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ANALYSIS OF PHASE MEASUREMENT ERROR FOR NULL GENERALIZED ELLIPSOMETRY USING THE PHASE COMPENSATOR

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An error analysis of null generalized ellipsometry for phase measurement using a Babinet compensator is presented. It turns out that there are errors only in a second order approximation, which means this technique provides high sensitivity for phase measurements. The sensitivity can be improved by equalizing the amplitudes of the x and y components of the field at the Babinet compensator but this complicates considerably the experimental procedure.

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1. Introduction

Phase measurement techniques, although known for more than a century, are increasingly used today for their sensitivity and accuracy [1-4]. On the other hand null measurements are reputed for being very sensitive [5]. Therefore null measurements of the phase using a Babinet compensator (null ellipsometry) are likely to provide high sensitivity. In our paper ellipsometry is used for the determination of the phase change in reflection for an anisotropic surface. This type of surfaces cannot be described just by two complex reflection coefficients r_p and r_s. Polarization conversion of light requires four complex reflection coefficients for a complete description of the optical properties of the surface, r_{pp}, r_{sp}, r_{ps} and r_{ss}, a formalism called generalized ellipsometry [6-8]. In principle, one derives an expression for the error of phase change measurement in these conditions and it turns out that there is an error only on a second order approximation, which is an eloquent argument in favor of the sensitivity of this measurement technique.

The experimental arrangement for phase measurement is illustrated in Fig. 1. The beam coming from left is linearly polarized by the polarizer P and becomes elliptically polarized after reflection on the grating. The compensator C compensates for the phase shift between x and y component and the beam is again linearly polarized. Another polarizer A (analyzer) is set perpendicular on the incident beam, which vanishes on transmission. The phase measurement is a null measurement and we nullify the light intensity by an appropriate combination of phase shift introduced by the compensator and polarization angle of the analyzer.

The ultimate goal is the determination of the optical properties of the material, i.e. the reflection coefficients. The measurable must be a function of these coefficients, $\Delta = \Delta\left(r_{pp}, r_{ps}, r_{sp}, r_{ss}\right)$.

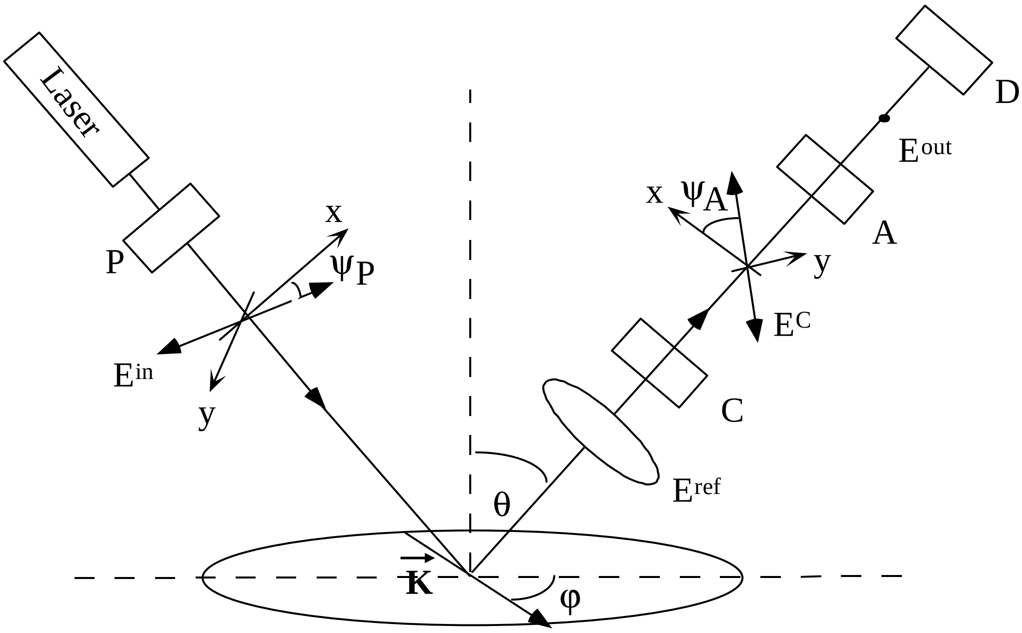

Fig. 1. Experimental arrangement for phase measurement using a Babinet compensator. P is the polarizer, A the analyzer, C the phase compensator, D the detector, **K** the grating vector, φ the azimuth angle, θ the incidence angle, ψ_P the polarizer angle, ψ_A the analyzer angle and E^{in}, E^{ref}, E^C and E^{out} are the light field at different positions in the system.

