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SHORT COMMUNICATION

DETERMINATION OF PROPAGATION CONSTANTS IN A Ti:LiNbO$_3$ OPTICAL WAVEGUIDE

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The WKB and variational methods are used to determine the propagation constants in a Ti : LiNbO$_3$ waveguide with the reconstructed refractive index profile (in depth and width) from the near field measurements.

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The Wentzel, Kramers, Brillouin (WKB) approximation is used for calculating energies and tunneling probabilities through potential which varies slowly over a distance. Also, this method is one of the most used approaches for approximating the TE and TM mode spectra in optical waveguides [1-3]. The errors of effective refractive indices of TE modes determined by this approximation were investigated by Janta and Čtyroký [1] for buried and unburied gaussian index profiles.

Recently [4], a modified Hermite – Gauss – exponential (MHGE) trial field has been used for obtaining the propagation characteristics of single – mode inhomogeneous planar waveguides, based on the variational method.

In this short communication the WKB method [1] and a variational method [4] are used to determine the propagation constants in an Er $^{3+}$ - doped Ti : LiNbO$_3$ waveguide with the reconstructed unburied gaussian refractive index profile (in depth and width) from the near field measurements [5]. The recording of the near field was performed using a standard optical fiber placed at a distance < 3λ, λ = 1.53 μm being the wavelength of the laser. For the displacement of the optical fiber we used an electrostrictive actuator controller commanded by a computer. In this case we have an inhomogeneous optical waveguide where the refractive index varies slowly over a distance comparable to a wavelength.

The scalar – wave equation of the waveguide is given by

$$\Delta\Psi(x,y)+\left[k_0^{\,2}n^2(x,y)-\beta^2\right]\Psi(x,y)=0, \tag{1}$$

where n(x,y) is the refractive index profile, β is the propagation constant and k_0 is the free space wave number. We take a separable variable solution for the transverse electric field Ψ(x,y) of the mode, [Ψ(x,y) = Ψ(x) Ψ(y)] and obtain two unidimensional wave equations

$$\frac{d^2\Psi(x)}{dx^2}+\left[k_0^{\,2}n^2(x)-\beta_x^{\,2}\right]\Psi(x)=0\,,\;\frac{d^2\Psi(y)}{dy^2}+\left[k_0^{\,2}n^2(y)-\beta_y^{\,2}\right]\Psi(y)=0\,, \tag{2}$$

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where the refractive indices n(x) and n(y) are given by the relations [5]

$$n(x) = n_s + \Delta_x \exp\left[-\left(\frac{x}{d_x}\right)^2\right], n(y) = n_s + \Delta_y \exp\left[-\left(\frac{y}{d_y}\right)^{1.85}\right], \tag{3}$$

$\Delta_x = 0.0012$ and $\Delta_y = 0.00108$ are a measure of the increase in refractive indices, $d_x = 3.5$ μm and $d_y = 6.5$ μm are the effective depth of diffusion and $n_s = 2.27$ is the index of the substrate for a wavelength of $\lambda = 1.53$ μm. From these parameters we compute the normalized frequencies V_x and V_y

$$V_x = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 d_x^{\,2}(n_{1x}^{\,2} - n_s^{\,2})} = 1.0609,\; V_y = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 d_y^{\,2}(n_{1y}^{\,2} - n_s^{\,2})} = 1.86914,$$
$$n_{1x}^{\,2} - n_s^{\,2} = 2n_s\Delta_x,\; n_{1y}^{\,2} - n_s^{\,2} = 2n_s\Delta_y, \tag{4}$$

where $n_{1x} = n_s + \Delta_x = 2.2712$ and $n_{1y} = n_s + \Delta_y = 2.27108$ are the maximum refractive indices.

