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MODELLING OF THE MAXIMUM REFRACTIVE INDEX DIFFERENCE PROFILE OF OPTICAL WAVEGUIDES OBTAINED BY DOUBLE ION EXCHANGE IN GLASS

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In this paper we present the modelling of the maximum refractive index difference profile of the optical waveguides obtained in glass by double ion exchange (Ag^{+}) using Gaussian and erfc functions. The dependences of the maximum refractive index difference, the center of the Gaussian and erfc functions in depth and width, respectively and the corresponding variances on the technological parameters: the width of the mask (the window), the first and the second in-diffusion time of the ions were evaluated. The obtained results can be used in the design of the optical integrated devices.

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1. Introduction

In the last years many theoretical and experimental papers using several methods treated the determination of the refractive index profile because its knowledge plays an important role in the characterization of the optical waveguides (i.e. bandwidth, spot size, single-mode propagation conditions, etc.) [1-5]. The direct profile measurement is usually destructive and very difficult to perform due to the narow guiding area and the low refractive index difference.

The improvement of the optical waveguides performances can be made by acting on the in-diffusion times and the width of the mask (the window). One of the most important characteristics of the optical waveguides is the maximum refractive index difference. Based on some experimental results concerning the measurement of the maximum refractive index difference of optical waveguides obtained in glass by double ion exchange (Ag^{+}) in this paper we report a new method for the evaluation of the maximum refractive index difference profile, the center of the Gaussian and erfc functions in depth and width, respectively and the corresponding variances on the the technological parameters: the width of the window, the first and the second in-diffusion times for ions using Gaussian and erfc functions.

The paper is organised as follows. In Section 2 are presented some theoretical considerations on the use of the Gaussian and erfc functions in the modelling of the maximum refractive difference profile of the optical waveguides obtained in glass by double ion exchange. Section 3 deals with the discussion of the obtained results in the numerical simulation and Section 4 is dedicated to the conclusions of this paper.

2. Theoretical considerations

The double ion exchange Ag^{+} - Na^{+} in glass, by diffusion is used for obtaining high-index region and it is performed in two consecutive steps, during wich the high-index region is formed by exposing the masked waveguide to $AgNO_3$ solution, in the first step, while the low-index region is formed during the second step by immersing the waveguides in a $NaNO_3$ bath, using an electric field.

Taking into account the model presented in papers [4, 5] the measured refractive index difference profile in the depth (y) and the width (x) of several waveguides obtained by double ion exchange (Ag^+) in glass for in-diffusion times t_1 = 0.75 min, t_2 = 0.75 min., 1.5 min., 2.5 min., 3.5 min. and for windows f = 0.6 μm, 1.2 μm, 2.4 μm, 4.8 μm, 10 μm, 20 μm, (represented schematically in Fig. 1), which assures the best fit with the experimental data, can be written in the form:

$$\Delta n = \Delta n_{\max} f(y) g(x) \tag{1}$$

where

$$f(y) = \exp\left(-\frac{(y - y_0)^2}{\sigma_y^2}\right) \tag{2}$$

and

$$g(x) = \exp\left(-\frac{x^2}{\sigma_x^2}\right). \tag{3}$$

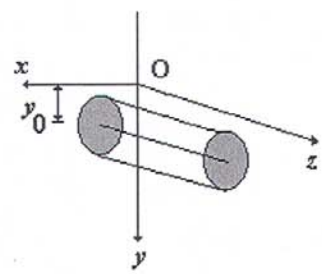

Fig. 1. Schematic representation of the waveguide structure obtained by double ion exchange.

In the Eqs. (1)-(3) $\Delta n_{\max}$ represents the maximum refractive index difference, y_0 defines the center of the Gaussian function in depth and σ_y, σ_x are the corresponding variances of the Gaussian functions (Eqs. (2)-(3)) in depth (Fig. 2) and width, respectively. The values of y_0, $\Delta n_{\max}$, σ_y, and σ_x depend on the technological parameters: the first and the second in-diffusion time of the ions, t_1, t_2, respectively and the window, f, according to the folowing equations:

$$\Delta n_{\max} = f_1(t_1, t_2, f) \tag{4}$$

$$y_0 = f_2(t_1, t_2, f) \tag{5}$$

$$\sigma_y = f_3(t_1, t_2, f) \tag{6}$$

$$\sigma_x = f_4(t_1, t_2, f). \tag{7}$$

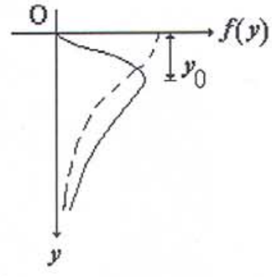

Fig. 2. The refractive index profile in depth (y) after the in-diffusion of the first ion in the time t_1 (dashed line) and after the diffusion of the second ion in the time t_2 (solid line).

