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NONIMAGING LIGHT CONCENTRATORS FOR PHOTOMULTIPLIER TUBES WITH NONUNIFORM RESPONSE OF THE PHOTOCATHODE

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The relative efficiency of the photocathode of a photomultiplier tube may not be uniform over its entire diameter. A special kind of concentrator should be designed in order to send more light onto the area of the photocathode that has a higher efficiency. A design method for obtaining such a concentrator is described and the two new concentrators obtained are analyzed by ray-tracing. The capacity of the classic compound parabolic concentrator to direct more light on the areas of higher efficiency is also analyzed and the results are compared to those of the two new concentrators proposed. The total efficiency of light conversion is evaluated for the new designs proposed and the classic CPC in the case of a Hamamatsu R-7056 photomultiplier tube to be used by the VERITAS project [1].

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1. Introduction

An optical concentrating system is designed to concentrate a beam of light of a certain divergence angle and a given cross-sectional area into the minimum possible area without loss of throughput. The optical concentrating system can be characterized by the optical concentration ratio which is defined as the amount of light collected at the exit aperture over the amount of light incident at the entrance aperture.

A nonimaging concentrator is essentially a funnel for light. Different designs of such concentrators (e.g. the compound parabolic concentrators [2], the straight-walled concentrators [3]) have been studied and used over the years in the fields of solar energy concentrating systems and the Cherenkov light detectors for high-energy physics and astronomy [4].

Sometimes, the relative efficiency of the photocahtode of the photomultiplier tube used as a light detector and placed behind the concentrating system, is not uniform over its entire diameter (see Hamamatsu R-7056 tube, Fig. 2 in [5]). The exterior region of the active area of the photocathode shows a maximum of the relative efficiency whereas the inner area shows a minimum.

A special kind of concentrator can be designed so that all the light that interacts with the wall of the concentrator will be reflected to the exterior area of the photocathode. Even if a certain amount of the incident light will go directly to the central region of the photocathode, the rest will be directed to the area of higher efficiency. The general design method and the results of the ray-tracing simulations will be described in the following sections.

2. The general design method for the concentrator profile

Let us consider that the active area of the photocathode has a smaller relative efficiency in the region V'V (Fig. 1). A'A is the diameter of the entire active area of the photocathode and the highest relative efficiency is in the ring bordered by A'V' and VA. We shall assume that the exit aperture of the concentrator fits exactly the diameter AA' and that A is the first point on the side profile of the concentrator.

The point A with the coordinates (X_A, 0) is always determined. Lets assume that point $C[X_C, h]$ is the k-1 point on the side profile of the concentrator and its coordinates are known. Then we will try to determine the coordinates of the k^{th} - $E(X_E, Y_E)$, point on the side profile of the concentrator (Fig. 1).

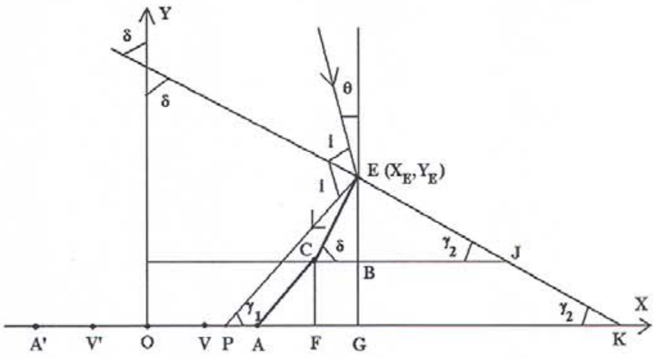

Fig. 1. The next (K-th) point $E[X_E, Y_E]$ on the side profile of the new concentrator. I_K is the angle of incidence, δ_K is the angle between the optical axis and the normal at the segment [CE], θ is the acceptance angle (the limit angle), P_K is the point of impact between the reflected ray and the photocathode.

