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SCHLIEREN METHOD FOR MEASURING THE TEMPERATURE COEFFICIENT OF THE REFRACTIVE INDEX OF OPTICAL GLASSES

A. Petris*, C. Popa[a], D. Popa[a], V. I. Vlad

Institute of Atomic Physics, NILPRP – Dept. of Lasers, ROCEP, Bucharest, Romania
[a]University "Politehnica", Dept. of Physics, 77206 - Bucharest, Romania

The measurement of the temperature coefficient of the refractive index of optical glasses using the schlieren method is presented. The heat propagation in two plane-parallel plates made from different optical glasses, with the same geometry and identically heated, is simultaneously investigated using the schlieren setup. One of the plates with known parameters describing the heat diffusion and the temperature coefficient of the refractive index provides the calibration of the experimental configuration. The theoretical model of heat diffusion, experimentally checked by us [1], allowes the determination of the unknown temperature coefficient of the refractive index for another plate. The experimental results are in good agreement with the values found in the literature.

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1. Introduction

The schlieren method provides a simple and noninvasive procedure for a real-time visualization of spatial changes in the refractive index gradient of transparent media. This gradient can be related to the density, temperature or pressure distributions within the transparent media [2 – 5].
In a previous paper [1] we studied the heat diffusion in a plane glass plate heated by a plane thermal source. The heat propagates along an axis perpendicular to the thermal source and the refractive index gradient, related to the temperature gradient, is along the direction of heat diffusion. The light intensity in the schlieren image, proportional to the square of the temperature gradient, was measured using a compact schlieren system with image acquisition facilities. We recorded the temperature gradient and explained the schlieren images in the frame of an appropriate theoretical model of heat diffusion [6]. In this paper we determine the temperature coefficient of the refractive index of a glass plate that was simultaneously observed with a reference glass plate for which the calibration of the experimental setup was achieved.

2. The schlieren method for heat diffusion studies

The test object is illuminated with a collimated light beam with uniform intensity. The problem is treated one-dimensionally, with the *x* axis along the direction of heat diffusion, perpendicular to the light rays. For small phase differences in the glass plate, the distribution in the transform plane (before knife-edge) is:

$$U(\xi) = \int [1 + i\phi(x)] \exp\{-ikx\xi / f\} dx = \delta(\xi / \lambda f) + i\tilde{\phi}(\xi / \lambda f) \qquad (1)$$

* Corresponding author: petris@ifin.nipne.ro

where $\tilde{\phi}(\xi)$ is the Fourier transform of $\phi(x)$.
The field distribution after the knife-edge is:

$$U'(\xi) = S(\xi)\left[\delta(\xi) + i\,\tilde{\phi}(\xi)\right] \tag{2}$$

The knife-edge acts as a step function *S(ξ)*,

$$S(\xi) = \begin{cases} 1, \xi \ge \xi_0 > 0 \\ 0, \xi < \xi_0 \end{cases} \tag{3}$$

removing the zero order of the Fourier transform of $\phi(x)$:

$$[S(\xi)][\delta(\xi)] = 0$$

Thus:

$$U'(\xi) = iS(\xi)\tilde{\phi}(\xi) \tag{4}$$

The complex amplitude distribution in the image plane, $\Psi(x')$ is the Fourier transform of $U'(\xi)$. After some calculations, the amplitude of the output field in the image plane is:

$$\psi(x') = \frac{i}{2}\left[\phi(x') + \frac{i}{\pi}\left(\frac{d\phi(x')}{dx'}\right)\right]. \tag{5}$$

If the phase object contains sharp discontinuities, $\frac{d\phi(x')}{dx'} >> \phi(x')$, Eq. (5) becomes:

$$\psi(x') = -\left(\frac{1}{2\pi}\right)\frac{d\phi(x')}{dx'} \tag{6}$$

and the corresponding intensity in the schlieren image will be:

$$I(x') \propto \left(\frac{d\phi(x')}{dx'}\right)^2 \tag{7}$$

In our experiment this condition is fulfilled due to the very strong and short thermal excitation of the glass plate.

3. Heat diffusion in a glass plate

Our experimental conditions, a short and strong heat pulse applied on a side of the glass plate, determined us to consider the model of heat propagation in an infinite medium with a temporal delta-type infinite plane thermal source. Considering the source placed in the origin of the *x*-axis, along which the heat diffusion takes place, the solution of the heat equation is [6]:

$$U(x,t) = \frac{Q}{c\rho}G(x,t) \tag{8}$$

where Q is the surface density of the thermal source, supposed to be constant on the heated surface, c is the specific heat and ρ is the density of the material. Here, the temperature influence function, $G(x, t)$, solution of the one dimensional equation of heat propagation, is the Green function, given by:

$$G(x,t)=\frac{1}{2\sqrt{\pi a^2 t}}\exp\left(-\frac{x^2}{4a^2 t}\right) \tag{9}$$

were $a^2=\frac{K}{c\rho}$ is the internal diffusion coefficient, with K - the thermal conduction coefficient of the material.

