

## PHASE SYNCHRONIZATION AND CODING CHAOS WITH SEMICONDUCTOR LASERS

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The synchronization phenomena that appear in nonlinear dynamics of two coupled external cavity semiconductor lasers can be used in communications systems. Numerically solving the Lang-Kobayashi system of equations for such a system we analyze the possibility of transmitting encoded information by modulating either the transmitter output light or the transmitter injected current. The synchronizing of lasers chaotic orbits are analyzed using both the phase for electric fields and Hilbert phase for intensities of the diode lasers light.

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### 1. Introduction

The nonlinear dynamics of laser diode devices that are very interesting as physical systems and very useful in all kind of engineering applications, especially under external optical feedback, have drawn great attention. When the laser output is redirected into the laser cavity as feedback from an external reflecting surface and the external cavity length is smaller than the output coherence length, the system will behave as a laser with a compound cavity. When the length is increasing chaotic behavior appears [1]. The level of optical feedback and intensity of injected current greatly influence the chaotic system evolution as well [2].

A chaotic system responds in complicated ways to external driving signals. Such a response can be a perfect match in amplitude of the signals or only a phase synchronization, where the amplitudes of slave and master device are poorly correlated but relationship becomes evident if a suitable phase is defined. Originally pointed out for a chaotic oscillator perturbed by an external periodic signal [3,4], the phase synchronism between coupled chaotic oscillators has been introduced recently [5]. The relative *optical* phase determines the intensity distributions in the far field but it is difficult to estimate in a real experiment by measuring the interference fringe visibilities. Hilbert phase (defined using analytic signal of a real intensities time series) [6] is experimentally accessible and allows quantitative detection of phase synchronization between coupled chaotic systems.

Some widely accepted methods devoted to the information transmission using chaotic dynamics properties are based on chaos masking [7], chaos modulation [8], or chaos shift keying [9]. The last is based on the definition of some clearly separated chaotic orbits, which the decoder has to detect separately. In the chaos modulation method, the message modulates the carrier. The masking scheme uses the idea of synchronized chaos [10,11], where a master is used to synchronize one or two slave subsystems. The message is added to the chaotic signal generated by the master and is transmitted to the slave receiver. The message signal has to be small compared with the masking chaotic signal. When the subsystems are synchronized, the chaotic part of the mixed signal can be subtracted and the message signal is reproduced. In our simulations we use a masking scheme when

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we modulate the light intensity from the master system, and a chaos modulation scheme if we modulate the master injection current.

In this paper we consider a model for chaos modulation and masking scheme for digital communications using as carrier the chaotic output of a continuous wave mono-mod laser diode operating with external cavity. Using the Lang-Kobayashi system of equations we numerically model the chaotic light emission and synchronization between two such systems – one, the so-called “master”, and another “slave”. The encoded digital message can be recovered coupling the transmitter with a similar chaotic laser diode receiver. The high frequency of the modulating signal and the impossibility of decoding the information without an appropriate receiver are carrying out important applications.

## 2. Chaotic dynamics of the laser diode

To model an external cavity semiconductor laser we use the Lang-Kobayashi type rate equations for the complex field with a feedback delay term [12,13]. These equations are generally considered to give a valid approximation of a single-mode semiconductor laser with moderate optical feedback. The rate equations for the complex field  $E$  and carrier density  $N$  are given by:

$$\frac{dE(t)}{dt} = (1 - i\alpha) \left( G(t) - \frac{1}{\tau_p} \right) \frac{E(t)}{2} + \gamma E(t - \tau) e^{i\omega\tau} + \sqrt{2\beta N(t)} \xi(t) \quad (1)$$

$$\frac{dN(t)}{dt} = \frac{J}{e} - \frac{1}{\tau_n} N(t) - G(t) |E(t)|^2 \quad (2)$$

