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FRACTALS AND DOMINANT UNIQUENESS PARAMETERS IN THE DESCRIPTIONS OF SOME ELECTRICAL ENGINEERING MATERIALS AND COMPONENTS

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Starting from the finding that the most intriguing feature of the fractals applications in physics refers to the dominant (prevalent) character of the specimen sizes in the description of some mechanical parameters or even of some electrical parameters (as e.g. the electrical capacitance of some capacitors), this work analyzes the presence of other dominant uniqueness parameters in the descriptions of some characteristic parameters of the materials intended to electrical engineering. There were pointed out the following dominant uniqueness parameters in the descriptions of some magnetic materials: a) the initial relative permeability μ_{ri}, which determines with sufficiently good accuracy – by means of some parabolic correlations in $\log \mu_{ri}$ - the parameters of the hysteresis cycle, as well as the dispersion parameters (the so-called "limit laws") of spinelic ferrimagnetic materials, b) the oscillations frequency, which determines (by means of some power laws) the viscous friction coefficient of the magnetization domains oscillations, as well as the quality factor of sound propagation through certain elastic media.

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1. Introduction

Unlike the mathematical systems (geometrical figures, symmetry groups, polynomes, etc), whose elements are determined by a given (specific) number of uniqueness parameters (e.g. the lengths of the sides, or the lengths of 2 sides and the angle between them, for arbitrary triangles), the number n_U of uniqueness parameters U_i corresponding to a physical state (or process) depends on the required accuracy, increasing with the accuracy level [e.g., the state of the air can be described roughly by the temperature and pressure, more accurately adding the humidity, with an increased accuracy adding also the CO_2 content, etc.].

If the physical dimension of a parameter specific to the studied state (or process) is:

$$[P] = \prod_{i=1}^{n_U} [U_i]^{\alpha_i} , \tag{1}$$

then 2 states (or processes) *Σ'*, *Σ"* are named similar if the values of the parameters $\{U_i \mid i = 1, n\}$ and *P* corresponding to these states fulfill the relation [1], [2]:

$$\frac{P'}{P''} = \prod_{i=1}^{n_U} \left(\frac{U_i'}{U_i''} \right)^{\alpha_i} . \tag{2}$$

Some of the uniqueness parameters could be similitude criteria, i.e. non-dimensional parameters: [s]=1, with equal values: *s' = s"* in all similar states or processes. In the macroscopic physics, the similitude indices α_i are integers or semi-integers, very seldom intervening other

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rational values (as 2/3 in the reversed Langmuir law). This finding is valid also for the relatively slow (laminar) flows, the Poiseuille-Hagen equation (valid for $\mathrm{Re} < \mathrm{Re}_{cr}$) giving:

$$f = \frac{2d}{\rho v^2} \cdot \frac{dp}{dx} = \frac{A}{\mathrm{Re}} \ , \quad \text{where:} \qquad \mathrm{Re} = \frac{\rho \cdot d \cdot v}{\eta} \ , \tag{3}$$

and A is a constant depending on the pipe cross-section shape (A = 64 for circumferences, etc).

For turbulent flows, the describing equations become considerably more intricate, the Darcy similitude criterion f depending concomitantly on the Reynolds number Re, on the relative roughness h/d, on the ratio of the dynamic viscosity in the fluid volume and at the pipe walls η_b/η_w, resp., etc. In certain particular turbulent flows, the flow parameters are described by means of a unique **dominant** parameter, e.g.: a) **only** by the Reynolds number for the smooth pipe flow regime [3], [4]:

$$f = (5000\,\mathrm{Re})^{-0.2} \ , \ f = 0.0032 + \frac{0.221}{\mathrm{Re}^{0.237}} \ , \qquad \text{or:} \qquad \frac{1}{\sqrt{f}} = 2\log_{10}\left(\mathrm{Re}\sqrt{f}\right) - 0.8 \tag{4}$$

(when the similitude criteria are used as uniqueness parameters, the basic relations are not necessarily expressed by power laws), or: b) **only** by the relative roughness h/d for very quick (squared) flows.