The point E will be connected to C by a straight line (segment [EC] in Fig. 1) with such a slope that all the light rays incident within the design collecting angle θ_{lim} will be reflected anywhere inside either region VA or A'V'. The rays incident on point E at 0° in respect with the optical axis will be reflected closer to the point A than to the point V, whereas the rays incident on point E at θ_{lim} will be reflected closer to V than to A. All the rays incident within the acceptance angle θ_{lim} on the segment [EC] will be reflected in the area VA closer to A than those incident on point E. If the coordinate X_P of the point of impact P between the reflected ray and the photocathode, obey the conditions:

$$X_V \le X_P \le X_A \tag{1}$$

or

$$X'_A \le X_P \le X'_V \tag{2}$$

then the entire light incident within the acceptance angle on the segment [EC] will be reflected inside the region of higher efficiency. From simple geometrical considerations, the angle of incidence is:

$$i_k = \arctan\left[\frac{K \cdot h}{(X_{K-1} - X_P) + \dfrac{h}{\tan\delta}}\right] + (\pi/2 - \delta) \tag{3}$$

As $i = \delta - \theta \ge 0$ (Fig.1) it follows that $\delta \ge \theta_{lim}$. Therefore we may say that:

$$\theta_{lim} \le \delta \le \pi/2 \tag{4}$$

If $\delta \leq \theta_{\lim}$ or $\delta \geq \pi/2$ the rays will be back reflected to the entrance aperture. From equation (3) it follows that:

$$X_P = X_{K-1} + \frac{h}{\tan\delta} + K \cdot h\tan(i+\delta) \tag{5}$$

We will replace i in the above formula by $\delta - \theta$. For the two extreme cases: $\theta = 0$ and $\theta = \theta_{\lim}$ a given value of h and the smallest value of δ in the angular range (4), two values will be obtained for X_P. If both of them satisfy the conditions (1,2), the coordinate of the K^{th} point on the side profile of the concentrator will be calculated as

$$X_K = X_{K-1} + \frac{h}{\tan\delta} \tag{6}$$

$$Y_K = K \cdot h \tag{7}$$

where $Y_0 = 0$ and $X_0 = X_A$.

If at least one of the two values of X_p do not satisfy conditions (1,2), then the value of δ_K should be incremented by one unit and the procedure repeated until both values of X_P fulfill the conditions (1,2). For that value of δ and the given h, the coordinate X_K will be computed according to (6). Then we can proceed to the next point on the side profile. Otherwise we must increase δ with one unit and repeat the procedure. If the conditions (1,2) are not satisfied for any δ in the angular range (4), then the procedure stops and no further point will be determined. The last point considered will be the k-1 point.

2.1 Choosing the height h

One of the most important problems is how large h should be in order to have an accurate evaluation of the point's coordinates. Two design options are possible:

a) h has the largest possible value for which the conditions (1,2) are still satisfied when $\delta = 89^{\circ}$ (the largest possible slope). Only one point is obtained in addition to A. The concentrator is a simple straight-walled, frustum collector having the radius of the entrance aperture given by X_C and the length h (Fig. 2).

b) h is as small as possible. The smaller h, the larger the number of points obtained the smaller the total height of the concentrator and the larger the radius of the entrance aperture. The radius of the entrance aperture is, for small and medium acceptance angles much larger than the radius of the entrance aperture of the concentrator described at point a) (Fig. 3). A larger radius of the entrance aperture will also mean a larger amount of light interacting with the wall and going to the area of highest efficiency on the photocathode. However h cannot be smaller than a certain value which makes the time necessary to compute the coordinates, prohibitive. The radius of the entrance aperture will be given by the X coordinate of the highest point.

We have developed computer programs in order to generate the side profile curves numerically. These were written in ASAP (Advanced System Analysis Program) [6] an optical programming language. The programs require as inputs the coordinates of the extreme point A on the photocathode, the coordinates of the point V (the extreme point of the small efficiency area), the acceptance angle $\theta_{\lim}$ and the height h. The selection of the best value of the height h in the case a) assumes $\delta = 89^{\circ}$ and considers a trial value at the beginning. Then, this value is gradually changed until is found the largest value of the height h for which the conditions (1,2) are still fulfilled. In the case b) a trial height of $h = 1$ was chosen at the beginning, a set of points was obtained and then the total primary height of the concentrator was divided by 100 to get the final value of h. This was a convenient value of h so that the time used by our Pentium II computer was acceptable. However, on faster computers the height h can be much smaller and the result will be a concentrator having a shorter length and a considerably larger entrance aperture.

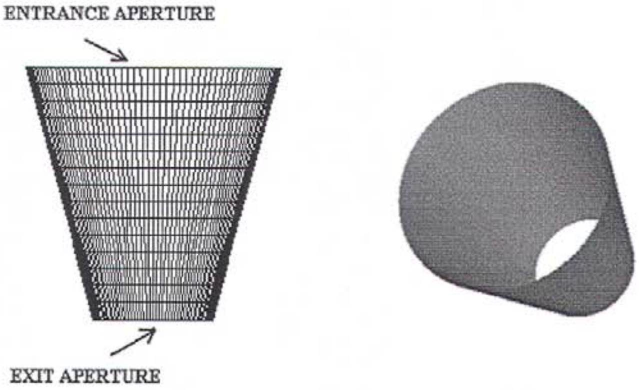

Fig. 2. The straight-walled concentrator (SWC).

