the symmetry axis of the device.

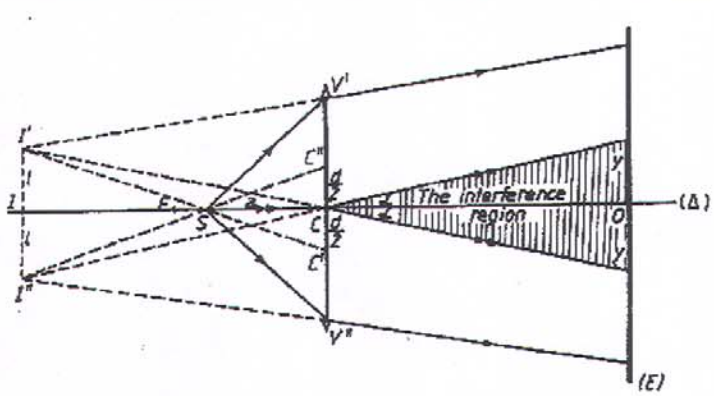

Fig. 5 Interference with Billet's bilens in the case SC = a < f.

The coherent sources are the virtual images (I' and I'') of the source S in the two halves of the lens. As for the case of Fresnel's biprism, the interference field is unlimited on the direction of the symmetry axis (Δ).

As previously, if SC=a and CI=b, from the lens equation in the Gaussian form (1/a-1/b=1/f) we should obtain b=af/(f-a). From the similarity of triangles SII' and SCC' we easily find the distance between the "Young-sources" I' and I'', namely

$$2\ell=\frac{ad}{f-a}. \tag{22}$$

For the angular opening of the interference region (the harrowed zone in the drawing) we can approximate

$$2\alpha \approx \frac{2\ell}{b} = \frac{d}{f}. \tag{23}$$

If the monitoring screen were installed at the distance L≡ CO, the field of interference on the screen would be

$$2y \approx 2\alpha L \approx \frac{Ld}{f} \tag{24}$$

and the width of an interference fringe is

$$i = \frac{\lambda(b+L)}{2\ell} = \frac{\lambda}{ad}\left[af + L(f-a)\right]. \tag{25}$$

Now, we can determine the number of interference bands on the screen as

$$N = \frac{2y}{i} = \frac{aLd^2}{\lambda f\left[af + L(f-a)\right]} \tag{26}$$

and we notice that, for $L \to +\infty$, the number N is finite,

$$N \to N_{max} = \frac{ad^2}{\lambda f(f-a)}. \tag{27}$$

We can also remark that all results obtained in this last section of our paper do not depend on the diameter D of the initial lens.

3. Conclusions

The "new" hypostasis of the Billet bilens described in this paper is able to enhance significantly the utility of this classical interference device. On this bases is possible to produce new passive elements for integrated optics in amorphous chalcogenide films with the technique developed in [13]. The laser or the electron beam can be used for tailoring Billet bilens for use in the infrared region of the electromagnetic spectrum.

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