Journal of Optoelectronics and Advanced Materials Vol. 6, No. 3, September 2004, p. 1005 - 1008

# FIRST ORDER REVERSAL CURVES DIAGRAM DEDUCED BY A SHEPARD METHOD FOR BIVARIATE INTERPOLATION OF SCATTERED DATA

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A new identification strategy of the distribution in Preisach-type models is described in this paper. The mixed second derivative of the First Order Reversal Curves (FORC) is evaluated after an interpolation in the weighted least-squares sense and a Shepard interpolation method is applied in order to replace the initial irregular grid with a regular one. The main advantages of this strategy are that it can deal with FORC curves on irregular grids and when the experimental errors are important, the weighted least-squares sense of this method increases the precision of the FORC diagram. The parameter set values given in this paper have been established using FORC curves obtained by simulations with a Complete-Moving-Hysteresis model on known Preisach distributions.

(Received April 19, 2004; accepted June 22, 2004)

*Keywords*: FORC, FORC diagram, Preisach model, Shepard algorithm, Irregular grids

## 1. Introduction

Magnetic interactions between particles are very important for both theoretical and experimental studies of magnetic recording media. For example, Preisach-type models can predict the noise level in such media [1] if the parameters of this model are well identified. Further information about the individual magnetization processes can easily be obtained via this interaction field. The classical method used in the characterization of magnetic interactions is the $\delta M$ method [2]. Accurate micromagnetic simulations of interacting Stoner-Wohlfarth particles suggest that this method is appropriate for the study of the mean interaction field [3]. Moreover, recent studies have demonstrated that measurement of first order reversal curves (FORC) can increase the precision of interaction measurements. Moreover, such measurements permit the decoupling of the effects of mean interaction field and of interaction field variance [4], which is not possible in a $\delta M$ -based method. Finally, the FORCs do not require an alternating magnetic field to demagnetize the sample, as is the case in the $\delta M$ method. Relatively few algorithms have been proposed in order to obtain the FORC diagram from a family of FORC curves. The first of these, proposed by C. R. Pike and co-workers [4] consists in a local bivariate polynomial interpolation with six coefficients on regular field grids [4]. Better results have been obtained on irregular field grids, based on the condition that each field step produce the same variation of magnetic moment in the major hysteresis loop [5]. Relatively recent Preisach models for strongly interacted ferromagnetic particulate systems use more complex distributions [6] which are more difficult to identify from a reasonable number of experiments. The FORC diagram strategy we propose here can deal with fully irregular grids and is based on a local interpolation of the FORC curves in the weighted least-squares sense, using 10-

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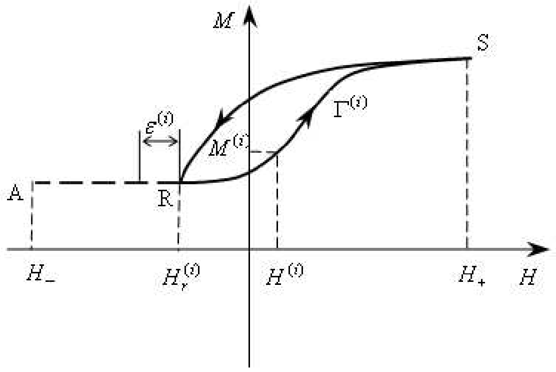
parameter polynomial functions. Using this strategy one can obtain accurate FORC diagrams with a minimal set of experimental measurements.

## 2. Measurement of FORC curves

Each FORC curve, $\Gamma^{(i)}$, begins with the saturation of the sample using a positive applied, $H_+$. The field is then ramped down to a reverse field $H_r^{(i)}$ (see Fig. 1 a)). The FORC $\Gamma^{(i)}$ consists of measurements of the magnetization $M^{(i)}$ at different increasing field values $H_1^{(i)}, H_2^{(i)}, \ldots, H_{N_i}^{(i)}$ back up to saturation (in the magnetization process R $\rightarrow$ S of Fig. 1 a)).



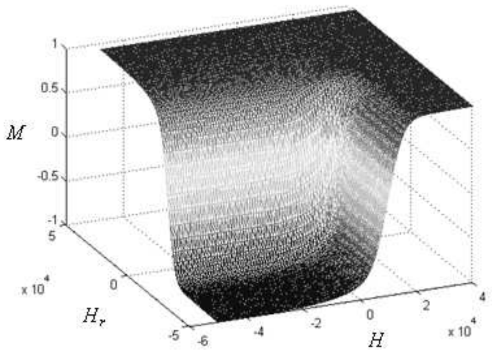




Fig. 1. a) Experimental measurement of a FORC. After a positive saturation (S) the field is ramped down to the reversal field $H_r^{(i)}$ ($S \rightarrow$R). The FORC $\Gamma^{(i)}$ consists of a measurement of the magnetization $M^{(i)}$ at different increasing field values $H^{(i)}$ back up to saturation (R$\rightarrow$S); b) Simulated FORC curves with a Complete – Moving – Hysteresis model.