Based on the theoretical model presented in paper [1] the depth efficiency, η_y, which characterizes the superposition between the measured and calculated refractive index difference is defined by the formula (similar to the overlap factor):

$$\eta_y = \frac{\int \frac{\Delta n_{\text{measured}}(0,y)}{\Delta n_{\max}} f(y)\mathrm{d}y}{\left(\int\left(\frac{\Delta n_{\text{measured}}(0,y)}{\Delta n_{\max}}\right)^2 \mathrm{d}y \int f^2(y)\mathrm{d}y\right)^{\frac{1}{2}}}. \quad (8)$$

In a similar way has been defined the width efficiency, η_x, which exihibits the same behaviour as η_y.

In Eq. (8) the integration limits are determined by the dimensions of the waveguides.

In the numerical evaluation of the above mentioned parameters we tested several functions and we found that for the description of the waveguides having the dimensions of few microns ("small" windows) the Gaussian functions (2), (3) are suitable for modelling, the depth and width efficiencies being unity. Some results are summarized in Table 1.

Table 1. Numerical evaluation of the optical waveguide parameters.

f (μm)	t_1 (min.)	t_2 (min.)	y_0 (μm)	$\Delta n_{\max}$ $\times 10^{-4}$	σ_y (μm)	σ_x (μm)	$\eta(y)$
0.6	0.75	0.75	5.1	43	2.661	3.1695	1
0.6	0.75	1.5	5.1	43	2.661	3.1695	1
0.6	0.75	2.5	7.2	27.17	3.4237	3.805	1
0.6	0.75	3.5	9.3	19.81	4.0593	4.4407	1
1.2	0.75	0.75	3.9	95.6	2.0254	2.5339	1
1.2	0.75	1.5	5.7	55.52	2.7881	3.1695	1

As can be seen from Figs. 3 a, b the same Gaussian functions (2), (3) are not suitable for the modelling of the waveguides having the width larger than 10 μm ("large" windows), both in depth and width, the errors being of about 10 %. In this last case for modelling in depth and width we used other Gaussian and erfc functions, respectively, defined by the relations:

$$f(y) = \exp\left(-\frac{(y-y_0)^2}{\left(\frac{\sigma_{y,\text{left}}+\sigma_{y,\text{right}}}{2}\,\text{sign}(y-y_0)+\frac{\sigma_{y,\text{left}}-\sigma_{y,\text{right}}}{2}\right)^2}\right) \quad (9)$$

$$g(x) = \frac{\frac{1}{2}\left[\text{erfc}\left[\frac{(x-x_0)}{\sigma_x}\right]+\text{erfc}\left[\frac{(-x-x_0)}{\sigma_x}\right]\right]-1}{\max\left\{\frac{1}{2}\left[\text{erfc}\left[\frac{(x-x_0)}{\sigma_x}\right]+\text{erfc}\left[\frac{(-x-x_0)}{\sigma_x}\right]\right]-1\right\}} \quad (10)$$

where $\sigma_{y,\text{left}}$ and $\sigma_{y,\text{right}}$ are the depth variances in the left and right side, with respect to maximum, respectively of the Gaussian function.

In order to model the depth and width maximum refractive index difference profile for "large" windows we have tested several functions [6]. We found that the Gaussian and erfc defined by Eqs. (9) and (10) assure the best fit with the experimental data in order to minimize the errors and also they smooth the refractive index difference profiles in their maximum points.