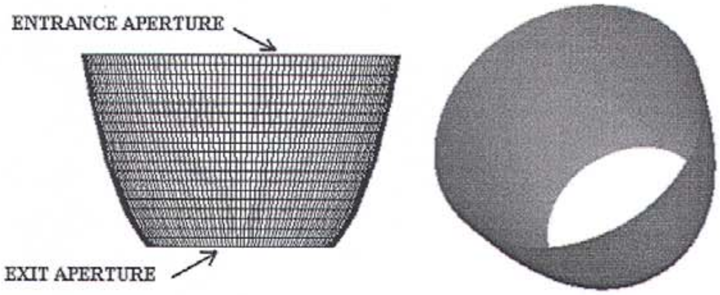

Fig. 3. The concentrator having a large entrance aperture (LEAC).

At the beginning the 2-D case is solved to obtain a number of points on the side profile of the concentrator. Then, they are fitted to obtain a 2-D curve. The 3-D concentrator is obtained by rotating the 2-D profile about its symmetry axis.

In order to manufacture the concentrator on a numerical "milling" machine, the coordinates of the points of 2-D side profile are introduced in the machine's memory, which interpolates them and then cuts the shape accordingly.

3. The optical properties of the new designed concentrators

The optical properties of the concentrators were evaluated using ray-tracing software codes written in ASAP. 2000 rays were uniformly generated over the entrance aperture of each of the designs analyzed. The flux of light was counted at the exit of the concentrator on two detectors. The first one was a circular detector with a radius equal to the radius of the inner, low efficiency area. The second detector was an annular one with an area equal to the exterior, high efficiency area on the photocathode. The inner radii considered were 15, 20 and 25 mm, while the exterior radius equal to the radius of the entire active area of the photocathode was 30 mm. Nonimaging concentrators were designed as was described above so that all the incident rays on the wall of the concentrators, within acceptance angles of 10°, 20°, 30° and 40° respectively, will be reflected towards the annular area of highest relative efficiency. The two possible geometries of the new concentrators described above at subsections a) and b) were considered for ray-tracing. The new straight-walled concentrator (Fig. 2) is referred to as SWC, while the new concentrator having a larger entrance aperture and a much larger number of points determined on its side profile (Fig. 3), will be referred to as LEAC. For the LEAC concentrators the coordinates of 40 points were determined on their side profile. These points were fitted by ASAP in a least square sense with a third order polynomial curve to obtain a 2-D profile. The optical properties of these new concentrators were compared to those of a classic compound parabolic concentrators (CPC) [2] designed for the above mentioned acceptance angles and a radius of the exit aperture of 30 mm. Their radii of the entrance aperture are in compliance with the thermodynamic limit (2).

The geometrical properties of all the above described designs are presented in Table 1 and 2.

Table 1. The height in millimeters of the nonimaging concentrators analyzed.

	CPC				SWC				LEAC			
	10°	20°	30°	40°	10°	20°	30°	40°	10°	20°	30°	40°
15 mm	1140.8	322.7	155.9	90.2	77.0	39.0	24.0	17.0	62.0	29.9	18.7	12.4
20 mm	1140.8	322.7	155.9	90.2	51.0	26.0	18.0	11.0	41.2	19.9	12.5	8.2
25 mm	1140.8	322.7	155.9	90.2	26.0	13.0	8.0	5.0	20.7	10.0	6.2	4.1

Table 2. The radius in millimeters of the entrance aperture for the nonimaging concentrators analyzed.

	CPC				SWC				LEAC			
	10°	20°	30°	40°	10°	20°	30°	40°	10°	20°	30°	40°
15 mm	172.7	87.7	60.0	46.6	31.3	30.7	30.4	30.3	50.9	40.2	35.8	33.3
20 mm	172.7	87.7	60.0	46.6	30.9	30.4	30.3	30.2	43.9	36.8	33.9	32.2
25 mm	172.7	87.7	60.0	46.6	30.4	30.2	30.1	30.1	37.0	33.4	31.9	31.1

In Table 3 it is shown the average ratio of the amount of light collected in the annular region of high efficiency over the total amount of light collected within the desired acceptance angles (10°, 20°, 30°, 40°) and for the three radii of the low efficiency area (15, 20, 25 mm). The reflectivity of the wall of the concentrators was considered 85% as this value describes pretty well the aging behavior of a coated aluminum surface over a couple of years of exploitation. The variation of the reflectivity of the wall with the angle of incidence was considered unimportant for the present evaluation (see [5]).