$$G(t) = \frac{g(N - N_0)}{(1 + s|E(t)|^2)} \quad (3)$$

The solitary laser oscillates in a single longitudinal mode with angular frequency  $\omega$  ( $1.2 \text{ rad}\cdot\text{fs}^{-1}$ ) under continuous wave (CW) operation, and it has the linewidth enhancement factor  $\alpha=5$ , gain  $G$  with the gain parameter  $g$  ( $1.5 \times 10^8 \text{ ps}^{-1}$ ) and gain saturation coefficient  $s$  ( $5 \times 10^{-7}$ ),  $\tau_p$  ( $2 \text{ ps}$ ) is the photon lifetime. The external cavity round-trip time is  $\tau = 2L/c$  ( $200 \text{ ps}$ ) –  $L$  is the distance from the laser facet to the external reflector and  $c$  is the speed of light in vacuum.  $N$  represents the carrier density averaged over the active region with  $N_0$  ( $1.5 \times 10^8$ ) the carrier at transparency and  $\tau_n$  ( $2 \text{ ns}$ ) carrier lifetime.  $J$  represents the bias current and  $e$  the electron charge. The random spontaneous emission is modeled by a complex not correlated Gaussian white noise term  $\xi$  of zero mean, and with a spontaneous emission rate  $\beta$  ( $1.1 \times 10^{-9} \text{ ps}^{-1}$ ). The feedback coefficient  $\gamma$  is related to the cavity parameters as

$$\gamma = \frac{1 - R}{\tau_i \sqrt{R}} \sqrt{\rho} = \frac{\Gamma}{\tau_i} \quad (4)$$

where  $R$  is the reflectivity of the laser exit face,  $\rho$  is the fraction of the laser output power coupled by the external cavity and the round-trip time within the laser resonator is  $\tau_i = 2\eta l/c$  ( $8 \text{ ps}$ ) with  $\eta$  being the refractive index of the laser cavity and  $l$  the internal cavity length.

The equations (1)-(3) are written in the reference frame where the frequency at transparency of the laser is zero, and neglecting the effect of lateral diffusion and spatial hole burning and we do not include multiple reflections. The outgoing field at the internal laser mirror facing the external cavity is written:

$$E(t)e^{-i\omega t} = |E(t)|e^{-i(\omega t + \Phi(t))} \quad (5)$$

The output power of the laser is

$$I(t) = \frac{hc\alpha\alpha_m}{4\pi\mu} |E(t)|^2 \quad (6)$$

where  $h$  is the Planck constant,  $\alpha_m$  ( $45 \text{ cm}^{-1}$ ) is the facet loss and  $\mu$  (4) is the group refractive index.

In equation (1) we can add another term responsible for coupling an injected external field into the laser cavity, i.e. the term responsible for the chaotic oscillators coupling,

$$kE_{ext}(t) \quad (7)$$

with  $k$  ( $0.1 \text{ ps}^{-1}$ ) the coupling parameter and  $E_{ext}$  the external field to be injected.

In order to numerically solve the (1-3,7) delay differential equations we used a Runge-Kutta (2,3) pair of Bogacki and Shampine integrator and a piecewise cubic Hermite interpolation scheme. Neglecting optical feedback ( $\gamma = 0$ ), a stable steady state solution can be found at a current greater than threshold current ( $I_{th} \approx 14.7 \text{ mA}$ ). Chaos can be reached only with an external feedback and without this the attractor evolves smoothly to a stationary point. After a short period of predominant spontaneous emission, a steady state laser emission is established at powers corresponding to the level of injected current. Coupling in the external cavity, feedback increased from zero and the system switches to an oscillating state which follows a chaotic trace, and sharp sidebands on each side of the main optical spectrum appears [14,15], characterizing the main laser system and depending of the injected current. Further increasing the feedback, the relaxation oscillation sidebands decrease in amplitude and are supplemented with external cavity mode beating features, depending on the length of the cavity. At important feedback values ( $\gamma > 3 \times 10^{10} \text{ ps}^{-1}$ ) the chaos is fully present. When external cavity mode spacing is equal to an integer sub-multiple of the relaxation oscillation resonance peak a period-doubling sequence to chaos is observed [14]. When the ratio between them is not an integer, at fixed injection current and external cavity length, the increasing of the feedback from external mirror pushes the system to a quasi-periodic sequence to chaos.