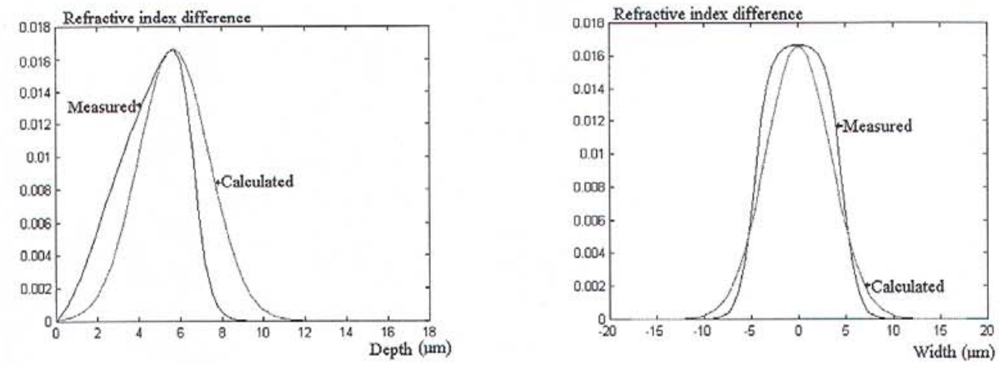

Fig. 3 a, b. The depth (a) and width (b) dependence of the refractive index difference for the "large" waveguide (f= 10 μm, $t_1 = t_2$ = 0.75 min.) using for modeling one Gaussian function.

For the waveguides having the width 10 μm and for the diffusion time t_1 = 0.75 min. some results are summarized in Table 2. The maximum and minimum values of the errors with respect to the measurements are defined as: $\max[\Delta n_{\text{measured}} - \Delta n_{\text{calculated}}]$ and $\min[\Delta n_{\text{measured}} - \Delta n_{\text{calculated}}]$. As can be seen from Figs. 4 a, b both in depth and width, respectively the Gaussian and erfc functions defined by the relations (9), (10) give the best fit with the experimental data and the results are improved.

Table 2. Numerical evaluation of the waveguide parameters for 10 μm width and 0.75 min. diffusion time

t_2 (min.)	$\sigma_{y,\text{left}}$ (μm)	$\sigma_{y,\text{right}}$ (μm)	σ_x (μm)	y_0 (μm)	x_0 (μm)	$\Delta n_{\max}$ $\times 10^{-4}$	Max. error $\times 10^{-2}$	Min. error $\times 10^{-2}$
0.75	3.6579	1.2692	1.8963	5.7	4.6268	166.09	0.4983	-0.243
1.5	4.3889	1.7097	2.5339	8.1	4.6984	119.46	0.2977	-0.1233
2.5	5.2143	2.7826	3.1695	10.5	5.0963	91.12	0.1507	-0.1023
3.5	6.0841	3.5	3.5508	12.9	5.3337	74.602	0.103	-0.109

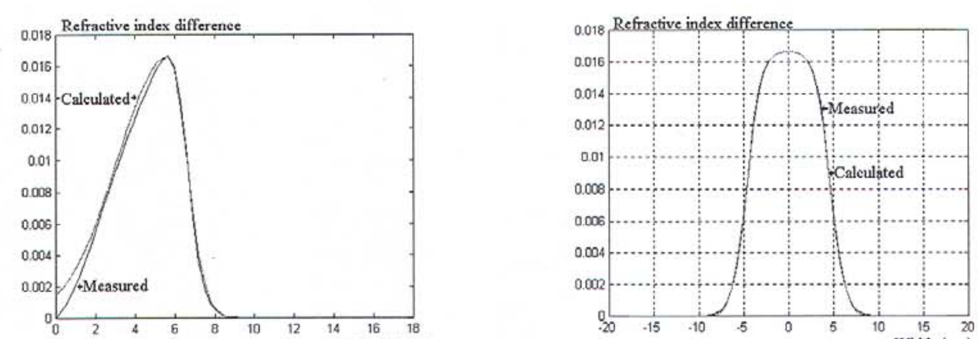

Fig. 4 a, b. The depth (a) and width (b) dependence of the refractive index difference for the "large" waveguide (f= 10 μm, $t_1 = t_2$ = 0.75 min.) using for modeling the Gaussian and erfc functions, respectively, defined by the relations (9), (10).

3. Discussion of the simulation results

Using the Gaussian and erfc functions, defined by the relations (9) and (10) for the medelling of the maximum refractive index difference profile in depth and width, respectively we evaluated numerically the above mentioned parameters.

The dependence of the center of the Gaussian function (which assures in the depth of the waveguide the best fit of the measured data), y_0, versus the window for four constant values of the in-diffusion time t_2 is presented in Fig. 5.

As can be seen from Fig. 5 for small values of the window, up to 5 μm there is a liniar dependence of the center of the Gaussian function on the window, while for high values, greater than about 10 micrometers, the dependence shows saturable behaviour, the increase of the diffusion time determining the increase of the depth of the center of the Gaussian function.