The adimensional form of the equations (2) can be written as eigenvalue equations

$$\left[-\frac{d^2}{dX^2} + V_x^{\,2}\left(1 - e^{-X^2}\right)\right]\Psi(X) = U_x^{\,2}\Psi(X),\ \mathrm{X} > 0, \tag{5a}$$

$$\left[-\frac{d^2}{dY^2} + V_y^{\,2}\left(1 - e^{-Y^{1.85}}\right)\right]\Psi(Y) = U_y^{\,2}\Psi(Y),\ \mathrm{Y} > 0, \tag{5b}$$

$$\left[-\frac{d^2}{dX^2} + V_x^{\,2}\left(1 + \frac{n_s}{2\Delta_x} - \frac{n_{ax}^{\,2}}{2n_s\Delta_x}\right)\right]\Psi(X) = U_x^{\,2}\Psi(X),\ \mathrm{X} < 0, \tag{5c}$$

$$\left[-\frac{d^2}{dY^2} + V_y^{\,2}\left(1 + \frac{n_s}{2\Delta_y} - \frac{n_{ay}^{\,2}}{2n_s\Delta_y}\right)\right]\Psi(Y) = U_y^{\,2}\Psi(Y),\ \mathrm{Y} < 0, \tag{5d}$$

$$\Psi(X) = (p_1 + p_2)e^{-\frac{(p_2+p_3)^2}{2} - (X-p_2)(p_2+p_3-\frac{1}{p_1+p_2})},\ \mathrm{X} > \mathrm{p}_2, \tag{6a}$$

$$\Psi(X) = (p_1 + X)e^{-\frac{(X+p_3)^2}{2}},\quad 0 \le \mathrm{X} \le \mathrm{p}_2, \tag{6b}$$

$$\Psi(X) = p_1 e^{-\frac{p_3^{\,2}}{2} + \frac{(1-p_1p_3)X}{p_1}},\quad \mathrm{X} \le 0, \tag{6c}$$

where $n_{ax} = n_s$, $n_{ay} = 1$, $X = x / d_x$, $Y = y / d_y$, $U_x^2 = d_x^2 (k_0^2 n_{1x}^2 - \beta_x^2)$, $U_y^2 = d_y^2 (k_0^2 n_{1y}^2 - \beta_y^2)$ and $b_x = 1 - U_x^2/V_x^2$, $b_y = 1 - U_y^2/V_y^2$ are the normalized propagation constants. U_x and U_y are the modal parameters of the index profile.

The trial MHGE wave function $\psi(X)$ is given by [4] where p_1, p_2, p_3 are variational parameters and the relations (6) satisfy the boundary conditions. We have similar relations for the variable Y.

The WKB – analysis yields the effective refractive index N_m of the m^{th} mode as the solution of TE mode equations [1, 2]

$$\frac{2\pi}{\lambda}\int_0^{x_t}\sqrt{n^2(x)-N_{mx}^2}\,dx = m\pi+\frac{\pi}{4}+\arctan\left[\sqrt{\frac{N_{mx}^2-n_{ax}^2}{n_{1x}^2-N_{mx}^2}}\right],\ \mathrm{x_t}=d_x\sqrt{\ln\frac{\Delta_x}{N_{mx}-n_s}}$$

$$\frac{2\pi}{\lambda}\int_0^{y_t}\sqrt{n^2(y)-N_{my}^2}\,dy = m\pi+\frac{\pi}{4}+\arctan\left[\sqrt{\frac{N_{my}^2-n_{ay}^2}{n_{1y}^2-N_{my}^2}}\right],\ \mathrm{y_t}=d_y\left(\ln\frac{\Delta_y}{N_{my}-n_s}\right)^{\frac{1}{1.85}} \quad (7)$$

where m = 0, 1, 2, …, M – 1 is the mode number, M is the total number of modes, x_t and y_t are the turning points defined by the relations

$$n^2(x)-N_{mx}^2=0\,,\,n^2(y)-N_{my}^2=0\,, \quad (8)$$

$n_{ax} = n_s$, $n_{ay} = 1$ are the superstrate refractive indices.

Our modes are unburied [1] because we have a single real turning point for each x or y directions. We have real solutions of the equations (7) only for m = 0, which confirm the single mode behaviour of our waveguide. The normalized propagation constants b_{mx} and b_{my} are given by the relations

$$b_{mx}=\frac{\left(\frac{\beta_{mx}}{k}\right)^2-n_s^2}{n_{1x}^2-n_s^2}\,,\,b_{my}=\frac{\left(\frac{\beta_{my}}{k}\right)^2-n_s^2}{n_{1y}^2-n_s^2}\,,\,\mathrm{N_{mx}}=\frac{\beta_{mx}}{k}\,,\,\mathrm{N_{my}}=\frac{\beta_{my}}{k}\,, \quad (9)$$

