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SOLITONS IN STIMULATED BRILLOUIN SCATTERING

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A study on the existence conditions for the temporal solitons in the stimulated Brillouin scattering process (SBS) is presented. The possibility of occurence of compensation and topological solitons from the interference of the three coherent fields is analytically demonstrated.

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1. Introduction

The aim of this work is the identification and characterization of optical solitons in stimulated Brillouin scattering (SBS). A large class of nonlinear processes is investigated using the inverse problem in the scattering theory, which was pioneered in the work of Gardner et al. [1] and Reed and Simon [2].

The study of the optical nonlinear processes using the inverse problem in the scattering theory revealed the physical conditions for the existence of optical solitons and allowed the description of their dynamics (the evolution in nonlinear media, collisions, interaction between several solitons etc). Calogero and Degasperis described the spectral analysis of the nonlinear equations of evolution that results from the inverse problem in the scattering theory [3]. Ablowitz and Segur [4] gave a rigorous characterization of solitons in the inverse method in the scattering theory Novikov et al. [5] analyzed the existence of solitons in the interaction process of several waves in nonlinear media.

We have built the nonlinear SBS equations using the derivatives along the characteristic directions of the solution of D'Alembert wave equation (the main mathematical technique in the inverse method in the scattering theory) [6]. This system of (first-order nonlinear autonomous) equations takes the general form:

$$\frac{dx^i}{dy} = a^i_j x^j + h^i_{jk} x^j x^k \tag{1}$$

where x^i are Riemann invariants (which contain the pump beam intensity and the Stokes beam intensity).

We found out the conditions for the Stokes soliton occurence in the SBS media with losses ($\alpha>0$) and without losses ($\alpha = 0$) in the form:

$$I_S \cong \left(\frac{\Delta\rho}{\rho_0}\right)^2 \cdot I_{LC} + \delta(I_{L0}) \tag{2}$$

where: α is the linear optical losses coefficient ;

I_S - the Stokes component;

I_{LC} - the incident component perturbed in the SBS process;

I_{L0} - the incident component that is not perturbed in the SBS process;

and $\Delta\rho/\rho 0$ - the acoustic field component, the relative variation of the nonlinear medium density.

We have analyzed the soliton which arises from the compensation of the dispersion by the SBS nonlinearity, the pulse duration and propagation velocity in the nonlinear medium. This soliton is called, in this paper, compensation soliton.

Finally, we have built the system of equations (1) supposing very low dispersion ($k = k(\omega) \approx 0$). In this case, the Stokes soliton occurs not as the result of the compensation between the dispersion and the nonlinearity (as in the case of the Korteweg - de Vries equations), but it is the result of a condition imposed on the SBS nonlinear equation system, which is given in our case by Eq. (2). This SBS soliton is called, in this paper, topological soliton and it is characterized by different parameters than those of the compensation soliton. Sagdeev et al. [7] describe a similar tentative for identifying solitons in hydrodynamics.

2. Nonlinear model of SBS using prime integrals on the characteristic curves

The hypotheses in which the model is constructed are following the SBS description given by Yariv [8]:

a) the electromagnetic field is described by the Maxwell's equations with nonlinear polarization, P_{NL}, in the Gauss system, (hypothesis introduced by Armstrong and Bloembergen);

b) the SBS process is produced by the variation of the dielectric permittivity of the medium, which is induced by the pressure fluctuations, P, at constant entropy, S: $\partial\varepsilon / \partial T = 0$. $\lambda_T = 0$; $\beta_T = 0$;

c) the conservation relations holds: $\theta_S = \pi$; $K = K_L + K_S \cong 2K_L$; $\omega = \omega_L - \omega_S$;

d) the geometry of the SBS process is that of Fig.1; where $\mathbf{K}_L$ is the wave vector of the pump optical field, $\boldsymbol{K}_S$ is the wave vector of the scattered (Stokes) field and $\mathbf{K}$ is the wave vector of the induced acoustical field

e) a progressive wave representation for the incident and for the reflected (scattered) fields is selected;

f) the field evolution is considered on the propagation axis only: $\vec{E} = \vec{E}(z', t)$; $\vec{B} = \vec{B}(z', t)$;

g) the electric $\vec{E}$ and magnetic $\vec{B}$ fields are linearly polarized vectors;

h) the scattered component of the field in the SBS process are the Stokes components.

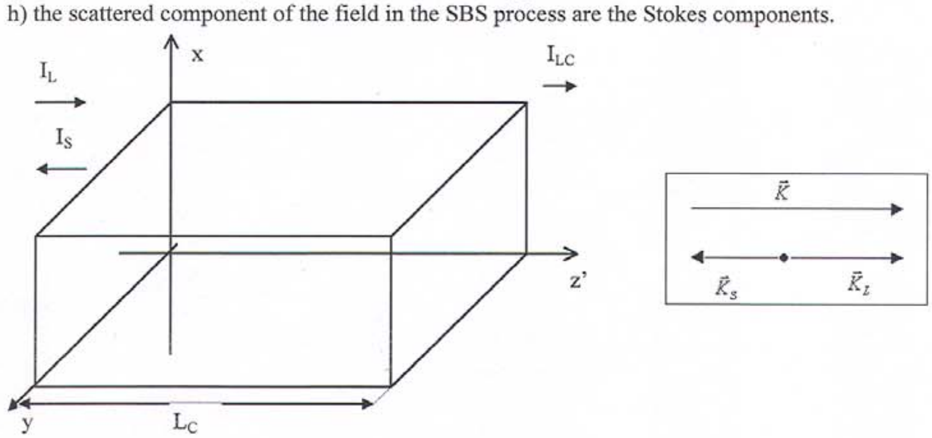

Fig. 1 The geometry of the SBS process; I_L - pump intensity; I_S - scattered intensity; I_{LC} - transmitted intensity.

With these hypotheses, we shall write the nonlinear equations of the evolution of the optical pump, scattered (Stokes) and acoustic fields. The selection condition for the Stokes component can be written as [13] and are defined in annex:

$$\xi_L - \xi_S = 2 \cdot \xi_{1f} \tag{3}$$

In order to simplify the equations, one can introduce the some new variables, with physical meaning of phases of the waves, which are defined along the three characteristics $\{\xi_L, \xi_S, \xi_{1f}\}$[8]:

$$\varphi_L = \omega_L t + K_L \cdot z' = \xi_L \cdot K_L, \varphi_S = \omega_S t - K_S \cdot z' = \xi_S \cdot K_S, \varphi_f = \omega t + K \cdot z' = \xi_{1f} \cdot K \tag{4}$$

where the ω_L is the frequency of the pump optical field, ω_S is the frequency of the scattered optical (Stokes) field and ω is the frequency of the induced acoustical field.

The prime integral along the $\{\xi_{1f}, \xi_L, \xi_S\}$ characteristic is (are derived in annex):

$$\begin{aligned}
\frac{\partial E_L}{\partial \xi_L} &= -\frac{\alpha}{4} E_L - \frac{\pi \cdot \gamma^e}{n^2} \cdot \frac{\partial}{\partial \xi_L}(E_S \cdot \rho') \\
\frac{\partial E_S}{\partial \xi_S} &= -\frac{\alpha}{4} E_S + \frac{\pi \cdot \gamma^e}{n^2} \cdot \frac{\partial}{\partial \xi_S}(E_L \cdot \rho') \\
\frac{\partial \rho'}{\partial \xi_{1f}} &+ \left(\frac{4k\omega}{\Gamma_B}\right)\rho' = \frac{\gamma^e \cdot k^3}{8\pi\rho_0\omega\Gamma_B}\left(E_L + E_S\right)^2
\end{aligned} \tag{5}$$

where E_L is (optical) field; E_S - the amplitude of the scattered electrical (optical) field and ρ' - the amplitude of the acoustical field.

Substitution of $\{\xi_L, \xi_S, \xi_f\}$ in eq. (5) leads to:

$$\begin{aligned}
&\frac{\partial E_L}{\partial \varphi_L} = \frac{-\alpha}{4K_L} \cdot E_L - \frac{\pi \gamma^e}{n^2} \cdot \frac{\partial}{\partial \varphi_L}(E_S \cdot \rho'), \quad \frac{\partial E_S}{\partial \varphi_S} = \frac{-\alpha}{4K_L} \cdot E_S + \frac{\pi \gamma^e}{n^2} \cdot \frac{\partial}{\partial \varphi_S}(E_L \cdot \rho'), \\
&\frac{\partial \rho'}{\partial \varphi_f} = -\frac{4\omega}{\Gamma_B} \cdot \rho' + \frac{\gamma^e \cdot K^2}{8\pi\rho_0\omega\Gamma_B}(E_L + E_S)^2
\end{aligned} \tag{6}$$

The system (6) can be normalized by means of the substitutions: (I_0 = the maximum value of the pump field intensity)

$$x = \sqrt{\frac{c \cdot n}{8\pi \cdot I_0}} \cdot E_L, \quad y = \sqrt{\frac{c \cdot n}{8\pi \cdot I_0}} \cdot E_S, \quad z = \frac{\pi \gamma^e}{n^2} \cdot (\rho') \quad A = \frac{2\omega}{\Gamma_B}; \alpha' = \frac{\alpha}{2K_L} \cong \frac{\alpha}{2K_S} = \frac{\alpha}{K} \tag{7}$$

$\Gamma_B = K^2 \eta_B / \rho_0$

where n is the linear refractive index of the propagation medium, c is the light velocity in free space, η_B is the viscosity of the propagation medium, and α- linear losses of the propagation medium.