2. Error analysis

For calculations we will use the Jones matrix approach [9]. The input components of the beam are

$$E_x^{in} = E^{in}\cos\psi_P,\quad E_y^{in} = E^{in}\sin\psi_P. \tag{1}$$

Upon reflection the field becomes

$$\begin{pmatrix} E_x^{ref} \\ E_y^{ref} \end{pmatrix} = \begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix}\begin{pmatrix} \cos\psi_P \\ \sin\psi_P \end{pmatrix} E^{in}, \tag{2}$$

or

$$E_x^{ref} = \left(r_{pp}\cos\psi_P + r_{ps}\sin\psi_P\right)E^{in},\quad E_y^{ref} = \left(r_{sp}\cos\psi_P + r_{ss}\sin\psi_P\right)E^{in}. \tag{3}$$

The phase shift due to reflection on the sample is

$$\Delta = \arg\left(\frac{E_y^{ref}}{E_x^{ref}}\right) = \arg\left(\frac{r_{sp}\cos\psi_P + r_{ss}\sin\psi_P}{r_{pp}\cos\psi_P + r_{ps}\sin\psi_P}\right). \tag{4}$$

The compensator will introduce a phase shift Δ_C which in the end is supposed to be equal to Δ but for now we assume is arbitrary. Then we have

$$E_x^C = E_x^{ref},$$
$$E_y^C = E_y^{ref}\exp(-i\Delta_C). \tag{5}$$

After the analyzer the beam is

$$\begin{pmatrix} E_x^{out} \\ E_y^{out} \end{pmatrix} = \begin{pmatrix} \cos^2\psi_A & \cos\psi_A\sin\psi_A \\ \sin\psi_A\cos\psi_A & \sin^2\psi_A \end{pmatrix}\begin{pmatrix} E_x^C \\ E_y^C \end{pmatrix} =$$
$$\left(\cos\psi_A E_x^C + \sin\psi_A E_y^C\right)\begin{pmatrix} \cos\psi_A \\ \sin\psi_A \end{pmatrix}. \tag{6}$$

The output intensity measured by the detector D is

$$I^{out} = \frac{1}{2}\left(\left|E_x^{out}\right|^2 + \left|E_y^{out}\right|^2\right) = \frac{1}{2}\left|\cos\psi_A E_x^C + \sin\psi_A E_y^C\right|^2 =$$
$$\frac{1}{2}\left|\cos\psi_A E_x^{ref} + \sin\psi_A E_y^{ref}\exp(-i\Delta_C)\right|^2 =$$
$$\cos^2\psi_A I_x^{ref} + \sin^2\psi_A I_y^{ref} + 2\sin\psi_A\cos\psi_A\sqrt{I_x^{ref}I_y^{ref}}\cos(\Delta - \Delta_C), \tag{7}$$

where

$$I_x^{ref} = I^{ref}\cos^2\alpha,$$
$$I_y^{ref} = I^{ref}\sin^2\alpha, \tag{8}$$

and

$$\tan(\alpha) = \left|\frac{r_{sp}\cos\psi_P + r_{ss}\sin\psi_P}{r_{pp}\cos\psi_P + r_{ps}\sin\psi_P}\right|. \tag{9}$$

I^{ref} is the intensity that can be measured by a detector right after the reflection on the sample and also happens to be the maximum output at the detector D in Fig. 1 (when the analyzer A is parallel to the linearly polarized output beam and if there are no pure transmission losses in the compensator and the analyzer). Obviously, from Eq. (3), I^{ref} is of the form

$$I^{ref} = \left(\left|r_{sp}\cos\psi_P + r_{ss}\sin\psi_P\right|^2 + \left|r_{pp}\cos\psi_P + r_{ps}\sin\psi_P\right|^2\right)I^{in}, \tag{10}$$

with

$$I^{in} = \frac{1}{2}\left|E^{in}\right|^2. \tag{11}$$

α is the polarization angle of the beam when the phase shift is compensated and also the angle at which there is one of the semiaxes of the polarization ellipse. Using Eqs. (8) and (9) we can rewrite (7) as

$$I^{out} = I^{ref}\left(\cos^2\psi_A\cos^2\alpha + \sin^2\psi_A\sin^2\alpha + 2\sin\psi_A\cos\psi_A\sin\alpha\cos\alpha\cos(\Delta_C - \Delta)\right). \tag{12}$$