The schlieren method gives information about the refractive index gradient which is related to the temperature gradient. According to Eq. (9) this is given by:

$$\frac{\partial T}{\partial x}\propto\frac{x}{\theta^{\frac{3}{2}}}\exp\left(-\frac{x^2}{4\theta}\right) \tag{10}$$

where $\theta=a^2 t$.
Consequently, we can write:

$$\frac{\partial T}{\partial x}\propto\frac{x}{a^3 t^{\frac{3}{2}}}\exp\left(-\frac{x^2}{4a^2 t}\right) \tag{11}$$

The light intensity in the schlieren image is related to the temperature gradient in the glass plate as:

$$I=C_{sch}\left(\frac{1}{4\pi^2}\right)\left(\frac{2\pi}{\lambda}\right)^2 d_p^2\left(\frac{\partial n}{\partial T}\right)^2\left(\frac{\partial T}{\partial x}\right)^2=C_{sch}\left(\frac{d_p}{\lambda}\right)^2\alpha^2\left(\frac{x}{a^3 t^{\frac{3}{2}}}\exp\left[\frac{x^2}{4a^2 t}\right]\right)^2 \tag{12}$$

where d_p is the thickness of the glass plate (assumed constant during the heat transfer), $\alpha=\frac{\partial n}{\partial T}$ is the temperature coefficient of the refractive index which is a tabulated constant for any specified glass in the studied temperature range and C_{sch} is the calibration constant, with a particular value for each experimental schlieren setup.

4. Experimental setup

The heat diffusion in the glass plates was visualized and recorded by the experimental arrangement shown in Fig. 1.

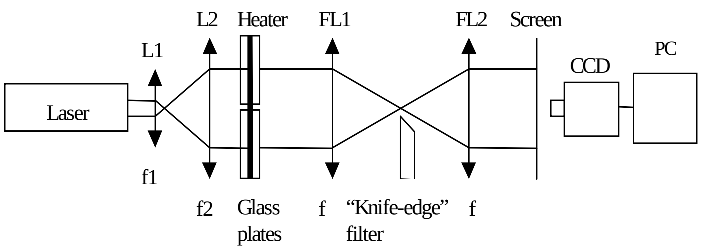


Fig. 1. Schlieren experimental setup.

The schlieren system has been set-up with a He-Ne laser as light source (P = 20mW, λ = 633nm) which provides an uniform and intense illumination. The Fourier lenses FL1 and FL2 have the same focal length, f = 430 mm. The "knife-edge" is placed in the common focal plane of the Fourier lenses, on a micrometric x-y translation stage that allows its precise positioning. This is an usual 4f spatial filtering system. The plane-parallel glass plates, made from different optical glasses, have the same dimension, 75 x 25 x 10 mm. A horizontal plane thermal source, placed on the face 75 x 10 mm of the plates, generates thermal fronts in both glasses. The schlieren patterns have been recorded using a CCD camera connected to the PC. The camera was used in a linear regime, to keep the proportionality between the gray level in the image and the input light intensity.

5. Experimental results and discussion

The schlieren images were analyzed in the frame of the theoretical model described above. For quantitative analysis of the recorded images, the proportionality between the gray level and the light intensity in the schlieren images is required. At the beginning of the heat propagation, the temperature gradient and the corresponding brightness in the schlieren images are very high close to the heater. In these conditions, the CCD camera is unable to record in the linear regime the whole range of spatial light distribution from the schlieren pattern. To avoid this inconvenient, only images recorded after sufficiently long time intervals from the beginning of the heating, were selected for quantitative analysis. In this case, the camera is not saturated for any pixel of the image. In the acquired images, the gray level was measured pixel by pixel along the heat propagation direction (x – axis), at different time intervals from the start of the heating.

We recorded and analyzed the schlieren images obtained when the glass plates were heated 3s and 5s, respectively, using the same heater. In Fig. 2 are shown the schlieren images recorded after 35s and 40s, from the start of the heating.

(a) (b)

Fig. 2. Schlieren images recorded at 35s (a) and 40s (b), from the beginning of 3s heating (a) and 5s heating (b).