In order to explain the rather strange similitude indices intervening in the description of turbulent flows parameters, Kolmogorov [5] proposed a hierarchical structure of vortices, the energy being injected firstly in the largest vortices and transferred in a cascade from the larger to the smaller vortices, up to the smallest ones, where the energy is dissipated. This hypothesis was strengthened by the contributions of Mandelbrot [6]. Taking into account that (for samples of different sizes) it is difficult to have exactly equal values of all uniqueness parameters (excepting the sample size), in order to fulfill the classical relation: $P = C \cdot L^D$ (with a non-rational similitude index D, called *the fractal dimension*) of the fractal theory [7], it results that the physical applications of the fractal theory correspond to *the prevalence (dominance) of the size (length) uniqueness parameter*. If for different size domains, the values of D belong to a set (discrete or continuous) of real numbers, the corresponding physical structure is called *multi (poly)fractal.*

In the last years, there were published several identifications of multi (poly)fractal structures, as those corresponding to: (i) the fracture surfaces of metals [8], (ii) fracture surfaces of concrete specimen [9], (iii) several parameters of disordered and porous media, aggregates, polymers and membranes [10], (iv) electrode surfaces (of fractal dimension $D_s \approx 2.6$) of some supercapacitors [11], [12], etc.

2. Experimental part

Mainly, the spinelic ferrimagnetic materials used in electronics were studied in this work. The accomplished analysis pointed out the possibility to classify many radio-frequency ferrimagnetic materials in technological series corresponding to the following manufacturing technologies:

a) classical manufacturing technology and widely variable (*Li-Zn, Ni-Zn, Mn-Zn*) *compositions*, involving the German (Siemens) materials 7U15, 20K12, 80K1, 300M11, 550M25, M1100, 1100N22, 2000T7, 2000N27, 2000T26, 2000N82 [13], the Hungarian materials ON10, M1100 [14], and the Romanian materials F1, D1, D2, D3, D12, D42, D5, E2, A1, A11, A21m A3, A41, A5 [15],

b) special technologies (programmed depressurization during the cooling, wet chemical preparation, densely sintering, etc) *in manufacturing MnZn ferrites* (named in following the HM series), involving the Dutch (Philips) materials of grain sizes: 0.21 μm ($\mu_{ri} \approx 120$), 0.87 μm ($\mu_{ri} \approx 500$), and 2.7 μm ($\mu_{ri} \approx 1200$) [16], the Russian materials 700 HM, 1000 HM3, 1500 HM2, 1500 HM3, 2000 HM, 3000 HM, HMC1, 4000 HM, 6000 HM [17], and the Romanian materials A22 and A7 [15],

c) stable composition ($Fe_2O_3 = 49.2 \pm 0.8$, NiO = 17.1 ± 1.1 , ZnO = 33.5 ± 0.5 molar percentage) *and variable* (T_s = 1160 … 1295^{0}C) *sintering temperatures* (named in following the HH series), involving the Russian materials: 200 HH, 400 HH, 600 HH, 1000 HH, and 2000 HH [17],

d) *special composition* (of the perminvar type), involving the materials: German (Siemens) U17 [13], the Russian materials: 10 ВЧ1, 30ВЧ2, 50 ВЧ2, 60 HH, 100 ВЧ, 100 HH, 150 ВЧ, 200 HH2, 300 HH [17], and the Romanian materials F2, D41, and F4 [15].

The hysteresis parameters, as well as the dispersion parameters of the Romanian ferrimagnetic materials were measured according to the experimental methods presented by the works [18].

3. The initial permeability – as a dominant uniqueness parameter

3.1. Descriptions of the magnetic dispersion parameters

Starting from the Mikami's model [19] of the magnetic permeability dispersion:

$$\chi' = \frac{\chi_{of}}{\left(1 - f^2 / f_0^2\right) + i \cdot f / f_{rel}} + \frac{\chi_{oC}}{1 + \left(i\, f / f_1\right)^{\beta}} \ , \tag{5}$$

Naito [20a] (see also Snoek [20b]) obtained the "limit" laws describing the "resonance" frequency f_0 and the frequency f_1 for which the imaginary part of the complex susceptibility, due to the spin rotation motion (second term in relation (4)) reaches its maximum, in terms of the contributions of the forced oscillations of the magnetization domains walls (χ_{of}) and the spin rotation motion (χ_{oC}), resp., to the static magnetic susceptibility, as:

$$f_0 = 11 \times \chi_{of}^{-1.37} \quad (GHz), \qquad \text{and:} \qquad f_1 = \frac{8\, GHz}{\chi_{oC}} \ . \tag{6}$$