In order to obtain

$$\begin{aligned}
&\frac{\partial x}{\partial \varphi_L} = -\frac{\alpha'}{2} \cdot x - \frac{\partial}{\partial \varphi_L}(y \cdot z), \quad \frac{\partial y}{\partial \varphi_S} = -\frac{\alpha'}{2} \cdot y + \frac{\partial}{\partial \varphi_S}(x \cdot z), \\
&\frac{\partial z}{\partial \varphi_f} = -2 \cdot A \cdot z + \frac{\omega_L^2 \gamma^{e^2} \pi v I_0}{c^3 n \rho_0 v \Gamma_B 4\omega} \cdot x \cdot y;
\end{aligned} \tag{8}$$

where γ^e is the isentropic compression coefficient of the nonlinear propagation medium and v is the velocity of the acoustical field in the nonlinear propagation medium.

Using the expression of the SBS gain defined by Kroll [9]:

$$g_B^e\left[\frac{cm}{W}\right]=\frac{\omega_L^e(\gamma^e)^2}{c^3 n\rho_0 v\Gamma_B} \tag{9}$$

the characteristic length of interaction and the normalized cross-section in the SBS process [10]

$$L_B^{Def}=4\pi\cdot\frac{v}{\omega} \tag{10}$$

and the normalized SBS gain in the form:

$$\sigma^{Def}=g_B^e\cdot L_B\cdot I_0=4\pi g_B^e\cdot\frac{v}{\omega}\cdot I_0 \tag{11}$$

we can obtain the complete set of equations which describe the SBS process with conditions (a) -(h):

$$\frac{\partial x}{\partial\varphi_L}=-\frac{\alpha'}{2}\cdot x-\frac{\partial}{\partial\varphi_L}(y\cdot z),\quad \frac{\partial y}{\partial\varphi_S}=-\frac{\alpha'}{2}\cdot y+\frac{\partial}{\partial\varphi_S}(x\cdot z),\quad \frac{\partial z}{\partial\varphi_f}=-2A\cdot z+\sigma\cdot x\cdot y; \tag{12}$$

We shall present some methods for solving SBS equation system (12). The Cauchy problem for the system (12) is:

$$\chi(\varphi_L)\big|_{\varphi_L=\varphi_{L_0}}=x_0(\varphi_{L_0})\quad y(\varphi_S)\big|_{\varphi_{S_0}=\varphi_{S_0}}=y_0(\varphi_{S_0})\quad z(\varphi_f)\big|_{\varphi_f=\varphi_{f_0}}=z_0(\varphi_{f_0}) \tag{13}$$

where:

$$\varphi_L\big|_{z'=0}=\varphi_{L_0},\quad \varphi_S\big|_{z'=0}=\varphi_{S_0},\quad \varphi_f\big|_{z'=0}=\varphi_{f_0} \tag{14}$$

In order to obtain an homogenous structure in the phase space, we introduce the scalar transform

$$x'=\sqrt{\frac{L_B}{L'_B}}\cdot x;\quad y'=\sqrt{\frac{L_B}{L'_B}}\cdot y;\quad z'\equiv z \tag{15}$$

where L_B is the interaction length in the (S.B.S.) process and the system (12) takes the form:

$$\frac{\partial x'}{\partial\varphi_L}=-\frac{\alpha'}{2}\cdot x'-\frac{\partial}{\partial\varphi_L}(y'\cdot z),\quad \frac{\partial y'}{\partial\varphi_S}=-\frac{\alpha'}{2}\cdot y'+\frac{\partial}{\partial\varphi_S}(x'\cdot z),\quad \frac{\partial z}{\partial\varphi_f}=-2A\cdot z+\sigma_1\cdot x'\cdot y'$$
$$\sigma_1=g_B^e L'_B I_0 \tag{16}$$

We are looking for solution of the system (16) in the form:

$$x'=x_1\cdot e^{i\varphi_L}+x_1^*\cdot e^{-i\varphi_L},\quad y'=y_1\cdot e^{i\varphi_S}+y_1^*\cdot e^{-i\varphi_S},\quad z=z_1\cdot e^{i\varphi_f}+z_1^*\cdot e^{-i\varphi_f} \tag{17}$$

From (16) and (17), one can derive:

$$\frac{\partial}{\partial\varphi_L}(x_1+y_1\cdot z_1)+i(x_1+y_1\cdot z_1)=-\frac{\alpha'}{2}\cdot x_1,$$
$$\frac{\partial}{\partial\varphi_S}(y_1-x_1\cdot z_1^*)+i(y_1-x_1\cdot z_1^*)=-\frac{\alpha'}{2}\cdot y_1 \tag{18}$$
$$\frac{\partial z_1}{\partial\varphi_f}+i_1\cdot z_1=-2A\cdot z_1+\sigma_1\cdot x_1\cdot y_1^*$$

where: x_1*, y_1* and z_1* means the complex conjugated of x_1, y_1 and z_1, respectively.

Using the complex conjugated equations of the system (18), a system describing a quasilinear - hyperbolic evolution is obtained.

For the beginning, the system (18) is put in a parametric form by means of the "projection" of the evolution along the $[\varphi_L]$ and $[\varphi_S]$ characteristics on the acoustic field characteristic $[\varphi_f]$. The configuration of the characteristics φ_L, φ_S, φ_f in the $\{z', t\}$ plane, is presented in Fig. (2).

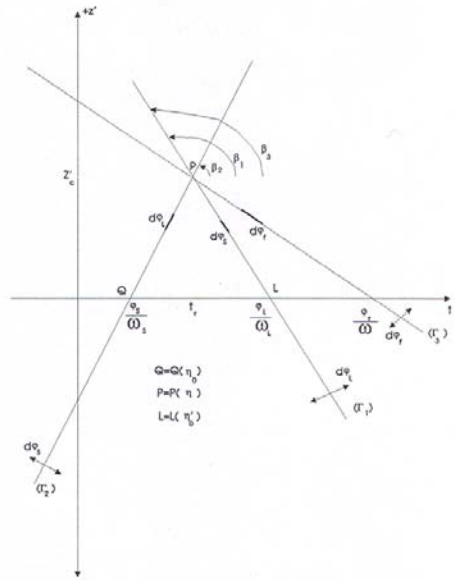

Fig. 2 The characteristics in the space-time plane.

These characteristics are defined in the form:

$$\Gamma_1 : \varphi_L = \omega_L \cdot t + K_L z', \ \Gamma_2 : \varphi_S = \omega_S \cdot t - K_S z', \ \Gamma_3 : \varphi_f = \omega \cdot t + K \cdot z' \qquad (19)$$

By using Fig. 2, one can write:

$$tg\beta_1 = -\frac{\omega_L}{K_L} = -\frac{c}{n}; \quad tg\beta_2 = +\frac{\omega_S}{K_S} = +\frac{c}{n}; \quad tg\beta_3 = -\frac{\omega}{K} = -\frac{v}{\sqrt{\gamma}}; \quad \frac{c}{n} \rangle\rangle \frac{v}{\sqrt{\gamma}}; \qquad (20)$$

If the coordinate system from Fig. 2 is normalized by: $z' \to K \cdot z', \quad t \to \omega \cdot t$ the characteristic equations take the form:

$$\Gamma_1 : \varphi_L = \frac{\omega_L}{\omega}(\omega \cdot t) + \frac{K_L}{K}(Kz'); \ \Gamma_2 : \varphi_2 = \frac{\omega_S}{\omega}(\omega \cdot t) - \frac{K_S}{K}(Kz'); \ \Gamma_3 : \varphi_f = (\omega \cdot t) + (K \cdot z') \qquad (21)$$

In this case, Eqs. (19) become:

$$tg\beta_1 = -\frac{2\omega_L}{\omega} \cong -\frac{c}{n} \cdot \frac{\sqrt{\gamma}}{v}; \quad tg\beta_2 = +\frac{2\omega_S}{\omega} \cong +\frac{c}{n} \cdot \frac{\sqrt{\gamma}}{v}; \quad tg\beta_3 = -1 \qquad (22)$$

