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SHORT COMMUNICATION

A MODIFIED HANNING WAVELET

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The wavelet theory is a powerful method for processing various signals. The Hanning wavelet is obtained from the Hanning window. In this brief report we present a new wavelet transform which is a generalisation of Hanning window. This transform is highly flexible one because it depends on three parameters and thus can cover more combinations of discrete and continuous parameter discretisations (hybrid cases).

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In processing signals, besides the well known Fourier methods, a more powerful instrument is offered by the wavelet theory [1,2]. The Hanning window [3] is a product of two Heaviside functions. Each Heaviside function is given [4] by:

$$ApproxH_a(t) = \frac{1}{2} + \frac{1}{\pi} arctg\left(t/a\right) \tag{1}$$

where a is the scale. In Fig. 1 the Heaviside function is represented for different values of a $\left(a = 0.2\,; a = 2\right)$. We construct the mother wavelet [5] which is the second derivative of the expression (1) :

$$h_{d,a}(t) = \frac{d^2}{dt^2}\left\{ApproxH_a(t+d) * ApproxH_a(d-t)\right\} = -2\frac{1}{\pi^2 a^2\left(1+\left(\frac{t+d}{a}\right)^2\right)\left(1+\left(\frac{t+d}{a}\right)^2\right)^2} -$$
$$-2\frac{\left(\frac{1}{2}+\frac{arctg\left(\frac{d-t}{a}\right)}{\pi}\right)(t+d)}{\pi a^3\left(1+\left(\frac{t+d}{a}\right)^2\right)^2} - 2\frac{\left(\frac{1}{2}+\frac{arctg\left(\frac{t+d}{a}\right)}{\pi}\right)(d-t)}{\pi a^3\left(1+\left(\frac{d-t}{a}\right)^2\right)^2} \tag{2}$$

where d is the window length. For the mother wavelet we have:

$$\int_{-\infty}^{\infty} h_{d,a}(t)dt = 0 \tag{3}$$

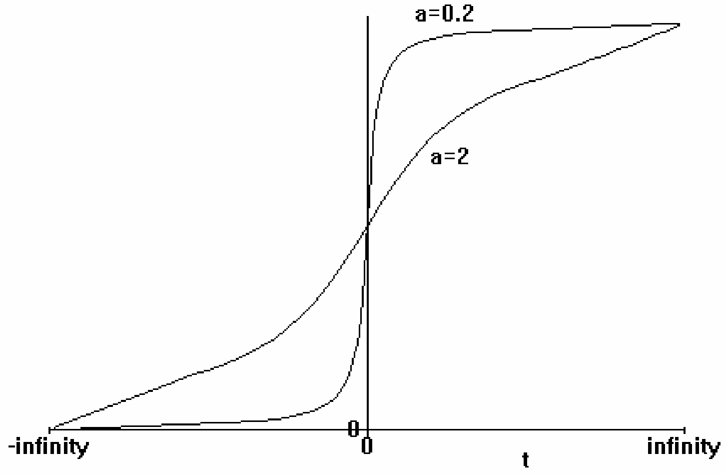

Fig. 1. The approximation of the Heaviside function for two values of the parameter a.

In Fig. 2a the mother wavelets $(a = 1.8, d = 2; a = 1.8, d = 4)$ are shown.

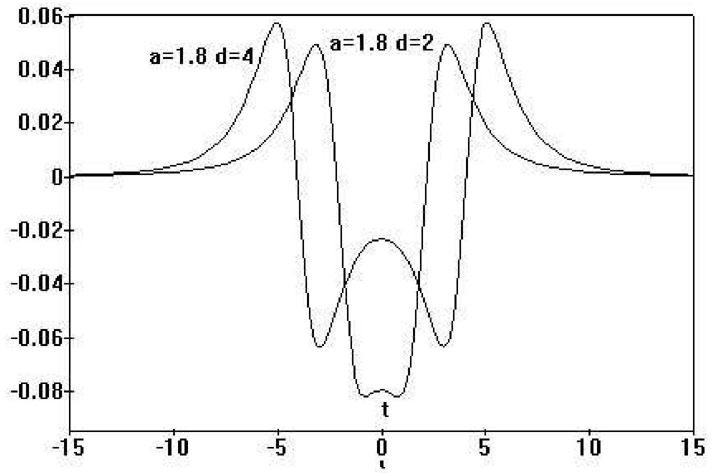

Fig. 2a. The modified Hanning wavelet.

In Fig. 2b the mother wavelets $(a = 1.0, d = 4; a = 1.5, d = 2.5)$ are shown.

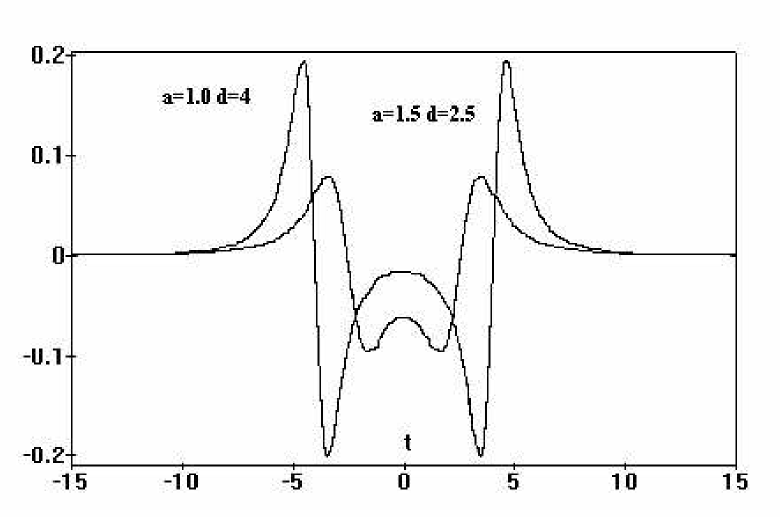

Fig. 2b. Modified Hanning wavelet.

The set of daughter wavelets are generated from the mother wavelet by shift operations: $h_{a,b,d}(t) = \frac{1}{\sqrt{F}} h_{d,a}(t-b)$ (4a) where b is the shift. The normalisation factor is given by: $F = \int_{-\infty}^{\infty} \left| h_{d,a}(t) \right|^2 dt$ (4b). The (1-D) wavelet transform of the f(t) is defined as:

$$W(a,b,d) = \int_{-\infty}^{\infty} f(t) h^{*}_{a,b,d}(t) dt \tag{5}$$

This is a correlation operation between the signal f(t) and the shifted and scaled mother wavelet $h_{a,b,d}(t)$. In the limit case $d \to 0$ is obtained the usual mexican wavelet (mexican hat) which depends of two parameters. In the limit case $b \to 0$ the Hanning wavelet is obtained. The $h_{a,b,d}(t)$ wavelet depends of three parameters (a,b,d). This wavelet generalizes the mexican and Hanninng wavelets and is more flexible than these ones.

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