The study of the magnetic dispersion parameters of the above indicated Dutch (Philips), German (Siemens), Hungarian, Russian and Romanian industrial soft ferrimagnetic materials, allowed us to find that the main technical characteristic frequencies (the "maximum" frequency f_{max} and the "critical" frequency f_{crit}, defined as frequencies for which the tangent of magnetic losses $tan\delta$ reaches the values: $tan\delta(f_{max})$ = 0.02 and $tan\delta(f_{crit})$ = 0.1, respectively) can be rather accurately described in terms of $\log \mu_{ri}$ (where μ_{ri} is the relative initial permeability) by means of the parabolic correlation:

$$\log f_{ch} = a_{ch} \log^2 \mu_{ri} + b_{ch} \log \mu_{ri} + c_{ch} \,. \tag{7}$$

The obtained results concerning the values of the parameters a_{ch}, b_{ch}, c_{ch} of the above correlation for different technological series, as well as the ratio R of the extreme values of the characteristic frequencies corresponding to the considered series, the slopes s and the curvatures K at the ends of the initial permeability range corresponding to each series, and the square mean error $\sigma(\Delta f_{ch} / f_{ch})$ - for each characteristic frequency and series – are indicated in Table 1.

Table 1. Characteristic Parameters of the Magneto-dispersive Correlations (7).

Defining features of correlations		Characteristic Parameters						$\sigma(\Delta f_{ch}/f_{ch})$, %
f_{ch}	Series of materials	R	a_{ch}	b_{ch}	c_{ch}	Slope s	Curvature K	
f_{max}	Classical technology	8000	- 0.28	- 0.16	+ 2.28	- 0.63 … … - 2.01	- 0.330 … … - 0.049	14.6 %
	HM	200	- 0.87	+ 3.2	- 1.70	- 1.77 … … - 3.10	- 0.208 … … - 0.051	31 %
	HH	67	- 1.0	+ 2.9	- 1.19	- 1.7 … … - 3.7	- 0.261 … … - 0.039	47 %
	Perminvar	22	- 0.21	- 0.35	+ 2.81	- 0.78 … … - 1.33	- 0.210 … … - 0.013	18 %
f_{crit}	HM	20	≅ 0	- 1.68	+ 5.38	≅ - 1.68	≅ 0	28 %
	HH	15	- 0.30	+ 0.45	+ 1.06	- 0.93 … … - 1.53	- 0.236 … … - 0.098	23 %
	Perminvar	20	- 0.29	- 0.01	+ 2.76	- 0.61 … … - 1.38	- 0.368 … … - 0.120	19 %

3.2. Descriptions of the magnetic hysteresis parameters

There were studied the dependencies (for different technological series) of the hysteresis parameters: a) the coercive magnetic field strength H_c , b) the remanent magnetic induction B_{rem}, and: c) the "closure" magnetic induction B_{cl}, defined as:

$$B_{cl} = \frac{B_d(H_{cl}) - B_r(H_{cl})}{2} \,, \qquad \text{where:} \qquad B_d(H_{cl}) - B_r(H_{cl}) = 0.001\ \mathrm{T}\,, \tag{8}$$

$B_d(H_{cl})$ and $B_r(H_{cl})$ being the magnetic inductions on the demagnetization and remagnetization branches, resp. of a stable hysteresis loop, corresponding to the same magnetic field strength H_{cl}.

We have found that the different magnetic hysteresis parameters p_h can be also sufficiently accurate described by means of some parabolic correlations of the type:

$$\log p_h = a_h \log^2 \mu_{ri} + b_h \log \mu_{ri} + c_h \ . \tag{9}$$

The obtained results were synthesized in Table 2. Some physical interpretations of the obtained results were reported in our preliminary work [21].

Table 2. The characteristic parameters of the correlations: $\log p_h = a_h \log^2 \mu_{ri} + b_h \log \mu_{ri} + c_h$.

The type of the hysteresis parameters	The type of technological series	Characteristic parameters			$\sigma(\Delta p_h / p_h)$, %
		A_h	b_h	c_h	
H_c	Classical technology	- 0.156	- 0.133	0.498	17.91 %
	HM	- 0.502	2.681	- 2.131	12.81 %
	HH	- 0.733	3.175	- 1.475	7.25 %
	Perminvar	- 0.021	- 0.804	4.094	11.93 %
B_{rem}	Classical technology	- 0.277	1.265	1.910	18.88 %
	HH	- 0.210	1.040	1.647	6.29 %
	Perminvar	- 0.583	2.247	1.174	18.77 %
B_{cl}	Classical technology	- 0.190	1.018	2.242	19.29 %
	HM	$\cong 0$	0.020	3.500	6.64 %
	HH	$\cong 0$	0.156	2.924	7.59 %
	Perminvar	- 0.510	2.076	1.499	9.17 %