Table 3. The average ratio of the amount of light collected in the annular region of high efficiency over the total amount of light collected within the desired a cceptance angles (10°, 20°, 30°, 40°) and for the three radii of the low efficiency area (15, 20, 25 mm), in %.

	CPC				SWC				LEAC			
	10°	20°	30°	40°	10°	20°	30°	40°	10°	20°	30°	40°
15 mm	67.6	71.9	76.9	77.5	75.5	75.7	75.3	75.1	90.5	85.2	81.8	79.2
20 mm	49.3	51.2	58.0	59.2	57.5	56.5	56.2	55.7	77.6	67.7	63.9	60.9
25 mm	25.6	27.7	30.2	33.2	32.4	31.3	30.9	30.8	51.8	42.5	37.8	34.9

As it can be seen from the Table 3 the amount of light collected in the region of high efficiency increases from small angles to large angles for the CPC, slightly decreases for SWC and decreases for LEAC. As it was expected the amount of light collected in the annular area decreases as this area shrinks for all three designs. Up to 20° (small angles) both the straight-walled concentrator (SWC) and the LEAC are superior to the CPC but as the angles increase the performance of the CPC of the CPC becomes closer to the SWC and even exceeds its performance. At 40° the LEAC's performance is still the best but the differences between it and the CPC are very small (~ 1%). However, the new concentrators have poorer baffling ability than the CPC and an additional baffling system should be used.

Under real conditions only the requirement of having the entire light incident on the walls of the concentrator sent to the region of the photocathode where the efficiency is higher may not suffice. If the difference between the values of efficiency across the photocathode is not large (less than 10% in average) and the reflectivity of the wall is poor (~ 80%) the improvements of these new designs may not be significant.

In order to have an evaluation of the performances of these new concentrators under real conditions, SWC and LEAC concentrators were designed and ray-traced assuming they would be placed in front of the Hamamatsu R-7056 tube. This tube was proposed to be used by the VERITAS project [1].

4. The optical properties of the new designed concentrators used with the Hamammatsu R-7056 tube

As inputs for the geometry building and the ray-tracing software programs that we wrote in ASAP, were considered the following parameters: the transmisivity of the photomultiplier's window - 96%, the width of the window - 1 mm, the acceptance angle imposed by the VERITAS design – 26°, the reflectivity of the wall - 85%, the radius of the exit aperture 11 mm, the radius of the central area of low efficiency - 5 mm. The programs took into account the Fresnel variation of the transmisivity and reflectivity of the photomultiplier's window. The relative efficiency of the photocathode of the R-7056 tube was considered to be 80% in the central region [5], whereas for the exterior region the average relative efficiency was evaluated at 89%. In addition to the SWC and LEAC designs, a compound parabolic concentrator was ray-traced. The CPC was designed to have an acceptance angle of 26° (required by VERITAS). The geometrical characteristics of these three concentrators are summarized in Table 4.

Table 4. The radius of the entrance aperture and the height for the four designs ray-traced in the case of the VERITAS project.

	CPC	SWC	LEAC
Radius [mm]	25.1	11.1	13.4
Height [mm]	72.5	9.9	7.2

In Fig. 4 it is shown the variation of the ratio $\eta_1(\theta)$ of the amount of light collected in the annular region of high efficiency (5 to 11 mm) over the total amount of light collected within the desired acceptance angle (26°) for the four designs considered. Fig. 5 shows the variation of the total efficiency of light conversion in photoelectrons at the photocathode $\eta(\theta)$ within the desired acceptance angle (26°) for the four designs considered. The total efficiency of light conversion in photoelectrons at the photocathode is defined by:

$$\eta(\theta) = \frac{A \cdot E_2(\theta) + E_3(\theta) \cdot D}{100} \qquad (8)$$

where A = 89% is relative efficiency of the photocathode in the high efficiency area, B = 80% is relative efficiency of the photocathode in the low efficiency area, $E_2(\theta)$ is the ratio of the amount of light collected in the high efficiency area over the incident amount of light and $E_3(\theta)$ is the ratio of the amount of light collected in the low efficiency area over the incident amount of light.