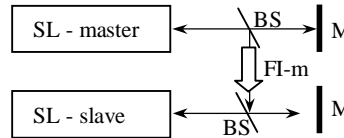


Fig. 1. Master and slave transmission arrangement (SL, semiconductor laser; BS, beam splitter; M, mirror - attenuator; FI-m, Faraday isolator - modulator).

The information transmission arrangement assumed in this paper is shown in Fig. 1. Two external cavity semiconductor lasers are coupled unidirectionally via a Faraday optical isolator. Attenuators are used to control feedback and coupling strength. Coherent light from the master drive system is modulated and injected into the second external cavity laser, i.e. the slave. Both systems are modeled by the set of equations (1-3). The light injected in the slave system, equation (7), is included in slave system equation (1) as a supplementary term.

In order to analyze the effect of synchronization to the data transmission capability of the system we use two kind of phase information. One is the complex field phase

$$\Phi(t) = \arctan\left(\frac{\text{Im}(E(t))}{\text{Re}(E(t))}\right) \quad (8)$$

and the other is the Hilbert phase [16] -  $\Phi_H$ , defined for the light intensity time series -  $I(t)$ , a real function, from the corresponding analytic signal

$$I_a(t) = I(t) + i \cdot HT[I(t)] = A(t) \exp(i\Phi_H(t)) \quad (9)$$

where HT denotes the Hilbert transform of  $I(t)$ ,

$$HT[I(t)] = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{I(t')}{(t'-t)} dt \quad (10)$$

and here  $P$  denotes taking the principal value of the integral. The Hilbert phase describes changes in the field envelope and can be experimentally evaluated from the laser intensity time series using fast Fourier transform. It is to be noted that the nature of the complex phase dynamics is distinct from the dynamics of the Hilbert phase. We consider the constructed phases as unbounded, not taken modulo  $2\pi$ , correcting the radian phase angles by adding multiples of  $\pm 2\pi$  when absolute jumps between consecutive elements are greater than the jump tolerance  $\pi$ .

The synchronization of chaotic systems is based on phenomena similar to the resonance of the linear oscillators. Coupling two chaotic oscillators with similar characteristics, the orbits of the two systems evolve in a very similar way. This behavior of chaotic systems was experimentally used in encoding communication using fiber ring lasers [17] and semiconductor lasers [18]. For the setup presented in Fig. 1 the external field injected in the “slave” laser is the output field of the “master” laser delayed with an arbitrary time simulating space propagation ( $T_c = 0.8 \text{ ns}$ ). Synchronization is possible if we have a solution of Eqs. (1-3,7) for both master and slave lasers and a condition for phase (complex or Hilbert) equivalence

$$\Phi(t - T_c) = \Phi'(t) \text{ and/or } \Phi_H(t - T_c) = \Phi'_H(t) \quad (11)$$

here apostrophe denote the slave variable. In real experiments the perfect equality (11) can not be satisfied due to the inherent error limit evolution of phases and is replaced by a hard connection in phase evolution. We can use as criterion the constancy of the synchronization error defined as

$$\Delta\Phi(t) = \frac{|\Phi'(t) - \Phi(t - T_c)|}{\langle \Phi'(t) \rangle} \quad (12)$$

where  $\langle \Phi'(t) \rangle$  is the temporal average of the slave phase.

### 3. Encoded communication and synchronization of chaotic oscillators

The information transmission using chaotic dynamics properties based on chaos masking can be obtained using a message  $M(t)$ , of very small amplitude; to assure privacy and to avoid large distortion in the master output, one modulates the transmitter output,  $E_{\text{Master}}$ , resulting in a signal  $E_{\text{mod}}$ , transmitted to the receiver:

$$|E_{\text{mod}}| = A |E_{\text{Master}}| (1 + M(t)) \quad (13)$$

where  $A$  is a constant attenuation factor. The decoding process is based on the synchronization of the receiver output,  $E_{\text{Slave}}$ , to the transmitter carrier field,  $E_{\text{Master}}$ , and not to the modulated injected signal, after corresponding space travel,  $E'_{\text{mod}}$ .