An artificial set of points corresponding to the horizontal segment AR (see Fig. 1 a)) is added in order to obtain the reversible part of the FORC diagram. In order to generate these points we can use a step $\varepsilon^{(i)}$ comparable with the average field step in the real RS process. All the points from all possible $\Gamma^{(i)}$ FORCs compose a surface $M(H_r, H)$ on the $[H_-, H_+]\times[H_-, H_+]$ Preisach-plane (Fig. 1 b)). In following sections we will describe a numerical algorithm which can be employed in order to obtain the FORC diagram

$$\rho(H_r, H) = -\frac{\partial^2 M(H_r, H)}{\partial H_r \partial H} \tag{1}$$

## 3. Evaluation of the FORC diagram

For each node of the Preisach plane we define a nodal function

$$f^k(H_r, H) = \sum_{j=1}^{10} c_j^k \cdot \varphi_j(H_r, H) \tag{2}$$

where the functions $\varphi_j \in \{1, H_r, H, H_r^2, H^2, H_r H, H_r^3, H^3, H_r^2 H, HH_r^2\}$. The nodal function $f^k$ is obtained via a weighted least-square (WLS) method using only the first $N_c$ neighbours of each $k$-node. If we denote with $\mathrm{N}^k$ the collection of first $N_c$ neighbours of the $k$-node,

$$Q_k = \sum_{i \in \mathrm{N}^k \cup \{k\}} w_k(H_{ri}, H_i)\left[M_i - \sum_{j=1}^{10} c_j^k \cdot \varphi_j(H_{ri}, H_i)\right]^2 \to \min \tag{3}$$

In contrast with the Shepard method which keeps the value $M_i$ for the nodal function in the $k$-node [7], we included the $k$-node in the minimization condition. The weight functions $w_k$ must be unitary in the $k$-node and must decrease gradually around this node. We obtained good results with gaussian weight functions with the same standard deviations for all the nodes but even the constant function $w_k(H_r, H) = 1$ can be used instead if the magnetization variations on the FORC curves are not very steep. Decreasing the standard deviation of the weight function can increase the local character of the interpolant and leads to a better identification of the steep regions on the FORC diagram. From (3) we obtain a linear system consisting of ten equations with $c_j^k$ , $j = \overline{1,10}$ as unknowns:

$$\frac{\partial Q_k}{\partial c_j^k} = 0\,,\ j = \overline{1,10} \tag{4}$$

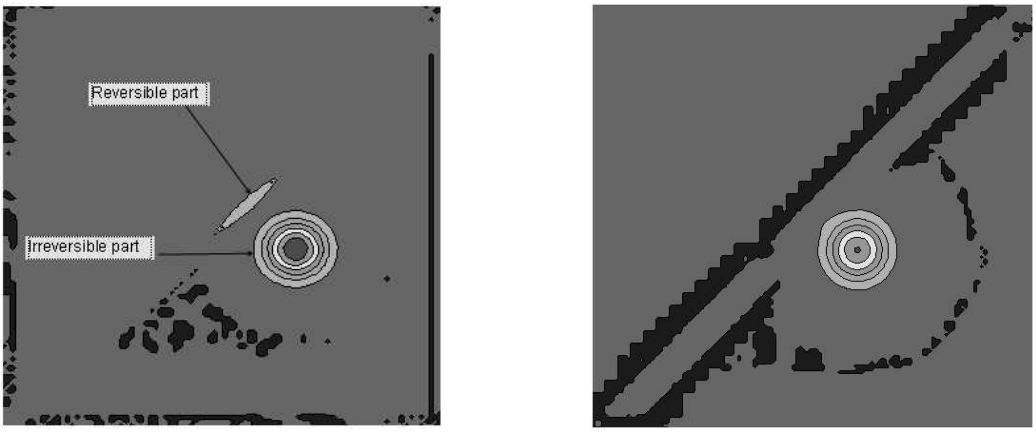
With these coefficients calculated, we can evaluate now the mixed second derivative (1) in the $k$-node:

$$\rho_k = -c_6^k - 2c_9^k H_{rk} - 2c_{10}^k H_k \tag{5}$$

In Fig. 3 we show the FORC diagrams obtained with a Pike method (b) and with the proposed WLS algorithm (a) for $w_k = 1$.

We can highlight some features of this WLS method relative to the Pike method: a) a versatile data input; b) a better identification of the distribution; c) one can try more weight functions $w_k$ and different values for $N_c$ until the desired precision is reached; d) the base functions $\varphi_j$ can also be replaced, for example with cosine series functions [8].

FORC diagram can be directly used in investigating the physical mechanisms giving rise to hysteresis in magnetic systems [4]. Moreover, we can use the operative diagram like a Preisach distribution for our magnetic system. For the systems with non zero moving parameter, the geometric transform of the FORC diagram into the operative plane which vanishes its negative regions and symmetrize it with respect to the second bisector of the Preisach plane can be successfully used to identify the moving parameter [5]. This new strategy for the identification of moving parameter in Preisach models leads to a complete nonparametric identification procedure for this kind of models. In order to predict different magnetization processes (which involve different regions in the Preisach plane) we need a relatively uniform distribution of the points in the Preisach plane. We can use any method to interpolate the initial FORC diagram but if we deal with complex systems which can have complicated FORC diagrams (multiple peaks, asymmetries) or if the nodal density varies widely in the Preisach plane, a flexible method is necessary. Very good results we obtained with a modified Shepard algorithm with 10-parameter cosine series nodal functions [8].




a) b)

Fig. 3. Calculated FORC diagram from the data of Fig. 2 with the weighted least-squares algorithm (a) and with a Pike-type algorithm (b). $w_k = 1$, $N_c = 28$.

## 4. Conclusions

In this paper, we have presented a strategy which improves the identification of FORC diagram using a Shepard algorithm. The use of a weighted least-square method on irregular grids can increase significantly the precision of the FORC diagram without increasing the number of experiments. Varying the standard deviation of the weight function in an appropriate manner we can obtain a better identification of the FORC diagram.

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