Differentiation of (21) yields:

$$\frac{d\varphi_L}{d\varphi_f} = \frac{\omega_L}{\omega} \cdot \frac{1 + \left(\frac{n}{c}\right)\left(\frac{dz'}{dt}\right)_{\varphi_L}}{1 + \frac{\sqrt{\gamma}}{v}\left(\frac{dz'}{dt}\right)_{\varphi_f}}; \quad \frac{d\varphi_S}{d\varphi_f} \cong \frac{\omega_L}{\omega} \cdot \frac{1 + \left(\frac{n}{c}\right)\left(-\frac{dz'}{dt}\right)_{\varphi_S}}{1 + \frac{\sqrt{\gamma}}{v}\left(\frac{dz'}{dt}\right)_{\varphi_f}} \qquad (23)$$

In the case of moving along *{Γ1}*, the initial condition is:

$$\left(\frac{dz'}{dt}\right)_{\varphi_S} = -\left(\frac{d\left(z_c' - z'\right)}{dt}\right)_{\varphi_S} \qquad (24)$$

In this case, from (23) and (24), one can obtain:

$$\delta=\left|\frac{1+\left(\frac{n}{c}\right)\left(\frac{dz'}{dt}\right)_{\varphi_L}}{1+\frac{\sqrt{\gamma}}{v}\left(\frac{dz'}{dt}\right)_{\varphi_f}}\right|=\left|\frac{1+\left(\frac{n}{c}\right)\left(\frac{d\left(z_c^{'}-z^{'}\right)}{dt}\right)_{\varphi_S}}{1+\frac{\sqrt{\gamma}}{v}\left(\frac{dz'}{dt}\right)_{\varphi_f}}\right|;\quad \frac{d\varphi_L}{d\varphi_f}=\frac{\omega_L}{\omega}\cdot\delta;\quad \frac{d\varphi_S}{d\varphi_f}=\frac{\omega_S}{\omega}\cdot\delta \tag{25}$$

Using (25), eqs. (18) take the form:

$$\begin{aligned}
&\frac{\partial}{\partial\varphi_f}\left(x_1+y_1\cdot z_1\right)+i\left(\delta\frac{\omega_L}{\omega}\right)\left(x_1+y_1\cdot z_1\right)=-\frac{\alpha'}{2}\left(\delta\frac{\omega_L}{\omega}\right)\cdot x_1\\
&\frac{\partial}{\partial\varphi_f}\left(y_1-x_1\cdot z_1^{*}\right)+i\left(\delta\frac{\omega_L}{\omega}\right)\left(y_1-x_1\cdot z_1^{*}\right)=-\frac{\alpha'}{2}\cdot\left(\delta\frac{\omega_L}{\omega}\right)\cdot y_1\\
&\frac{\partial}{\partial\varphi_f}\left(z_1+i_1\cdot z_1\right)=-(2A)z_1+\sigma_1\cdot x_1\cdot y_1^{*}
\end{aligned} \tag{26}$$

A nonlinear transform is used to bring the system (26) into a real form. In this case, the solutions of the real (differential hyperbolic quasilinear) system are implicit functions of Riemann invariants associated to the system (26) [11]. The Cauchy problem for the new system is defined. Thus, the system (26) takes form:

$$\begin{aligned}
&\frac{\partial N_1}{\partial\eta}=-4\gamma_1\cdot N_1+(2\gamma_1)\cdot N_2N_3-(\gamma_2);\quad \frac{\partial N_2}{\partial\eta}=-4\gamma_1\cdot N_2-2\gamma_2\cdot N_3-8\gamma_1\cdot N_1N_3\\
&\frac{\partial N_3}{\partial\eta}=+4\gamma_1N_3+\left(\frac{1}{2}\right)\cdot N_2+2\cdot N_1N_3-2\cdot N_2N_3^2
\end{aligned} \tag{27}$$

The invariants {N1(η), N2(η), N3(η)} are function of { $x_1x_1^*$; $y_1y_1^*$; $W(\eta)=z_1z_1^*(\eta)$ } and the following equations can be written:

$$\begin{aligned}
&x_1x_1^*-y_1y_1^*=\frac{4\left(1-N_3^2\right)}{1+3N_3^2}\left(N_1+\frac{\gamma_2}{4\gamma_1}-\frac{N_2\cdot N_3}{1-N_3^2}\right)\\
&\left(x_1x_1^*\right)\cdot\left(y_1y_1^*\right)=\left[\frac{N_2}{1-N_3^2}+\frac{4N_3}{1+3N_3^2}\left(N_1+\frac{\gamma_2}{4\gamma_1}-\frac{N_2\cdot N_3}{1-N_3^2}\right)\right]^2\\
&z_1\cdot z_1^*=N_3^2
\end{aligned} \tag{28}$$

These are precisely the components of the algebraic invariants calculated in [13].

If we write:

$$x_1x_1^*-y_1y_1^*=\varphi_1,\quad (x_1x_1^*)\cdot(y_1y_1^*)=\varphi_2 \tag{29}$$

then:

$$x_1x_1^*=\frac{2\varphi_2}{\sqrt{\varphi_1^2+4\varphi_2}-\varphi_1},\quad y_1y_1^*=\frac{2\varphi_2}{\sqrt{\varphi_1^2+4\varphi_2}+\varphi_1} \tag{30}$$

The numerical solutions of the equations (27-30) are given in [13].

3. Solitons in nonstationary SBS process

3.1. Compensation solitons

We try to find solution in the form of "sech^2 " using the hypothesis of " isospectral evolution" described in [3, 17,18]:

$$V_0(\eta)=\pm\Delta_0(\eta)\big|_{\eta\to\pm\infty} \tag{31}$$

In terms of the $\{N_1, N_2, N_3\}$ invariants, the condition (31) becomes:

$$N_2(\eta)=0 \tag{32}$$

Eq. (32) may be written in the form:

$$\left(x_1x_1^* - y_1y_1^*\right)^2 = \left(\frac{1-N_3^2}{N_3}\right)^2 \left(x_2x_1^*\right)\cdot\left(y_1y_1^*\right) \tag{33}$$

Using (28), (30) and (32), one can derive the functions $\{\varphi 1, \varphi 2\}$ as:

$$\varphi_1\Big|_{N_2=0} = \frac{4(1-N_3^2)}{1+3N_3^2}\left(N_1+\frac{\gamma_2}{4\gamma_1}\right); \quad \varphi_2\Big|_{N_2=0} = \frac{16N_3^2}{(1+3N_3^2)^2}\left(N_1+\frac{\gamma_2}{4\gamma_1}\right)^2 \tag{34}$$

From (30) and (34), we can write $\{ x_1x_1^*; \cdot y_1y_1^* \}$ in the form:

$$x_1x_1^* = \frac{4\cdot\left(N_1+\dfrac{\gamma_2}{4\gamma_1}\right)}{\left(1+3N_3^2\right)}; \quad y_1y_1^* = \frac{4N_3^2\cdot\left(N_1+\dfrac{\gamma_2}{4\gamma_1}\right)}{\left(1+3N_3^2\right)} \tag{35}$$

The equations of evolution for $\{N_1, N_3\}$, with (32) and (27), take the form:

$$\frac{\partial N_1}{\partial\eta} = -4\gamma_1\cdot N_1 - (\gamma_2); \quad \frac{\partial N_3}{\partial\eta} = +4\gamma_1 N_3 + 2\cdot N_1N_3 \tag{36}$$

Eqs. (35) lead to the implicit form:

$$y_1y_1^*(\eta) = N_3^2 x_1x_1^*(\eta) \tag{37}$$

Eqs. (37) express the dependence of the normalized Stokes field intensity (yy^*) on the normalized intensities of the optical pump and acoustic fields. This is the strong condition for the de existence of optical solitons in the scattered field. It can be mentioned also that Eq. (37) describes exactly the usual amplification regime of the Stokes field, with the difference that the normalized intensity of the optical pump field (xx^*) is perturbed (it is affected by the feedback induced by the other two fields, which is described by Eqs. (16)). We can also add that (yy^*, xx^*, N_3^2) are algebraic invariants that characterize the system (26) [12,13].

The Cauchy problem for the system (36) can be written in the form:

$$N_1(\eta)\Big|_{\eta=\eta'} = N_{10}(\eta'); \quad N_3(\eta)\Big|_{\eta=\eta'} = N_{30}(\eta') \tag{38}$$

where:

$$N_{10}(\eta') = N_{10}(x_{10}'(\eta'); \quad y_{10}'(\eta')); \quad N_{30}(\eta') = N_{30}(x_{10}'(\eta'); \quad y_{10}'(\eta')) \tag{39}$$

The solutions of the system (36), (that are not difficult to be found) and the Cauchy problem (38) and (39), show that Eqs. (35) are the evolution algebraic equations for $\{ x_1x_1^*; \cdot y_1y_1^* \}$ on the characteristics $\{\Gamma 1, \Gamma 2\}$.