$$D(t) = \sqrt{\frac{|E'_{\text{mod}}|^2}{|E_{\text{Slave}}|^2}} - 1 \quad (14)$$

where  $D(t)$  is the detected signal. The decoded message must be analogous to the encoded one in the case of chaotic synchronization of the oscillators. The information transmission based on chaos modulation can be obtained with a modulating message added directly to the master injected current, rather than through an electro-optic modulator introduced in the master output optical field,

$$|J_{\text{mod}}| = |J_{\text{Master}}|(1 + M(t)) \quad (15)$$

where  $J$  is the injected current. In these case the detection scheme can be similar with the one used for chaos masking.

Let's take the bias current of the laser diode  $J=40 \text{ mA}$ , the feedback coefficient  $\Gamma=0.3$  and assume that the master is identical with the slave. The light emission of the correspondent master is characterized by a high chaotic state (see Fig. 2a). After first identical trajectory steps, at the moment when the master signal reaches the slave, after a short period of adaptation, the slave leaves its own trajectory and follows a system chaotic trajectory dictated by the master.

In Fig. 2b is presented for comparison the delayed modulating signal and the detected one. We can easily observe the synchronicity of signals and the way in which a coded signal, of about 4% from chaotic carrier, can be recovered in good conditions. A similar good recovering signal is obtained even if the ratio of the coded signal is about 1%, but information is lost if the modulation ratio is below 0.2%. The noisy recovered signal may be filtered using an appropriate band-pass filter, a Fabry-Perot with an adequate bandwidth. The time accuracy in signal recovery is to be noted; the first signal pulse of about  $50 \text{ ps}$  is very good temporally resolved.