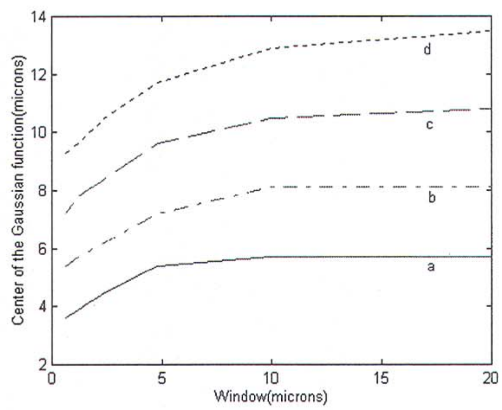

Fig. 5. The center of the Gaussian function versus the window for the folowing values of the in-diffusion time: $t_2 = 0.75$ min. (curve a), $t_2 = 1.5$ min. (curve b), $t_2 = 2.5$ min. (curve c) and $t_2 = 3.5$ min. (curve d).

Fig. 6 shows a quasilinear dependence of the center of the Gaussian function versus the in-diffusion time t_2 for six constant values of the window.

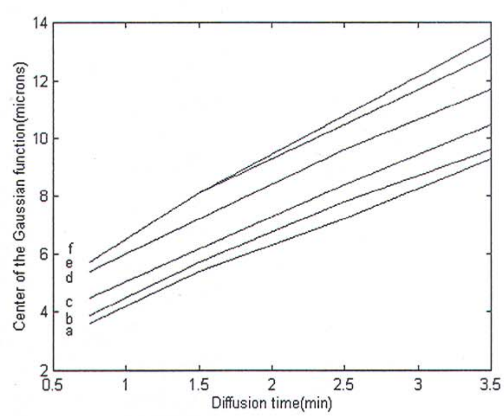

Fig. 6. The center of the Gaussian function versus the diffusion time t_2 for the folowing values of the window: $f = 0.6$ μm (curve a), $f = 1.2$ μm (curve b), $f = 2.4$ μm (curve c), $f = 4.8$ μm (curve d), $f = 10$ μm (curve e) and $f = 20$ μm (curve f).

The dependences of the maximum refractive index difference versus the window for four constant values of the in-diffusion time t_2 and versus the in-diffusion time t_2 for six constant values of the window are presented in Figs. 7 and 8, respectively.

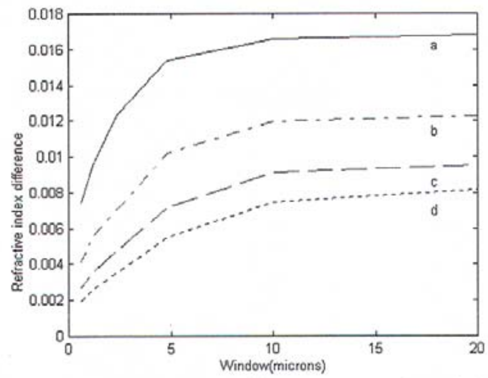

Fig. 7. The maximum refractive index difference versus the window for the folowing values of the d iffusion time: t_2 = 0.75 min. (curve a), t_2 = 1.5 min. (curve b), t_2 = 2.5 min. (curve c) and t_2 = 3.5 min. (curve d).

From Fig. 7 it can be seen that for high values of the window, greater than about 10 μ m, the dependence of the maximum refractive index difference values shows saturable behaviour. Fig. 8 shows that when the in-diffusion time t_2 is increased the maximum refractive index difference decrease.

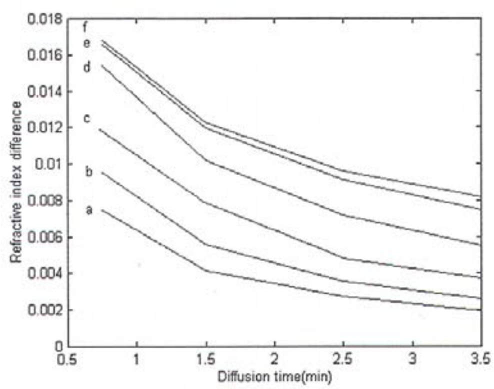

Fig. 8. The maximum refractive index difference versus the diffusion time t_2 for the folowing values of the window: f = 0.6 μ m (curve a), f = 1.2 μ m (curve b), f = 2.4 μ m (curve c), f = 4.8 μ m (curve d), f = 10 μ m (curve e) and f = 20 μ m (curve f).