One can express the prime integrals along the $\{\Gamma 1\}$ and $\{\Gamma 2\}$ characteristics as following: along $\{\Gamma 2\}$ one can calculate the prime integral for $\{ x_1x_1^*(\eta) \}$, while $\{ y_1y_1^*(\eta) \}$ remains constant at the intensity level of the spontaneous scattered field (Fig. 2):

$$\Gamma_2:\begin{cases} x_{10}'(\eta_0)\Big|_Q \longrightarrow x_1x_1^*(\eta)\big|_P \\ y_1y_1^*(\eta_0)\big|_Q \rightarrow y_1y_1^*(\eta_0) = y_S\big|_P \end{cases} \tag{40}$$

In these conditions, and taking into account that:

$$x_{10}^*(\eta_0) \gg y_S \quad , \tag{41}$$

eq. (40) yields:

$$x_1x_1^*(\eta)\big|_P \cong x_{10}'(\eta)\cdot e^{-4\gamma_1(\eta-\eta_0)} \tag{42}$$

Along $\{\Gamma 1\}$, one can calculate the prime integral for $\{ y_1 y_1^*(\eta_0^{''}) \}$, while $\{ x_1 x_1^*(\eta) \}$ will take the form (42), which remains unchanged along this characteristic:

$$\Gamma_1 : \begin{cases} x_1 x_1^*(\eta)\big|_P \rightarrow x_1 x_1^*(\eta)\big|_L \\ y_S\big|_P \rightarrow y_1 y_1^*(\eta_0^{''})\big|_{L;\eta_0^{''}-\eta=\eta_0^{'}} \end{cases} \tag{43}$$

From (42) and (43), one can obtain:

$$y_1 y_1^*\left(\eta_0^{\bullet}\right) \cong \frac{1}{12} x_{10}^{'}\left(\eta_0^{\bullet}\right) \cdot e^{-4y_1\left(\eta_0^{\bullet}-\eta\right)} \cdot ch^{-2}\left[\ln\sqrt{\frac{3 \cdot y_S}{x_{10}^{'}\left(\eta_0^{\bullet}\right)}} + \frac{8y_1^2 - y_2}{2y_1}\left(\eta_0^{\bullet} - \eta\right) + x_{10}^{'}\left(\eta_0^{\bullet}\right) \cdot \frac{1 - e^{-4\eta\left(\eta_0^{\bullet}-\eta\right)}}{8y_1}\right] \tag{44}$$

When $\alpha' = 0$, eq. (44) becomes:

$$y_1 y_1^*(\eta_0^{''}) = \left(\frac{x_{10}^{'}(\eta_0^{''})}{12}\right) ch^{-2}\left[\ln\sqrt{\frac{3 \cdot y_S}{x'_{10}(\eta_0^{''})}} + x'_{10}\,\frac{\eta_0^{''} - \eta}{2} \cdot - \frac{A}{2\sigma}(\eta_0^{''} - \eta)\right] \tag{45}$$

One remark's that:

$$y_1 y_1^*(\eta_0^{''}) = \Big|_{\eta_0^{''}=\eta} = y_S \approx e^{-G} \tag{46}$$

For $\alpha' \neq 0$, eq. (44) leads to:

$$y_1 y_1^*(t + \frac{n}{c} z_c^{'}) \cong \frac{\frac{1}{12} x_{10}^{'}(t + \frac{n}{c} z_c^{'}) \cdot e^{-8\gamma_1 \frac{\sigma\omega}{\delta}(t + \frac{n}{c} z_c^{'})}}{ch^2\left[\ln\left(\sqrt{\frac{3 \cdot y_S}{x'_{10}(t + \frac{n}{c} z_c^{'})}}\right) + \frac{8\gamma_1^2 - \gamma_2}{\gamma_1} \cdot \left(\frac{\sigma_1 \omega}{\delta}\right) \cdot (t + \frac{n}{c} z_c^{'}) + x'_{10}(t + \frac{n}{c} z_c^{'}) \cdot \frac{1 - e^{-8\gamma_1 \frac{\sigma\omega}{\delta}(t + \frac{n}{c} z_c^{'})}}{8\gamma_1}\right]}. \tag{47}$$

One can notice that, under the conditions for a "square hyperbolic secant " solution, the condition $\{N_2 = 0\}$ can be put in the form:

$$(x_1 x_1^* - y_1 y_1^*)^2 = \left(\frac{1 - N_3^2}{N_3}\right)^2 (x_1 x_1^*) \cdot (y_1 y_1^*) \tag{48}$$

If $N_3^2 \langle\langle 1$, eq.(48) becomes:

$$(x_1 x_1^*) \cdot (y_1 y_1^*) = N_3^2 (x_1 x_1^* - y_1 y_1^*)^2 \tag{49}$$

In addition, for:

$$x_1 x_1^* \rangle\rangle y_1 y_1^* \tag{50}$$

eq. (49) takes the form:

$$y_1 y_1^* \cong N_3^2 \cdot x_1 x_1^* \tag{51}$$

which corresponds to eq. (37).

If, in Eq. (49): $$x_1 x_1^* \langle\langle y_1 y_1^* , \tag{52}$$

then, one can find:

$$x_1 x_1^* \cong N_3^2 \cdot y_1 y_1^* \tag{53}$$

Eq. (51) and (53) explain the "reciprocal" character of the SBS process. This is confirmed experimentally and by numerical calculus in [12,13].

In the case $\{N_2 = 0\}$, the eqs. (36) lead to a nonlinear Schrödinger equation [4]:

$$\frac{\partial^2 N_3^{-1}}{\partial \eta^2} = -\left[4(N_1 + 3\gamma_1)^2 - 20\gamma_1^2 + 2\gamma_2\right] \cdot N_3^{-1} \tag{54}$$

where:

$$N_1(\eta) = c_1 \cdot e^{-4\gamma_1 \eta} - \frac{\gamma_2}{4\gamma_1}; \tag{55}$$

and c_1 is a constant of integration. From eqs. (28), ($N_3^2 << 1$), one obtains:

$$N_1(\eta) + \frac{\gamma_2}{4\gamma_1} = \frac{1}{4}(x_1 x_1^* - y_1 y_1^*) \tag{56}$$

One can be remark that the solution of eq. (54) is a hyperbolic secant only if:

$$4(N_1 + 3\gamma_1)^2 - 20\gamma_1^2 + 2\gamma_2 > 0 \tag{57}$$

Eqs. (56) and (57) lead to:

$$x_1 x_1^* - y_1 y_1^* \rangle \frac{\gamma_2}{\gamma_1} - 12\gamma_1 + \sqrt{80\gamma_1^2 - 8\gamma_2} \tag{58}$$

Thus, the necessary condition for a solitonic solution is given by Eq. (58) on the $\{\Gamma 1\}$ and $\{\Gamma 2\}$ characteristics and the sufficient condition is given by the Cauchy problem for $\{N3^{-1}(\eta)\}$:

$$N_3^{-1}(\eta)\big|_{\eta=\eta_0} = N_{30}^{-1}(\eta_0) \neq 0; \tag{59}$$

Eq. (58) may be written as:

$$x_1 x_1^*(\eta) - y_1 y_1^*(\eta) \rangle \frac{2\omega}{\sigma_1 \Gamma_B} - \frac{1}{2}\frac{\alpha'}{\sigma_1}\left(\delta\frac{\omega_L}{\omega}\right)\left\{1 - \sqrt{1 - \frac{1}{\delta}\cdot\frac{8\omega^2}{\Gamma_B \omega_L}}\right\} \tag{60}$$

If $\delta \approx 1; \quad 8\omega^2 \langle \Gamma_B \cdot \omega_L,$ (61)

eq. (60) takes the form :

$$x_1 x_1^*(\eta) - y_1 y_1^*(\eta) \rangle \frac{2\omega}{\sigma_1 \Gamma_B}(1 - \alpha') \tag{62}$$

Taking into account that:

$$x_1 x_1^* \approx \frac{I_L}{I_0}, y_1 y_1^* \approx \frac{I_S}{I_0}, \tag{63}$$

$$\frac{\omega}{K} = \frac{v}{\sqrt{\gamma}}; \alpha' = \frac{\alpha}{K}; \sigma_1 = g_B^e \cdot L_B' \cdot I_0; L_B = \pi \cdot \frac{v}{4\omega}\ , \tag{64}$$

with L'_B - the total section of interaction, the condition for a solitonic solution (62) becomes:

$$I_L - I_S \rangle \frac{8}{\pi} \frac{\omega\left(\omega - \alpha \cdot \frac{v}{\sqrt{\gamma}}\right) \cdot L_B}{g_B^e \cdot \Gamma_B \cdot v \cdot L'_B}, \quad I_S = \left[\frac{\pi \cdot \gamma^e}{n^2}\left(\frac{\Delta\rho}{\rho_0}\right)\right]^2 \cdot I_L \tag{65}$$

From Eq. (47) one can calculate the velocity of the compensation soliton:

$$v_s = \omega \cdot L_B \cdot \frac{g_B^L I_0 L_B - 4\omega\tau}{2\delta} \tag{66'}$$

and the soliton duration :

$$\Delta t_S = \frac{2\delta}{\omega\left(g_B^L I_0 L_B - 4\omega \cdot \tau\right)} \tag{66''}$$

For the nonlinear medium CS_2, commonly used in SBS [14-16], the interaction length is $L_B = 8.005$ cm and consequently, the soliton velocity is $v_s = 6.15\times10^9$ cm/s and its duration takes the value Δt_S=1.3 ns, for a peak intensity of 500 MW/cm^2.