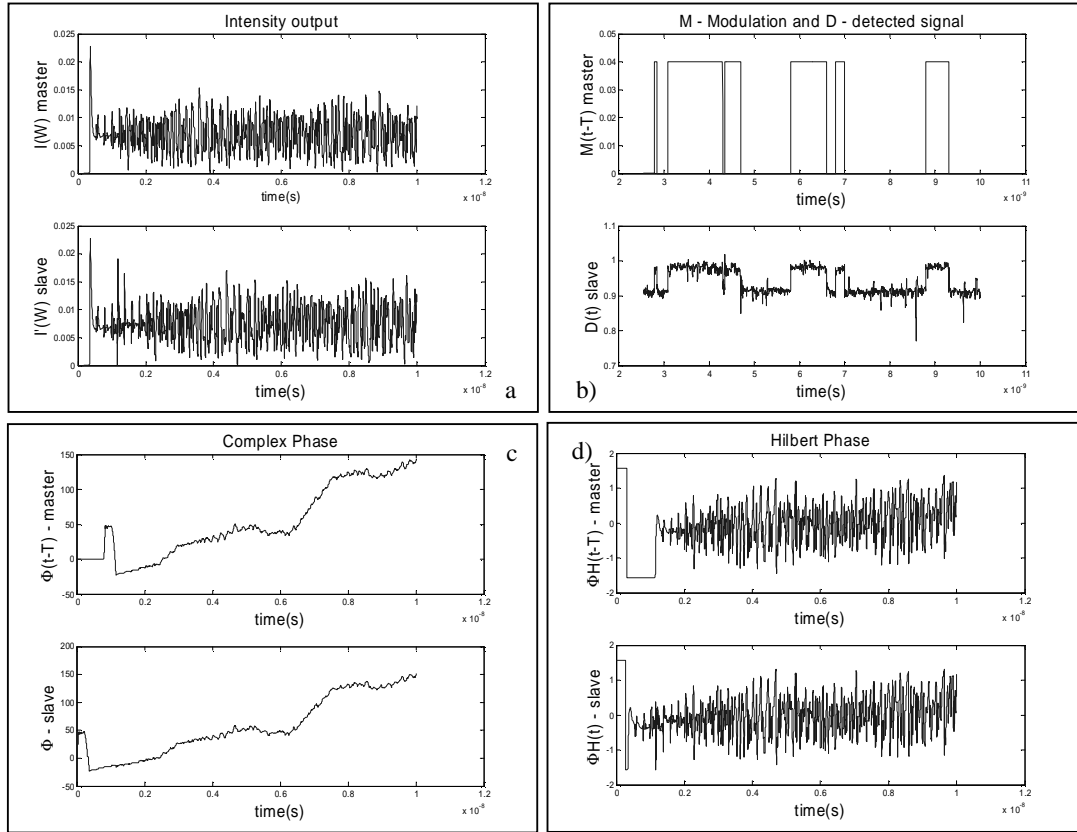


Fig. 2. A master chaotic semiconductor laser coupled with an identical slave, a) the intensity output; b) the modulation delayed and detected signal; c) complex phase; d) Hilbert phase.

The evolution of complex and Hilbert phase for master and slave diode laser light emission is presented in Fig. 2c and 2d. For both types of phases a clear following of the slave to the driving master is observed. We can say that systems are coupled in phase. In Fig.3 the phase synchronization error between master and slave present an important decrease, and this is an obvious mark for obtaining accurate data recovery conditions.

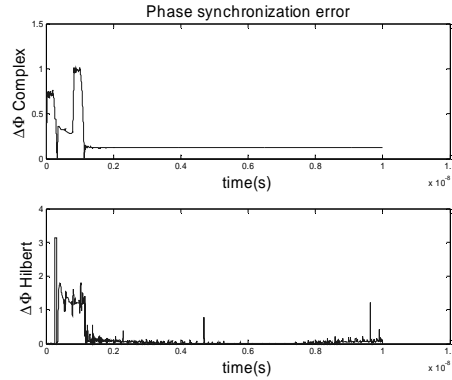


Fig. 3. The complex and Hilbert phase synchronization error between master and slave in a chaos masking carrier data transmission arrangement.

An interesting situation is achieved when different types of chaotic oscillators are coupled in a transmission scheme. For instance if we take the slave parameters: gain parameter  $g = 8 \cdot 10^{-8} \text{ ps}^{-1}$ , carrier lifetime  $\tau_n = 6 \text{ ns}$  and the feedback coefficient  $\Gamma = 0.2$ , and the master feedback coefficient  $\Gamma = 0.1$  with a coupling parameter  $k = 0.04 \text{ ps}^{-1}$  we obtain the signals presented in Fig.4.

In this case the light emission of the master and the slave are characterized by a low chaotic state (quasi-periodic oscillations), see Fig. 4a. After a relative long period of adaptation the slave light emission follow synchronously the master driving signal –the slave is forced to evolve on a non-characteristic orbit. For a digital carrier modulation of 10% (the coded signal - Fig. 4b) the same synchronicity can be observed in the decoded signal. After the adaptation period the decoded signal is evidently modulated by the data structure.