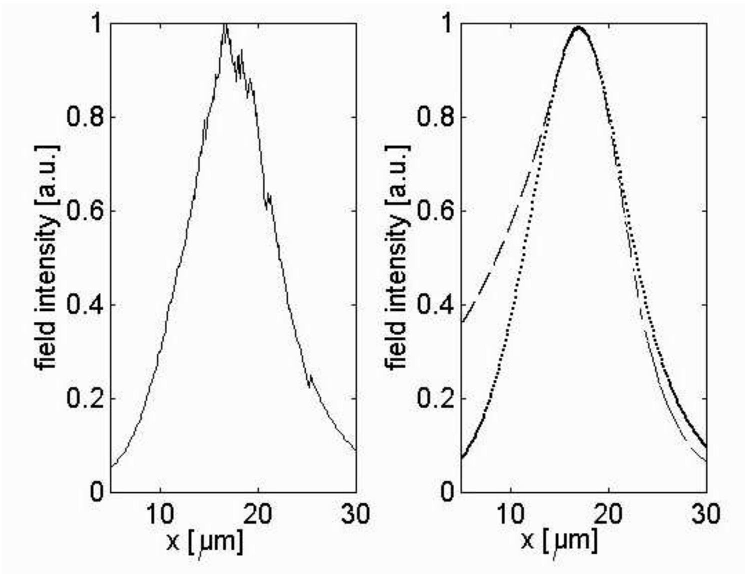

Fig. 1. Measured intensity field (solid line) I and superposition between the processed smoothing curve (dotted line) and variational calculated (dashed line) intensity fields, versus the width distance x.

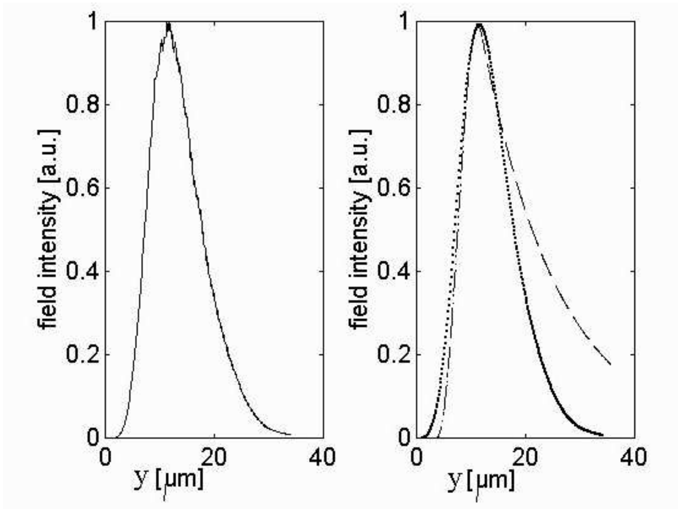

Fig. 2. Measured intensity field (solid line) I and superposition between the processed smoothing curve (dotted line) and variational calculated (dashed line) intensity fields, versus the depth distance y.

We have calculated for an Er^{3+} - doped Ti : $LiNbO_3$ waveguide with the reconstructed unburied gaussian refractive index profile (in x - width and y - depth) from the near field measurements [5], the modal parameters U_x , U_y of the index profile, the effective refractive indices N_x ,N_y of the m = 0 mode, the normalized propagation constants b_x , b_y, and the propagation constants β_x , β_y , by using WKB (N_x = 2.27009073, N_y = 2.27000443, b_x = 0.0756098, b_y = 0.0041019, β_x = 9.32248 μm^{-1}, β_y = 9.32213μm^{-1}) and MHGE variational (U_x^2 = 0.9961, U_y^2 = 3.4226, N_x = 2.27014, N_y = 2.27002, b_x = 0.1150, b_y = 0.0203, β_x = 9.3227μm^{-1}, β_y = 9.3222μm^{-1}) methods.

The variational values of the calculated parameters are more accurate in comparison with those given by WKB method [4] due to the abrupt change in the refractive index at x = 0, y = 0 [2]. One observe that in both approximations $\beta_x > \beta_y$.

A comparison between the measured [5], processed [5] and calculated (variational) intensity fields is shown in Fig. 1 and Fig. 2 (the intensity field I is proportional with ψ^2). The discordance from Fig. 1 is only from the X< 0 contribution to the wavefunction.

The agreement between the measured [5] and our variational intensity is quite good in the range of the maximum intensity fields.

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