In order to simulate the behaviour of the center of the Gaussian function in depth y_0, the maximum refractive index difference $\Delta n_{\max}$ and the variances of the Gaussian functions in depth and width, σ_y and σ_x, on the technological parameters: the in-diffusion time $t = t_2$ and the window f we considered the folowing dependence:

$$z = a_1 f + a_2 t + a_3 ft + a_4 f^2 + a_5 t^2 + a_6 f^2 t + a_7 ft^2 + a_8 (ft)^2 + a_9 f^3 + a_{10} t^3 + a_{11} f^3 t + a_{12} ft^3 + a_{13} f^3 t^2 + a_{14} f^2 t^3 + a_{15} (ft)^3 + a_{16} t^4 \quad (11)$$

where z stands for y_0, $\Delta n_{\max}$, $\sigma_{y,\text{left}}$, $\sigma_{y,\text{right}}$, σ_x.

We stoped the expansion in Eq. (11) at the fourth order terms because for high order terms the improvement of the precision is not significant and also the calculations began more difficult to be performed.

Solving numerically the system of equations (11) we obtained the values of the coefficients a_i, $i = 1,...,16$ for the waveguides characterized by the in-diffusion times t_1 = 0.75 min., t_2 = 0.75 min., 1.5 min., 2.5 min., 3.5 min. and the values of the windows for each in-diffusion time t_2, f = 0.6 μm, 1.2 μm, 2.4 μm, 4.8 μm (Table 3).

Table 3. Numerical values of the coefficient a (see text).

y_0 (μm)	$\Delta n_{\max}$	$\sigma_{y,\text{left}}$ (μm)	$\sigma_{y,\text{right}}$ (μm)	σ_x (μm)
4.0176	0.0122	7.3468	-0.7117	0.0154
4.5746	0.0158	-0.0421	3.4290	3.2203
-4.2038	-0.0089	-9.2958	1.8036	0.6611
-0.7377	-0.0021	-1.4234	-0.0591	-0.1221
-0.2115	-0.0159	4.8386	-2.3201	-2.0989
0.0010	0.0000	0.0207	-0.0155	0.0069
0.5589	0.0013	1.4449	-0.3445	-0.0894
1.1448	0.0016	2.3073	-0.4607	-0.1791
-0.0064	0.0001	-0.0088	0.0535	0.0291
-0.2836	0.0058	-2.5325	0.8603	0.6690
0.2124	0.0003	0.4301	-0.1018	-0.0358
0.3052	0.0003	0.5323	-0.1775	-0.0714
-0.2878	-0.0004	-0.5813	0.1295	0.0466
-0.4420	-0.0005	-0.8626	0.2258	0.0828
0.0846	0.0001	0.1662	-0.0433	-0.0154
0.0310	-0.0007	0.3201	-0.0917	-0.0684

Knowing the values of the coefficients a_i, $i = 1,...,16$, it is possible to evaluate using (11) the maximum refractive index difference, the center of the Gaussian and erfc functions in the depth and the width of the waveguide, respectively and the corresponding variances on the technological parameters: the windows and the in-diffusion times. The precision of the evaluation depends on the number of the terms of the Eqs. (11). For instance, the measured values: $\Delta n_{\max}$ = 0.00522, y_0 = 5.7 μm, $\sigma_{y,\text{left}}$ = 3.263 μm, $\sigma_{y,\text{right}}$ = 2.615 μm and σ_x = 2.279 μm, which characterize a waveguide fabricated with a window f = 0.6 μm and an in-diffusion time t_2 = 1.5 min. are in very good agreement with the corresponding values evaluated using (11): $\Delta n_{\max}$ = 0.00548, y_0 = 5.699 μm, $\sigma_{y,\text{left}}$ = 3.264 μm, $\sigma_{y,\text{right}}$ = 2.613 μm and σ_x = 2.278 μm.

4. Conclusions

Using Gaussian and erfc functions we performed the modelling of the maximum refractive index difference profile in the depth and the width of the waveguides obtained in glass by double ion exchange (Ag^+).

Based on the above mentioned model the dependences of the maximum refractive index difference, the center of the Gaussian and erfc functions in depth and width, respectively and the corresponding variances on the technological parameters: the window and the first and the second in-diffusion times for ions were evaluated. The experimental and the theoretical results are in very good agreement.

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