3.2. Topological solitons

The conditions for the solitonic solution can start also from the eqs. (16) in the case α' = (no losses):

$$x + y \cdot z = c_1(\varphi_S), \quad y - x \cdot z = c_2(\varphi_L), \quad \frac{\partial z}{\partial \varphi_f} = -(2 \cdot A) \cdot z + \sigma_1 \cdot x \cdot y \tag{67}$$

where $c_1(\varphi_S)$ is a prime integral on the characteristic (φ_S) and $c_2(\varphi_L)$ is a prime integral on the (φ_L) characteristic. The third equation represents the equation of evolution of the field acoustic amplitude, z, on the (φ_f) characteristic.

According to the theory of the algebraic invariants which characterize the autonomous nonlinear differential equations, the two prime integrals defined in (67) are proportional to the linear combination of the algebraic invariants. The simplest case is:

$$c_1(\varphi_S) = x_0(\varphi_S)\,,\ c_2(\varphi_L) = 0 \tag{68}$$

In these conditions, the eqs. (67) become:

$$x(\varphi_L) = x_0(\varphi_S) - y(\varphi_S)\cdot z(\varphi_f),\quad y(\varphi_S) = z(\varphi_f)\cdot x(\varphi_L)$$
$$\frac{\partial z(\varphi_L)}{\partial \varphi_f} = -(2\cdot A)\,z(\varphi_f) + \sigma_1 x(\varphi_L) y(\varphi_S) \tag{69}$$

After some substitutions, the eqs. (69) take the form:

$$x = \frac{x_0}{1+z^2},\quad y = \frac{x_0\cdot z}{1+z^2},\quad \frac{\partial z}{\partial \varphi_L} = -(2A)z + \frac{\sigma_1\cdot x_0^2}{(1+z^2)^2}\cdot z \tag{70}$$

with:

$$2A = 4\omega t. \tag{71}$$

If the quality factor for the acoustic field is high, the system (70) accept the following prime integrals:

$$I_{LC}(\varphi_L) = I_{LO}\frac{\exp\left[-2\cdot\int_{\varphi_{LO}}^{\varphi_L} g_B^e\cdot L_B\cdot I_{LO}\,d\varphi_L + 8\omega\,\tau\,\varphi_L\right]}{4ch^2\left[\int_{\varphi_{LO}}^{\varphi_L} g_B^e\cdot L_B\cdot I_{LO}\,d\varphi_L - 4\omega\,\tau\,\varphi_L\right]}$$
$$I_S(\varphi_S) = \frac{I_{L_0}(\varphi_S)}{4ch^2\left[\int_{\varphi_{S_O}}^{\varphi_S} g_B^e\cdot L_B\cdot I_{L_0}\,d\varphi_L - 4\omega\,\tau\,\varphi_S\right]} \tag{72}$$
$$\frac{\pi\gamma^e}{n^2}\cdot\frac{\Delta\rho}{\rho_0} = \exp\left[\int_{\varphi_{fo}}^{\varphi_f} g_B^e\cdot L_B\cdot I_{L_0}\,d\varphi_L - 4\omega\,\tau\,\varphi_f\right]$$

The prime integrals from (72) exist if:

$$I_S = \left[\frac{\pi\cdot\gamma^e}{n^2}\cdot\frac{\Delta\rho}{\rho_0}\right]^2\cdot I_{L_C},\quad \alpha = 0,\quad \omega\cdot\tau >> 0 \tag{73}$$

Thus, the conditions (73) are the conditions for existence of the solitonic solution for the Stokes field. The prime integrals from (72) are plotted in Fig. 3. From Eq. (72), the velocity of the Stokes solitons (at the output of the nonlinear medium) can be obtained:

$$V_{soliton} = \frac{\omega_S}{K_S}\left|\frac{4\omega\tau}{g_B^e\cdot L_B\cdot I_{L_0} - 4\omega\tau}\right| \tag{74}$$

The velocity of the Stokes soliton defined in (74) is valid only in the case of the non stationary isentropic S.B.S.compression, which requires:

$$\int_{\varphi_{L_0}}^{\varphi_L} g_B^e\cdot L_B\cdot I_{L_o}\,d\varphi'\rangle 4\omega\tau\varphi_L \tag{75}$$

In the case of the isentropic expansion, defined by:

$$\int_{\varphi_{L_0}}^{\varphi_L} g_B^e\cdot L_B\cdot I_{L_o}\,d\varphi'\langle 4\omega\tau\varphi_L\,, \tag{76}$$

one can notice from (72) and (73) that solitons can also exist. The amplitude of these solitons is in the neighborhood of the spontaneous Stokes field, and their velocity is close to the phase velocity of the Stokes field.

Thus, the velocity of the topological soliton is larger than that of the compensation soliton; in CS_2, it is approx. 1.8×10^{10} cm/s, in comparison to 6.15×10^{9} cm/s for the compensation soliton. The duration of the topological soliton, in CS_2, is 1.45 ps, much smaller than that of the compensation soliton (1.3 ns). The amplitude of the topological soliton is much smaller than that of the compensation one.

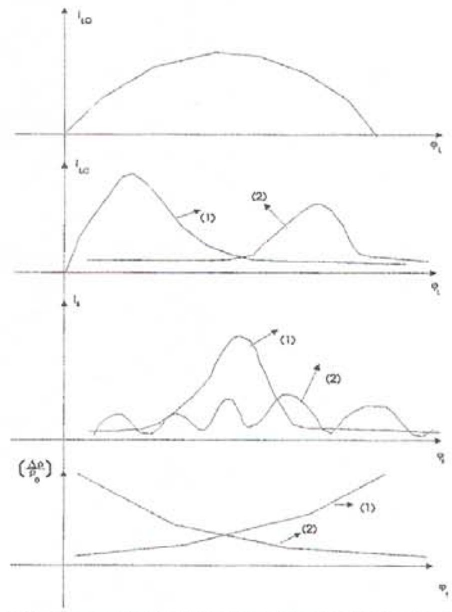

Fig. 3 Evolution of the intensities in the SBS process in the case of a Stokes soliton Solution for the two states of the acoustic field.

$$(1).\int_{\varphi_{f0}}^{\varphi_f} g_B^e \cdot L_B \cdot I_{L_0} d\varphi' \rangle 4\omega\tau\varphi_f, \quad (2).\int_{\varphi_{f0}}^{\varphi_f} g_B^e \cdot L_B \cdot I_{L_0} d\varphi' \langle 4\omega\tau\varphi_f$$

Let write the eqs. (16) in the form:

$$\frac{\partial}{\partial\varphi_L}\left(x+yz\right)=-\frac{\alpha'}{2}x, \quad \frac{\partial}{\partial\varphi_S}\left(y-xz\right)=-\frac{\alpha'}{2}y, \quad \frac{\partial z}{\partial\varphi_f}=-\left(2A\right)\cdot z+\sigma_1\cdot x\cdot y \tag{77}$$

If the pump optic pulse envelope, $C_1\,\{\varphi f\}$, satisfy the boundary condition:

$$\left.\frac{\partial C_1}{\partial\varphi_f}\right|_{z'=0} \neq 0 \tag{78}$$

where: ($x \leftrightarrow X$, $y \leftrightarrow Y$, $z \leftrightarrow Z$)

$$X+Y\cdot Z=C_1, \quad Y-X\cdot Z=C_2 \tag{79}$$

the eqs. (16) take the form:

$$\frac{\partial C_1}{\partial\xi}=-\frac{C_1-C_2\cdot Z}{1+Z^2}, \quad \frac{\partial C_2}{\partial\xi}=-\frac{C_2+C_1\cdot Z}{1+Z^2}, \quad \frac{\partial Z}{\partial\xi}=-\gamma_1\cdot Z+\gamma_2\cdot\frac{\left(C_1-C_2Z\right)\cdot\left(C_2+C_1Z\right)}{\left(1+Z^2\right)^2} \tag{80}$$

with:

$$\gamma_0=+\frac{\alpha'}{2}\cdot\frac{\omega_L}{\omega}\cdot\delta, \quad \xi=\gamma_0\cdot\varphi_f, \quad \gamma_1=\frac{2A}{\gamma_0}, \quad \gamma_2=\frac{\sigma_1}{\gamma_0} \tag{81}$$

One can mention that in contrast with the generation of the classical soliton, when the nonlinearity compensate dispersion, in this case, we have imposed a restrictive condition on the phase space topology in order to obtain the soliton, namely:

$$C_2\left(\varphi_f\right)=0 \tag{82}$$

that can be written in the form:

$$Y=XZ \tag{83}$$

It can be noticed that eq. (83) is identical with the first equation from (73) and this is the restrictive condition for the existence of the topological soliton. This condition has the physical meaning of the interference of all three fields in the SBS interaction.