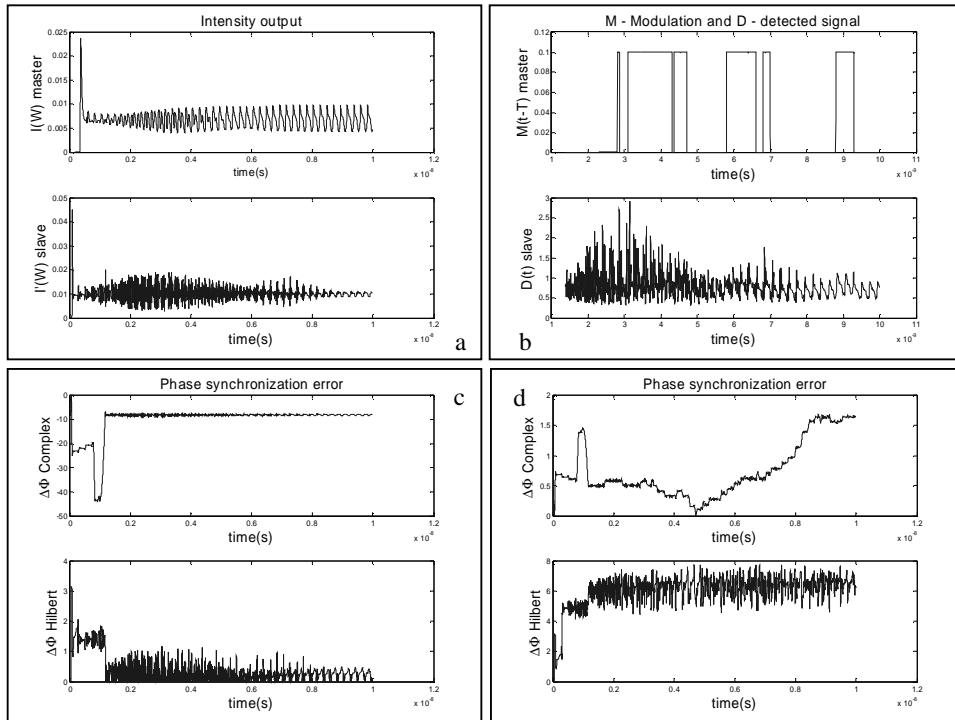


Fig. 4. A master chaotic semiconductor laser ( $\Gamma=0.1$ ) coupled with a different type of slave, a) the intensity output; b) the modulation delayed and detected signal; c) the complex and Hilbert phase synchronization error; d) the complex and Hilbert phase synchronization error in case of master feedback coefficient is  $\Gamma=0.3$

The synchronous coupling effect is also evident in the phase error graphics - Fig. 4c – where a apparent error reduction is present. Because in this case the slave is forced to a non-characteristic orbit the phases are no longer identical, and  $\Delta\Phi$  functions present oscillations correlated with the master driving signal.

It is important to note that this forced synchronization is absent when the master oscillates on a chaotic non-periodic orbit. If the master feedback coefficient becomes  $\Gamma=0.3$ , the system is decoupled and the phase error clearly show this – see Fig. 4d. The phase synchronization error in this case is important and the detected signals do not carry data information any longer.

When a master chaotic oscillator ( $\Gamma=0.1$ ) is coupled ( $k = 0.1 \text{ ps}^{-1}$ ) in a transmission scheme with an identical slave but without external cavity ( $\Gamma=0$ ), the situation is somehow similar. In Fig. 5a we can see how slave is forced to follow the master driving signal and in Fig. 5b the corresponding detected signal with a good recovery of the data structure is shown.