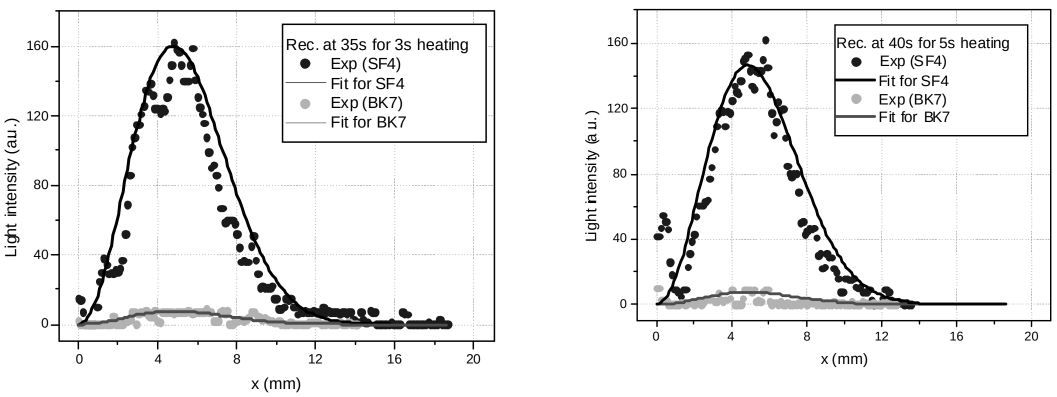
The ratio of the light intensities in the schlieren images (Eq. (12)) for the two glasses simultaneously investigated is:

$$I_1/I_2 = \left\{ \alpha_1^2 \left[\frac{x}{a_1^3 t^{\frac{3}{2}}} \exp\left(\frac{x^2}{4a_1 t} \right) \right]^2 \right\} \Bigg/ \left\{ \alpha_2^2 \left[\frac{x}{a_2^3 t^{\frac{3}{2}}} \exp\left(\frac{x^2}{4a_2 t} \right) \right]^2 \right\}. \qquad (13)$$

Using this ratio, it is possible to determine the temperature coefficient of the refractive index of one of the plates, knowing this parameter for the other one.

In our experiment, a BK7 glass plate, with K= 0.762 W/mK, ρ = 2.707 Kg/m^3 and c = 837 J/KgK, giving $a_2^2 = 3.4\times10^{-7}\, m^2 s^{-1}$, was taken as reference and a SF4 glass plate, with $K = 0.64095\, W/mK$, $\rho = 4.78\times 0^3\, kg/m^3$, $c = 410\, J/kgK$, giving $a_1^2 = 3.27\times 0^{-7}\, m^2 s^{-1}$, was studied in order to determine the temperature coefficient of its refractive index.

In Fig. 3 are shown the experimental results obtained from the analysis of the recorded schlieren images (Fig. 2) and the plot of the theoretical curves (Eq. (11)). Both the measured gray level distributions and the theoretical curves are normalized to their maxima. One can observe a small peak of the light intensity located close to the top face of the plates (x = 0) that is given by the derivative of the heated edge of the glass plates.



(a) (b)

Fig. 3. The experimental results obtained from the analysis of the schlieren images recorded at 35s (a) and 40s (b), from the beginning of 3s heating (a) and 5s heating (b) (dots), fitted with Eq. (12) (solid line).

The best fit of the light intensity in the schlieren image for the BK7 reference plate provided the value of the expression $C_{sch}\left(d_p/\lambda\right)^2\alpha_2^2$, where $C_{sch}\left(d_p/\lambda\right)^2$ is a factor common for both plates. Thus, from the best fit of the light intensity in the schlieren image of the SF4 plate, we obtained the temperature coefficient of the refractive index for this glass as $\alpha_1 = \Delta n/\Delta T = 6.6\times10^6\ K^{-1}$. This is in the range of the values given in the literature for this sort of glass, $\Delta n/\Delta T = (6.6-6.9)\times10^6\ K^{-1}$, for temperatures between 0 and 80°C, at λ=643.8 nm.

6. Conclusions

A schlieren system has been setup and used to visualize the simultaneous heat diffusion in two glass plates with the same geometry. The schlieren images were analyzed using a specific software and compared with the theoretical model of the heat diffusion in an infinite medium at a very strong and short thermal excitation by an infinite plane thermal source. By this method, taking as reference the BK7 glass plate, we determined the temperature coefficient of the refractive index for the SF4 glass plate. Despite the differences between the experimental conditions and the theoretical hypotheses (the heated medium is not infinite, the thermal excitation is not exactly a Dirac type function), a good agreement between experimental and theoretical results was found.

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