In this case, the eqs. (80) become:

$$\frac{\partial C_1^2(\eta)}{\partial \eta} = -\frac{C_1^2(\eta)}{1+Z^2(\eta)}, \quad \frac{\partial Z^2(\eta)}{\partial \eta} = -\gamma_1 \cdot Z^2(\eta) + \gamma_2 \frac{C_1^2(\eta) Z^2(\eta)}{\left(1+Z^2(\eta)\right)^2} \tag{84}$$

where:

$$\eta = 2 \cdot \xi = \frac{\alpha \cdot C \cdot \delta}{2 \cdot n \cdot \omega} \varphi_f \tag{85}$$

The eqs. (83, 84) have a prime integral in the form:

$$X^2(\eta) = \frac{C_1^2(\eta)}{\left(1+Z^2(\eta)\right)^2}, \quad Y^2(\eta) = \frac{C_1^2(\eta) \cdot Z^2(\eta)}{\left(1+Z^2(\eta)\right)^2}$$
$$Z^2(\eta) = Z_0 \cdot \exp\left[-\gamma_1 \cdot \eta + 4 \cdot \gamma_2 \cdot \int \left(\frac{dC_1(\eta)}{d\eta}\right)^2 d\eta\right] \tag{86}$$

with the initial condition:

$$Z(\eta)\Big|_{\eta=\eta_0} = Z_0, C_1(\eta)\Big|_{\eta=\eta_0(Z'=0)} = C_{1_0} \tag{87}$$

Eliminating Z from (84), one obtains the equation, which defines the envelope of the optical pump pulse:

$$C_1(\eta) \cdot \frac{d^2 C_1(\eta)}{d\eta^2} = \gamma_1 \cdot C_1(\eta) \cdot \frac{dC_1(\eta)}{d\eta} + (1+2\gamma_1) \cdot \left(\frac{dC_1}{d\eta}\right)^2 -$$
$$-4\gamma_2 C_1(\eta) \cdot \left(\frac{dC_1(\eta)}{d\eta}\right)^3 - 8\gamma_2 \cdot \left(\frac{dC_1}{d\eta}\right)^4 \tag{88}$$

From the eqs. (86), one obtains also the Stokes field amplitude:

$$Y(\eta) = \frac{C_1(\eta)}{ch\left[\ln(Z_0) - \gamma_1 \cdot \eta + 4\gamma_2 \int \left(\frac{dC_1}{d\eta}\right)^2 d\eta\right]} \tag{89}$$

A particular solution of eq. (88) is:

$$C_1(\eta) = C_{1_0} \cdot e^{-\frac{|\eta|}{Z}} \tag{90}$$

The Stokes field take the form of the topological soliton:

$$Y(\eta) = \frac{C_{1_0} \cdot e^{-\frac{|\eta|}{2}}}{ch\left[ln Z_0 - \frac{\gamma_1}{2}\eta - \frac{\gamma_2}{2} C_{1_0}^2 \cdot e^{-|\eta|}\right]} \tag{91}$$

or in non-normalized quantities:

$$E_S(\eta) = E_{10} \cdot \frac{e^{-\frac{|\eta|}{2}}}{ch\left[ln(Z_0) - \frac{\gamma_1}{2}\eta - \frac{\gamma_2}{2} E_{10}^2 \cdot e^{-|\eta|}\right]} \tag{92}$$

We notice that, if one considers that equation (88) gives the spectral data of "the potential" in the nonlinear - Schrödinger equation (in terms of the inverse problem in scattering theory), these spectral data are obtained for:

$$S = \left[R(K) = 0; \quad -\infty < K < +\infty, \quad \rho = C_{1_0}, \quad P = \frac{1}{2} \right] \tag{93}$$

The G. L. M. equation [3] becomes:

$$K(\eta, \eta') + M(\eta + \eta') + \int_{\eta}^{+\infty} d\alpha \cdot K(\eta, \alpha) \cdot M(\alpha + \eta') = 0 \tag{94}$$

From (94), one obtains:

$$K(\eta, \eta') = -\frac{1}{2} \cdot \frac{e^{\frac{1}{2}\left(ln\, C_{1_0} - \eta'\right)}}{ch \frac{1}{2}\left(ln\, C_{1_0} - \eta\right)} \tag{95}$$

if one defines:

$$W(\eta) = 2 \cdot K(\eta, \quad \eta' = \eta + 0), \tag{96}$$

the potential of interaction in the nonlinear Schrödinger equation is found as:

$$U(\eta) = -\frac{d}{d\eta} W(\eta) = -\frac{1}{2} \frac{1}{ch^2 \left[\frac{1}{2}\left(\eta - \ln C_{1_0}\right) \right]} \quad . \tag{97}$$

Thus, the "topological soliton" is a soliton of the nonlinear Schrödinger equation with the potential defined in (97).

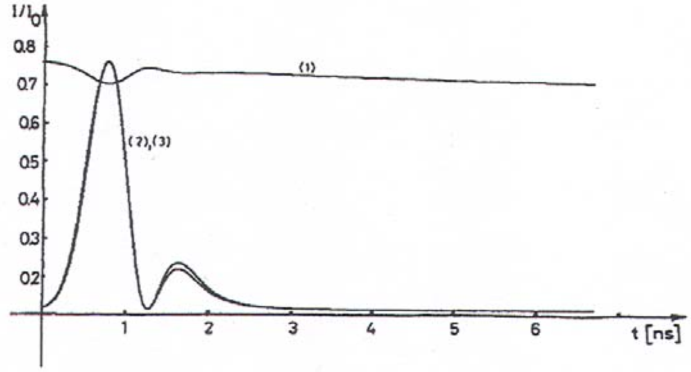

Fig. 4 Time evolution of: Normalized pump intensity (1); Stokes intensity (compensation soliton) (2); Acoustic field intensity (3).

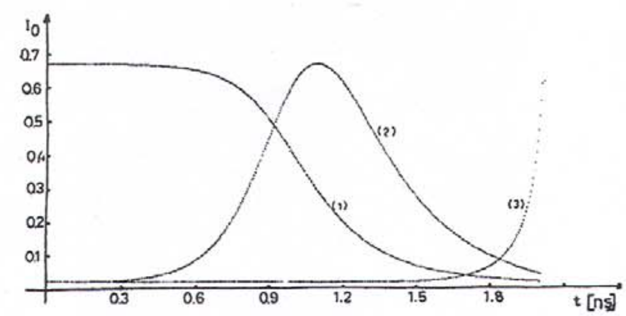

Fig. 5 Time evolution of: Normalized pump intensity (1); Stokes intensity (topological soliton) (2); Acoustic field intensity (3).

4. Conclusions

In this work the necessary and sufficient conditions for the temporal soliton generation in SBS were derived. We demonstrated that, in this nonlinear process, two types of solitons can occur: the usual compensation soliton (arising from the compensation of the dispersion by the nonlinearity (see Fig. 4) and the topological solitons (defined by some conditions imposed to the nonlinear equations, in the phase space- see Fig. 5).

The topological soliton appears on the increasing and decreasing slope of the isentropic compression of the nonlinear medium. The acoustic soliton generates the compensation soliton. The acoustic soliton and the optical compensation soliton result from the interaction of the singular oscillations of the acoustic field and of the interacting topological solitons.

The generation of these solitons is related to the mechanisms and to threshold conditions. The amplitude, velocity and duration of the compensation and topological solitons are different. Both types of solitons could be observed simultaneously in a space-time window, in the limits imposed by their parameters.

Appendix

The direction derivatives for the D'Alembert solution of the wave equation

In this section we will build the S.B.S. equations [16]. Regarding the S.B.S. process as a scattering process of the optical field on an induced "potential", we will use a "technique" paper to the "inverse method" in the scattering theory, namely the derivation on the characteristic directions of the general D'Alembert solution of the wave equation.

We consider the wave equation, which describes the S.B.S. process (Gauss system of measurement):

$$\left(\frac{n}{c}\right)^2 \frac{\partial^2 E(t,z)}{\partial t^2} - \frac{\partial^2 E(t,z)}{\partial z'^2} = -\frac{n}{c}\cdot\frac{\partial}{\partial t}\left[\alpha \cdot E(t,z') + \frac{4\pi}{nc}\frac{\partial P^{NL}(t,z')}{\partial t}\right] \tag{A.1}$$

We define the restricted Cauchy problem (initial conditions) in the form:

$$E(t,z')\big|_{t=0} = \varphi(z') = 0; \quad \left.\frac{\partial E(t,z')}{\partial t}\right|_{t=0} = \psi(z') = 0 \tag{A.2}$$

In the case of regular distributions, the general D'Alembert solution of the equation (A.1) with initial conditions (A.2) is written in the form:

$$E(t,z') = \frac{c}{2n}\int_0^t d\tau \int_{z'-\frac{c}{n}(t-\tau)}^{z'+\frac{c}{n}(t-\tau)} F(\xi,\tau)d\xi \tag{A.3}$$

where:

$$F(z',t) = -\frac{n}{c}\frac{\partial}{\partial t}\left[\alpha \cdot E(z',t) + \frac{4\pi}{n \cdot c}\frac{\partial P^{NL}(z',t)}{\partial t}\right] \tag{A.4}$$

is the inhomonogeneous term of the wave equation which contains, through the nonlinear polarization (P^{NL}), the nonlinear S.B.S. terms.