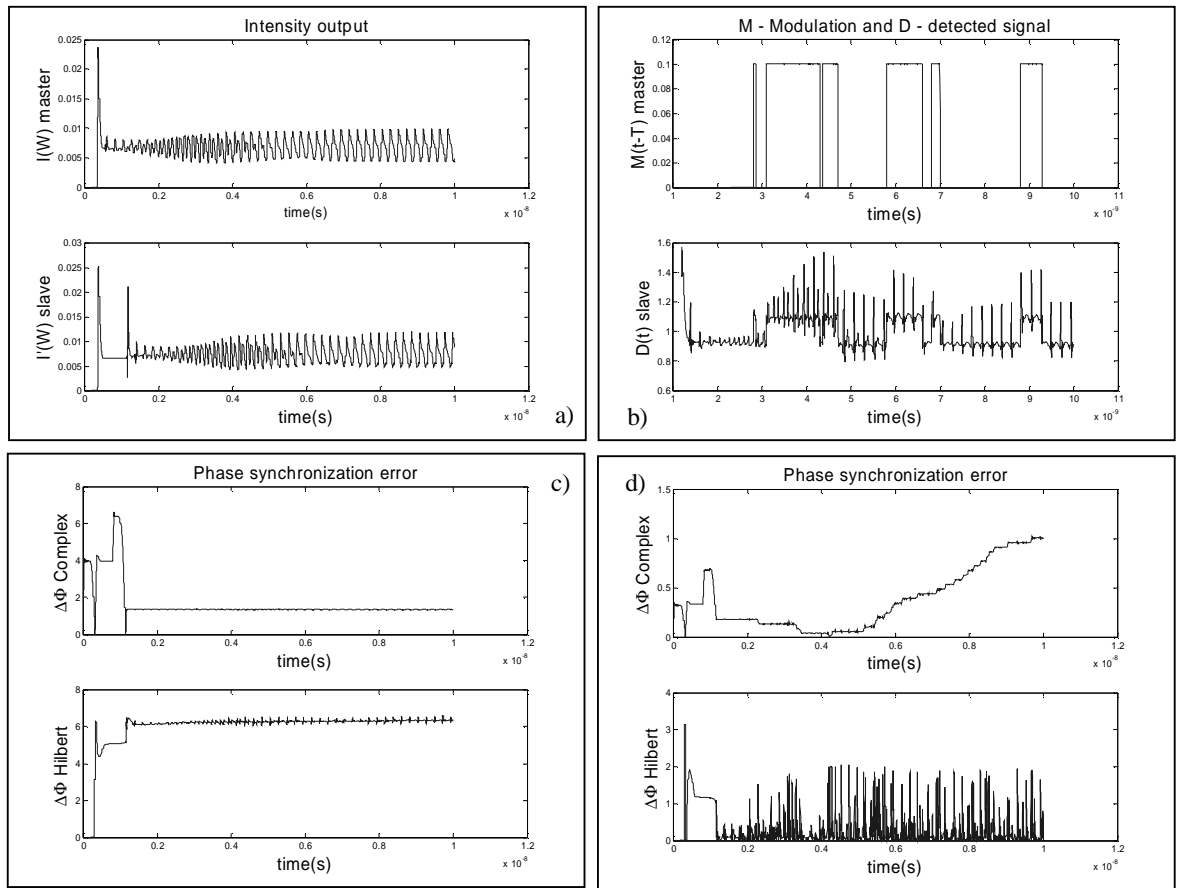


Fig. 5. A master chaotic semiconductor laser ( $\Gamma=0.1$ ) coupled with a slave without external cavity ( $\Gamma=0$ ), a) the intensity output; b) the modulation delayed and detected signal; c) the complex and Hilbert phase synchronization error; d) the complex and Hilbert phase synchronization error in case of master feedback coefficient is  $\Gamma=0.3$

In this case the phase error graphics - Fig. 5c – show a low level of oscillations correlated with the master driving signal, indicating a phase synchronicity for these signals. The synchronization is absent when the master oscillates in a chaotic non-periodic orbit (master feedback coefficient  $\Gamma=0.3$  – see Fig. 5d), and for these situation the detected signals cannot be used as a source for the encoded data information recovery, see Fig. 6.

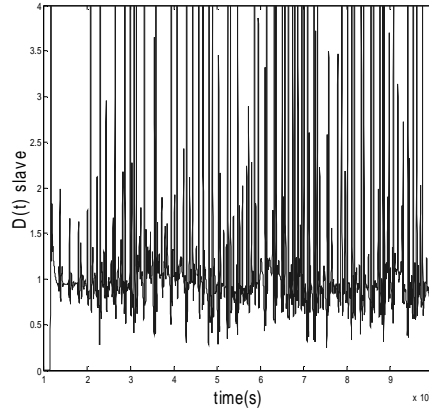


Fig. 6. The decoded signal for data presented in Fig. 5d.

Another technique used in data transmission with chaotic carrier is chaos modulation. The message modulates one of the parameters involved in the master chaotic process and the resonance of the coupled slave assures a close orbit trace similar to one of the master [19]. In our case the most accessible parameter is the injection current. If we assume two identical laser diodes ( $I=0.3$ ) coupled ( $k = 0.1 \text{ ps}^{-1}$ ) in a chaos modulation scheme with a digital modulation of 4% for the injected current, the intensity output and the modulation and detected signal data are presented in Fig. 7.