Table 5. The average ratio $\tilde{\eta}_1(\theta)$ of the amount of light collected in the annular region of high efficiency (5 to 11 mm) over the total amount of light collected and the average variation $\tilde{\eta}(\theta)$ of the total efficiency of light conversion in photoelectrons at the photocathode within the desired acceptance angle (26°) for the four designs considered in the case of the VERITAS Project.

	CPC	SWC	LEAC
$\tilde{\eta}_1(\theta)$ [%]	80.1	71.3	85.3
$\tilde{\eta}(\theta)$ [%]	66.7	81.2	79.1

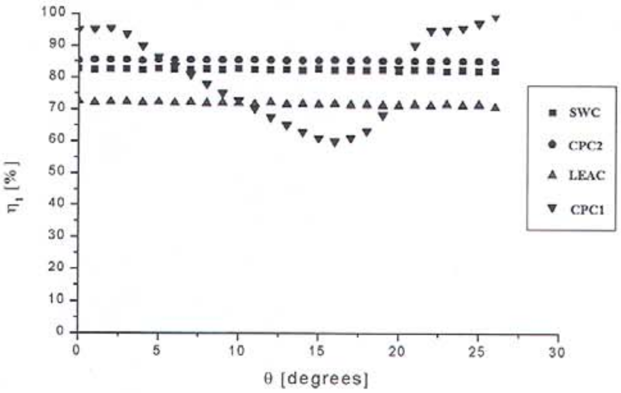

Fig. 4. The variation of the ratio $\eta_1(\theta)$ of the amount of light collected in the annular region of high efficiency (5 to 11 mm) over the total amount of light collected within the desired acceptance angle (26°) for the designs considered.

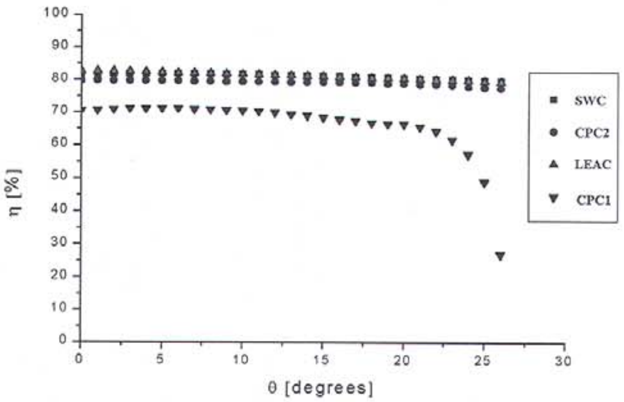

Fig. 5. The variation of the total efficiency of light conversion in photoelectrons at the photocathode $\eta(\theta)$ within the desired acceptance angle (26°) for the designs considered.

The average value of $\tilde{\eta}_1(\theta)$ and $\tilde{\eta}(\theta)$ within the design acceptance angle are given in the Table 5. As it can be seen from Table 5 the best performance in terms of light collected in the region of higher efficiency belongs to the LEAC. However this does not necessary result in the best performance of the total efficiency of light conversion in photoelectrons at the photocathode. This is offered by the straight wall concentrator (SWC) which also has the simplest design to manufacture. However the largest entrance aperture, small length and pretty good overall performance is given by the LEAC design. The last three designs in Table 5, offer a very good total efficiency of light conversion in photoelectrons at the photocathode within the proposed acceptance angle, have small lengths (therefore are cheap to manufacture) but have real acceptance angles much larger than desired. If an extra baffling system is not available for the camera, the classic CPC design seems to remain the only option even if its performances are ~ 14% lower than in the case of the new proposed designs.

5. Conclusions

A photomultiplier tube having a nonuniform relative efficiency across the active area of the photocathode requires a light concentrator to reflect most of the incident light over the area with a higher efficiency. A method to design a specialized nonimaging light concentrator for the above purpose was described. Two different designs were obtained and their optical performances were evaluated by ray-tracing for different acceptance angles and different dimensions of the high efficiency area. The smaller the acceptance angle and the area of high efficiency, the larger the amount of light reflected toward the high efficiency area by the two new designs proposed in comparison with the classic CPC. However the new designs have poor cut-off ability and therefore an additional baffling system is required whenever they are used.

For the practical case of the VERITAS project, the two new designs placed in front of a Hamamatsu R-7056 tube showed to be valuable options in order to improve the total efficiency of light conversion in photoelectrons at the photocathode. A gain of 12.4% to 14.5% in comparison with the CPC design for the VERITAS acceptance angle, resulted from simulations for the two new proposed designs.

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