Using the following transformations of coordinates:

$$\xi_L = \frac{c}{n} \cdot t + z'; \quad \xi_S = \frac{c}{n} \cdot t - z' \tag{A.5}$$

the integral equation (A.3) takes the form:

$$E(\xi_L, \xi_S) = \frac{c}{2 \cdot n} \int_0^{\frac{n}{2c}(\xi_L + \xi_S)} d\tau \int_{-\xi_S + \frac{c}{n}\tau}^{\xi_L - \frac{c}{n}\tau} F(\xi, \tau) d\xi \tag{A.6}$$

The partial derivation of the scalar field $E(\xi_L, \xi_S)$ are the derivatives on the characteristic directions of the wave equations, directions materialised in the system of coordinate transformation (A.5).

So, we may write:

$$\frac{\partial E(\xi_L, \xi_S)}{\partial \xi_L} = \frac{c}{2n} \int_0^{\frac{n}{2c}(\xi_L + \xi_S)} F\left[\left(\xi_L - \frac{c}{n}\tau\right), \tau\right] d\tau$$

$$\frac{\partial E(\xi_L, \xi_S)}{\partial \xi_S} = \frac{c}{2n} \int_0^{\frac{n}{2c}(\xi_L + \xi_S)} F\left[\left(-\xi_S + \frac{c}{n}\tau\right), \tau\right] d\tau \tag{A.7}$$

We make explicit $F(z,t)$ in the form:

$$F(z',t) = -\frac{n}{c} \cdot \frac{\partial}{\partial t} g(z',t) \tag{A.8}$$

where:

$$g(z',t) = \alpha \cdot E(z',t) + \frac{4\pi}{nc} \cdot \frac{\partial P^{NL}(z',t)}{\partial t} \tag{A.9}$$

Using relations (A.8) and (A.9), we make explicit the prime integral from (A.7).
Let be the coordinate transformation (transformation which defines the "movement" on "direction" ξ_L):

$$z' = \xi_L - \frac{c}{n}\tau; \quad t = \tau \tag{A.10}$$

From (A.10) it results:

$$\frac{\partial}{\partial t} = \frac{\partial \tau}{\partial t} \cdot \frac{\partial}{\partial \tau} + \frac{\partial \xi_L}{\partial t} \cdot \frac{\partial}{\partial \xi_L} = \frac{\partial}{\partial \tau} \tag{A.11}$$

$$F\left[\left(\xi_L - \frac{c}{n}\tau\right), \tau\right] = -\frac{n}{c}\frac{\partial}{\partial \tau} g\left[\left(\xi_L - \frac{c}{n}\tau\right), \tau\right]_{\xi_S = const} \tag{A.12}$$

In this way, the first integral equation from (A.7) takes the form:

$$\frac{\partial E}{\partial \xi_L} = -\frac{1}{2}\left[g\left[\frac{1}{2}(\xi_L - \xi_S), \frac{n}{2c}(\xi_L + \xi_S)\right] - g[\xi_L, 0]\right] \tag{A.13}$$

From condition (A.2) it results:

$$g[\xi_L, 0] = 0 \tag{A.14}$$

and then, equation (A.13) takes the form:

$$\frac{\partial E}{\partial \xi_L} = -\frac{1}{2}\left\{g\left[\frac{1}{2}(\xi_L - \xi_S), \frac{n}{2c}(\xi_L + \xi_S)\right]\right\}_{\xi_S = const.} \tag{A.15}$$

Equation (A.15) defines the "movement" on the (ξ_L) characteristic, where (ξ_S) is a constant (parameter). In these conditions, equation (A.15) takes the form:

$$\frac{\partial E}{\partial \xi_L} = -\frac{1}{2} g\left(\frac{\xi_L}{2}, \frac{n}{c}\frac{\xi_L}{2}\right) \tag{A.16}$$

By expliciting *(g)*, we obtain for the (E_L) component of the field, equation:

$$\frac{\partial E_L}{\partial \xi_L} = -\frac{\alpha}{4} E_L - \frac{\pi}{n^2}\frac{\partial}{\partial \xi_L} P^{NL}(E_L, E_S) \tag{A.17}$$

We analyze now the second integral equation from (A.7). In this case, we make the coordinate transformation:

$$z' = -\xi_S + \frac{c}{n}\tau; \quad t = \tau \tag{A.18}$$

This transformation defines the movement along the direction (ξ_S); in this case, ξ_L is a parameter. In this conditions, we make explicit F from (A.7) and obtain:

$$F\left[\left(-\xi_S + \frac{c}{n}\tau\right), \tau\right] = -\frac{n}{c}\frac{\partial}{\partial \tau} g\left[\left(-\xi_S + \frac{c}{n}\tau\right), \tau\right]_{\xi_L = const.} \tag{A.19}$$

Even in these conditions, the second integral equation takes the form:

$$\frac{\partial E}{\partial \xi_S} = -\frac{1}{2}\left\{ g\left[\frac{1}{2}(\xi_L - \xi_S), \frac{n}{2c}(\xi_L + \xi_S)\right] - g[0, \xi_S]\right\} \tag{A.20}$$

Similar with (A.14), from condition (A.2) it results:

$$g[0, \xi_S] = 0 \tag{A.21}$$

and then, equation (A.20) takes the form:

$$\frac{\partial E}{\partial \xi_S} = -\frac{1}{2}\left\{ g\left[\frac{1}{2}(\xi_L - \xi_S), \frac{n}{2c}(\xi_L + \xi_S)\right]\right\}_{\xi_L = const.} \tag{A.22}$$

Equation (A.22) defines the "movement" on the (ξ_S) characteristic, where (ξ_L) is a constant (parameter). In these conditions, equation (A.22) takes the form:

$$\frac{\partial E}{\partial \xi_S} = -\frac{1}{2} g\left(-\frac{\xi_S}{2}, \frac{n}{c}\frac{\xi_S}{2}\right) \tag{A.23}$$

By expliciting function *(g)* we obtain for the (E_S) component of the field, equation:

$$\frac{\partial E_S}{\partial \xi_S} = -\frac{\alpha}{4} E_S - \frac{\pi}{n^2}\frac{\partial}{\partial \xi_S} P^{NL}(E_L, E_S) \tag{A.24}$$

So, the equations obtained (A.17) and (A.24) describe the behavior of the optical field in the S.B.S. process. The same algorithm may be applied also to the wave equation of the acoustical field, induced in the S.B.S. process.

In conclusion, we mention that this method may be described so: in the wave equation (A.1) we make the coordinate transformation (A.5); the wave equation becomes:

$$\frac{\partial^2 E(\xi_L, \xi_S)}{\partial \xi_L \partial \xi_S} = \alpha_i \frac{\partial E}{\partial \xi_i} + \beta_{ij}\frac{\partial^2 P^{NL}}{\partial \xi_i \partial \xi_j}$$
$$i = L, S; \quad j = L, S \tag{A.25}$$

By integrating this equation on (ξ_L) or on (ξ_S) we obtain equations (A.17) and (A.24). Why have we used this method?

- in the given case of the S.B.S. process, the equations on the characteristic curves form a system of equations cvasilinear, coupled, situation which allows a continuation of the analytical investigation of the S.B.S. process.

In the case of the polistochastic S.B.S. process the equations on the characteristic curves allow an average of F - P - K type

- the equations on the characteristic lines allow the simple use of the initial conditions.

We mention, however, that the Maxwell system of equations which describes the behaviour of the optical field and the Euler system of equations, which describes the behaviour of the acoustical field are systems of equations of first order.

We will describe these equations using the variables on the characteristic curves.

After making these systems of equations compact we will obtain a result identical with those obtained trough the derivation of the general D'Alembert solution.