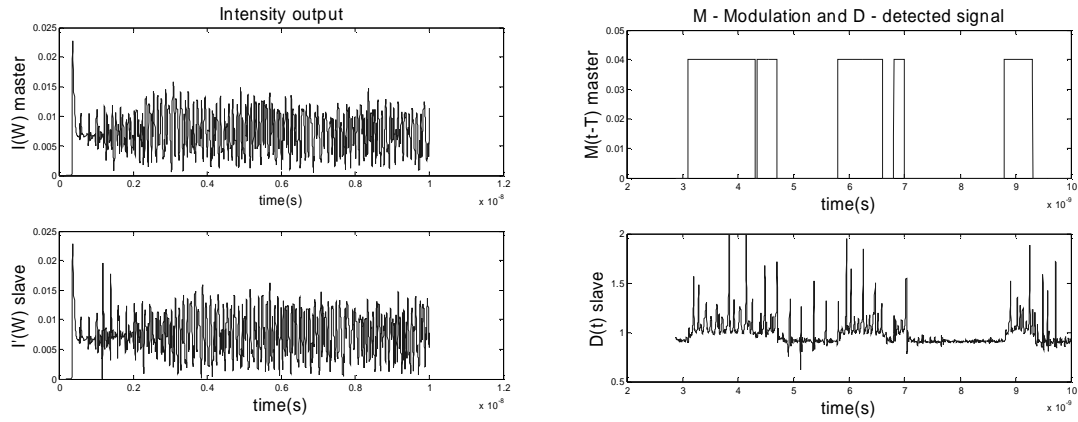


Fig. 7. Two identical chaotic semiconductor lasers coupled in a data transmission arrangement using chaos modulation technique, a) the intensity output; b) the modulation delayed and detected signal.

The slave is practically coupled with the master signal (see Fig.8, as the complex phase synchronization error indicates). Even for a high coherent state of the master emission, the slave is synchronized in complex phase with the master, but not exactly in Hilbert phase. This situation indicates an inadequate condition for a perfect data recovery. The transmitted signal clearly carries the encoded information but not in an accurate state, due to the relatively low time response of the system.

As we can see, from Figs. 4 to 8, any attempt to force the slave oscillator to evolve in a non personal orbit – i.e. chaotic orbits characterized by different parameters – produces a decrease in the quality of the decoded signal.



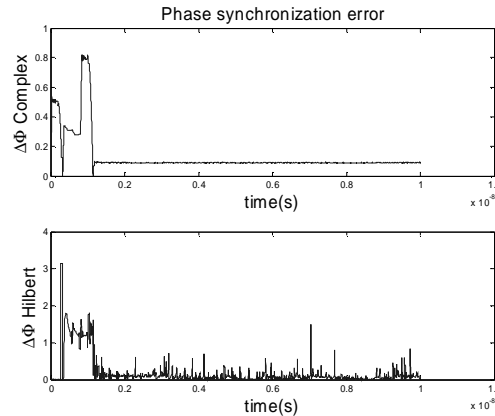


Fig. 8. The complex and Hilbert phase synchronization error between master and slave in a chaos modulation data transmission arrangement.

#### 4. Conclusions

In this paper we have presented numerical simulations of synchronized chaotic semiconductor lasers that are unidirectional coupled by their electric fields, and we have analyzed the variation of the phase synchronization error between master and slave in a chaos masking and modulation data transmission scheme. A good decoding of the message is provided by those systems where the chaotic orbits of the master and the slave are as close as possible. The chaos masking system can achieve theoretically data transfer rates up to 10 Gbps. An acceptable restoration of the signal is obtained in all cases where the coupling factor between the oscillators is strong and low chaotic states are used. If we use a high chaotic state for the master emission and small amplitude of the modulation the detected signal cannot be decoded. The same situation appears when the master and the slave signals are no longer in phase; the message cannot be longer decoded. The error of phase synchronization almost vanishes for the complex phase in cases for which the slave system is allowed to evolve in orbits very close to original ones. The synchronization error for the Hilbert phase is small for the cases where the complex phase is coupled, and it is major for the cases where the complex phase is not coupled. Although these results indicate that the Hilbert phase can be used as an experimental criterion to determine the quality of the chaotic systems coupling and data transmission recovery.

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