To obtain an expression for the phase measurement error we derive the output I^{out} on the phase Δ:

$$\frac{\partial I^{out}}{\partial\Delta} = -\frac{1}{2}\sin(2\psi_A)\sin(2\alpha)\sin(\Delta - \Delta_C)I^{ref}\,. \tag{13}$$

At null conditions one has $\psi_A \approx \alpha \pm \pi/2$ and $\Delta \approx \Delta_C$. Then, it follows

$$\delta I^{out} = \frac{1}{2}\left|\sin^2(2\alpha)I^{ref}\right|\delta\Delta^2\,, \tag{14}$$

or

$$\delta\Delta = \sqrt{\frac{2\delta I(0)}{\sin^2(2\alpha)I^{ref}}}\,, \tag{15}$$

where $\delta I(0)$ is the measurement error for output intensity when this intensity is zero. Generally, this is the minimum value of the error.

3. Discussion

It is important to note that the error appears only in the second order approximation. There is no measurement error for the phase in a first order approximation. This is a direct consequence of the fact that a null measurement type technique is used. It must be also mentioned that these considerations apply to precision not to accuracy. They are valid only if random, unbiased errors are assumed.

This relation makes sense, and we can derive it just from intuitive considerations. Since the phase measurement is a null measurement, the larger is the input to nullify, the more precise is the measurement of the phase. And we can see that for large values of I^{ref}, $\delta\Delta$ is indeed smaller. Also, the precision of the phase compensation is better when the components E_x^{ref} and E_y^{ref} have comparable amplitudes and is maximum when they are equal. The precision of the phase compensation is obviously related to how large is the difference between the maximum and the minimum output at the detector, because a large difference allows for a better discrimination of the minimum. Or, this difference is a maximum when

$$\left|E_x^{ref}\right| = \left|E_y^{ref}\right| \tag{16}$$

and is minimum when one of the components is zero. Indeed the smaller error is for α=45°, when Eq. (16) is fulfilled and becomes infinite for α=0° or α=90°, i.e. E_x^{ref} or E_y^{ref} is zero. From Eq. (12) if we allow the compensating phase shift Δ_C to vary we obtain

$$I^{\max} - I^{\min} = \left|\sin(2\psi_A)\sin(2\alpha)\right|I^{ref}\,. \tag{17}$$

This quantity is obviously maximum when α=45° and is minimum when α=0° or α=90°.

Therefore, an important enhancement of the precision of the phase measurement can be done if we rotate the axes of the compensator with an amount θ necessary to equilibrate the x and y components. In this case Eq. (15) becomes

$$\delta\Delta = \sqrt{\frac{2\delta I(0)}{I^{ref}}}\,. \tag{18}$$

However, Δ expressed by Eq. (4) becomes

$$\Delta = \arg\left(\frac{E_x^{ref}{}'}{E_y^{ref}{}'}\right) = \arg\left(\frac{E_x^{ref}\cos\theta + E_y^{ref}\sin\theta}{-E_x^{ref}\sin\theta + E_y^{ref}\cos\theta}\right), \tag{19}$$

where $E_x^{ref}{}'$ and $E_y^{ref}{}'$ are the components of the reflected beam in the rotated frame. Although the rotation of the compensator axes can dramatically improve the precision, in practice is difficult to implement and it may not be worth the trouble. To find the angle θ, you need to scan the output at the detector for various angles ψ_A with the compensator inactive until you find two positions say ψ_1 and ψ_2 separated by 90° where the output is equal. Then you have to rotate the compensator so that the x and y axes matches the two positions. Then θ must be noted for further use in calculations. This procedure must be repeated for each measurement and for each measurement we have a different value for θ.

One must be aware that Eq. (15) is valid only in the approximation that there are no pure transmission losses in the compensator and the analyzer. Generally, we do have this kind of losses and Eq. (15) must be modified accordingly

$$\delta\Delta = \sqrt{\frac{2\delta I(0)}{\sin^2(2\alpha) T_C T_A I^{ref}}}\,, \tag{20}$$

where T_C is the transmission of the compensator and T_A is the maximum transmission of the analyzer, when the beam is linearly polarized and the analyzer is parallel to the beam polarization.

4. Conclusion

We have shown using quantitative arguments that null ellipsometry using a Babinet compensator is a highly sensitive phase measurement technique. The first order approximation of the error is zero and we have to go to the second order approximation. By rotating the compensator to a position, where the x and y components are of equal amplitude, the sensitivity can be considerably increased but this complicates the experimental procedure and the computations and introduces supplementary sources of errors.

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