Using relations (Cummins and Gamman/966) for the variation of the dielectric permitivity {ε} with the conditions of the (B) hypothesis we will have:

$$\left(\frac{\partial \varepsilon}{\partial P}\right)_S = \rho_0 \cdot K'_S \left(\frac{\partial \varepsilon}{\partial \rho}\right)_T \tag{A.26}$$

where: K'_S - the adiabatic coefficient of compressibility;

We define the coefficient of electrostictive coupling $\{\gamma^e\}$, described in [19], in the form:

$$\gamma^e = \rho_0 \left(\frac{\partial \varepsilon}{\partial \rho}\right)_T \tag{A.27}$$

For expliciting the nonlinear polarization (induced in the nonlinear medium), we use the relation from [20] in the form:

$$P_{NL}(E_{L,S}) = \Delta\varepsilon(\rho) \cdot (E_L + E_S) \tag{A.28}$$

where:

$$\Delta\varepsilon(\rho) = \left(\frac{\partial \varepsilon}{\partial \rho}\right)_T \Delta\rho; \tag{A.29}$$

From relations (A.27) and (A.29) it results:

$$\Delta\varepsilon(\rho) = \gamma^e \frac{\Delta\rho}{\rho_0}; \tag{A.30}$$

The Euler equations which describe the evolution of the quantity $\{\Delta\rho / \rho_0\}$, have the form [21] and [22]:

$$\begin{gathered} \frac{\partial}{\partial t}\left(\frac{\Delta\rho}{\rho_0}\right) + \frac{\partial V}{\partial z'} = 0; V = \frac{dz'}{dt} \\ \frac{v^2}{\gamma} \cdot \frac{\partial}{\partial z'}\left(\frac{\Delta\rho}{\rho_0}\right) + \frac{\partial V}{\partial t} = \frac{\partial}{\partial z'}\left[\left(\frac{\eta_B}{\rho_0}\right)\frac{\partial V}{\partial z'} + \frac{\gamma^e}{8\pi\rho_0}(E_L + E_S)^2\right] \end{gathered} \tag{A.31}$$

The hydrodynamic equations (A.31) type were written in hypothesis (B), where v is the sound velocity in the nonlinear medium and γ is the adiabatic coefficient $\{\gamma = c_p / c_v\}$.

We shall use the following relations:

$$\frac{v^2}{\gamma} = \frac{\omega^2}{K^2}; \quad \frac{\eta_B}{\rho_0} = \frac{\Gamma_B}{K^2}; \tag{A.32}$$

where $\Gamma_B = \dfrac{1}{\tau}$ has the significance of a spectral width for the acoustic field and equations (A.31) takes the form:

$$\begin{gathered} \frac{\partial}{\partial t}\left(\frac{\Delta\rho}{\rho_0}\right) + \frac{\partial V}{\partial z'} = 0; V = \frac{dz'}{dt} \\ \omega^2 \cdot \frac{\partial}{\partial z'}\left(\frac{\Delta\rho}{\rho_0}\right) + K^2\frac{\partial V}{\partial t} = \frac{\partial}{\partial z'}\left[(\Gamma_B)\frac{\partial V}{\partial z'} + \frac{\gamma^e K^2}{8\pi\rho_0}(E_L + E_S)^2\right] \end{gathered} \tag{A.33}$$

The characteristic lines equations for the acoustical field has the form:

$$\xi_{1f} = \frac{\omega}{K}t + z'; \quad \xi_{2f} = \frac{\omega}{K}t - z' \tag{A.34}$$

The operator equations associated to the characteristic lines equations (A.34) are:

$$\frac{K}{\omega}\cdot\frac{\partial}{\partial t}=\frac{\partial}{\partial\xi_{1f}}+\frac{\partial}{\partial\xi_{2f}};\quad \frac{\partial}{\partial z'}=\frac{\partial}{\partial\xi_{1f}}-\frac{\partial}{\partial\xi_{2f}}$$
$$2\cdot\frac{\partial}{\partial\xi_{1f}}=\frac{K}{\omega}\cdot\frac{\partial}{\partial t}+\frac{\partial}{\partial z'};\quad 2\cdot\frac{\partial}{\partial\xi_{2f}}=\frac{K}{\omega}\cdot\frac{\partial}{\partial t}-\frac{\partial}{\partial z'} \qquad (A.35)$$

With the new variables $\{\xi_{1f}, \xi_{2f}\}$ equations (A.33) have the form $\{\Delta\rho/\rho_0=\rho'\}$:

$$\frac{\partial}{\partial\xi_{1f}}(\rho'+\frac{K}{\omega}V)+\frac{\partial}{\partial\xi_{2f}}(\rho'-\frac{K}{\omega}V)=0$$
$$\frac{\partial}{\partial\xi_{1f}}(\rho'+\frac{K}{\omega}V)-\frac{\partial}{\partial\xi_{2f}}(\rho'-\frac{K}{\omega}V)= \qquad (A.36)$$
$$=\left(\frac{\partial}{\partial\xi_{1f}}-\frac{\partial}{\partial\xi_{2f}}\right)\left[-\frac{\Gamma_B}{\omega K}\left(\frac{\partial}{\partial\xi_{1f}}+\frac{\partial}{\partial\xi_{2f}}\right)\rho'+\frac{\gamma^e\cdot K^2}{8\pi\rho_0\omega^2}(E_L+E_S)^2\right]$$

Adding the two expressions from (A.36) results:

$$2\cdot\frac{\partial}{\partial\xi_{1f}}(\rho'+\frac{K}{\omega}V)=\left(\frac{\partial}{\partial\xi_{1f}}-\frac{\partial}{\partial\xi_{2f}}\right)\left[-\frac{\Gamma_B}{\omega K}\left(\frac{\partial}{\partial\xi_{1f}}+\frac{\partial}{\partial\xi_{2f}}\right)\rho'+\frac{\gamma^e\cdot K^2}{8\pi\rho_0\omega^2}(E_L+E_S)^2\right]; \qquad (A.37)$$

for which:

$$\frac{\partial}{\partial\xi_{2f}}(\rho'-\frac{K}{\omega}V)=0;\qquad (=f(\xi_{1f})) \qquad (A.38)$$

We substract the two expressions from (A.36) and it results`:

$$2\cdot\frac{\partial}{\partial\xi_{2f}}(\rho'-\frac{K}{\omega}V)=-\left(\frac{\partial}{\partial\xi_{1f}}-\frac{\partial}{\partial\xi_{2f}}\right)\left[-\frac{\Gamma_B}{\omega K}\left(\frac{\partial}{\partial\xi_{1f}}+\frac{\partial}{\partial\xi_{2f}}\right)\rho'+\frac{\gamma^e\cdot K^2}{8\pi\rho_0\omega^2}(E_L+E_S)^2\right] \qquad (A.39)$$

for which:

$$\frac{\partial}{\partial\xi_{1f}}(\rho'+\frac{K}{\omega}V)=0; \qquad (A.40)$$

The equations of evolution on the two characteristic lines $\{\xi_{1f}, \xi_{2f}\}$ of the quantity $\{\rho'=\Delta\rho/\rho_0\}$ have the form:

$$4\cdot\frac{\partial\rho'}{\partial\xi_{1f}}=\frac{\partial}{\partial\xi_{1f}}\left[-\frac{\Gamma_B}{\omega K}\cdot\frac{\partial\rho'}{\partial\xi_{1f}}+\frac{\gamma^e\cdot K^2}{8\pi\rho_0\omega^2}(E_L+E_S)^2\right]$$
$$4\cdot\frac{\partial\rho'}{\partial\xi_{2f}}=\frac{\partial}{\partial\xi_{2f}}\left[-\frac{\Gamma_B}{\omega K}\cdot\frac{\partial\rho'}{\partial\xi_{2f}}+\frac{\gamma^e\cdot K^2}{8\pi\rho_0\omega^2}(E_L+E_S)^2\right] \qquad (A.41)$$

One can notice that the two equations have the same form and we shall use the prime integral on the characteristic line $\{\xi_{1f}\}$ forms the point of view of the Stokes line selection, namely:

$$\xi_L=\frac{c}{n}t+z'=\frac{1}{K_L}(\omega_L t+K_L z')$$
$$\xi_S=\frac{c}{n}t-z'=\frac{1}{K_S}(\omega_S t-K_S z') \qquad (A.42)$$
$$\xi_{1f}=\frac{1}{K}(\omega t+Kz')$$

In conclusion, the equation of evolution of the S.B.S. process, function of the characteristic variables, defined in (A.42), has the form:

$$\begin{aligned}
\frac{\partial E_L}{\partial \xi_L} &= -\frac{\alpha}{4} E_L - \frac{\pi \cdot \gamma^e}{n^2} \cdot \frac{\partial}{\partial \xi_L}(E_S \cdot \rho') \\
\frac{\partial E_S}{\partial \xi_S} &= -\frac{\alpha}{4} E_S + \frac{\pi \cdot \gamma^e}{n^2} \cdot \frac{\partial}{\partial \xi_S}(E_L \cdot \rho') \\
\frac{\partial \rho'}{\partial \xi_{1f}} + \left(\frac{4k\omega}{\Gamma_B}\right)\rho' &= \frac{\gamma^e \cdot k^3}{8\pi\rho_0\omega\Gamma_B}\left(E_L + E_S\right)^2
\end{aligned} \qquad (A.43)$$

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