4. The oscillations frequency – as a dominant uniqueness parameters

4.1. The validity intervals of the frequency power laws

It is well-known that the dielectric materials, as well as the elastic materials, present multiple distinct relaxations. E.g., in the case of metals (elastic materials), Cl. Zener [22] has shown that there are at least 6 such relaxations, due to: (i) pairs of solute atoms, (ii) grain boundaries, (iii) twin boundaries, (iv) interstitial solute atoms, (v) transverse thermal currents, and: (vi) intercrystalline thermal currents, covering the frequency domain between 10^{-12} *Hz* and 10^{6} *Hz*. Taking into account that the increasing and decreasing sides of the relaxation maximums of the plot: $\log(\tan\delta) = F(\log f)$ are rather abrupt, the frequency fields located around the inflexion points of the $\log(\tan\delta) = F(\log f)$ plots have an approximate linear shape and correspond to frequency intervals of 1 … 2 magnitude orders. These are the validity intervals of the frequency power laws of the type: $\tan\delta$ (or Q) $\propto f^n$. For the seismic waves, Anderson and Minster [23] have shown that the validity interval of the frequency power law: $Q \propto f^n$ corresponds to the condition: $\tau_M^{-1} << \omega << \tau_m^{-1}$, where

$\tau_M \approx 10^4$s and $\tau_m \sim 1$ s, therefore the amplitude of this interval corresponds to approximately 2 magnitude orders (10^2) (see also [24]).

The accomplished study [25] pointed out that: a) the validity interval of a frequency power law for the magnetic dispersion has the amplitude of almost 1 magnitude order, b) the best agreement for a frequency power law corresponds to the viscous friction coefficient corresponding to the oscillations of the magnetization domains walls: $r \propto f^n$, c) the validity domains of the frequency power laws corresponding to the viscous friction coefficient of the magnetization domain walls present the maximum technical interest in Electronics (as well as the frequency intervals corresponding to the seismic waves in the case of the elastic waves).

4.2. Are the frequency power laws a "fingerprint" of some fractal structures?

The frequency power laws met for some frequency intervals of the magnetic permeability dispersion, as well as for some frequency intervals (e.g. the "seismic" ones) of the elastic dispersion, are somewhat similar to the power laws corresponding to the size effects of the extensive parameters corresponding to some fractal structures.

Despite this similitude, it is not an easy task to explain the frequency power laws. There are though some arguments to interpret these frequency power laws as a consequence of a certain (restricted) fractal-type structure:

a) the geometrical similitude (and self-similarities, i.e. fractals) *represents a particular case of the general theory of the physical similarities* [1], [2]

b) the existence of a fractal structure could imply some frequency power laws

Taking into account that for the oscillations of frequency f propagating with the velocity v in a sample of characteristic size L, formed by grains, pores or magnetic domains of characteristic size l, the 2 irreducible similarity criteria predicted by the Buckingham's theorem [1] (the number of irreducible similarity criteria is equal to the difference of the numbers of uniqueness parameters and of active fundamental quantities, respectively) are: L/l and $f.l/v$, the expressions of the state parameters (extensive parameters or intensive ones, as the losses tangent, etc) can be written as: $p = F(L/l, f.l/v)$.

If the function $F(x)$ is differentiable and self-similar in both its arguments:

$$F(x_2)/F(x_1) = \Phi(x_2/ x_1) \ , \tag{10}$$

then [2], [26]:

$$p = (L/l)^{n_1} \cdot (f \cdot l/v)^{n_2} \ , \tag{11}$$

therefore a fractal-type power law for the size effects corresponding to the extensive parameters, and a frequency power law for the intensive parameters and even extensive ones of the same system subject to oscillations of different frequencies.

c) *the presence of some classical (geometrical) fractal structures could lead to the generation of harmonics and/or subharmonics of the basic oscillations*

It is well-known also that: (i) any discontinuity in an electrical transmission line will excite certain higher-order waves [27], (ii) the Cantor-like (fractal) structures determine a generation of subharmonics [28]. That is why the generation of some "fractal" frequency structures leading to the found frequency power laws has to be studied in following.

5. Conclusions

The accomplished study pointed out the possibility to use some dominant uniqueness parameters: (i) the initial magnetic permeability, and: (ii) the oscillations frequency, resp. in order to describe accurately some technical parameters of the magnetic dispersion and of the hysteresis cycle, as well as the viscous friction coefficient r of the magnetization domains walls oscillations (by means of a frequency power law), the real permeability and the tangent of magnetic losses, etc.

While the rather accurate descriptions corresponding to the initial permeability prevailing role are due to the classical "magnetic" order, the frequency power law descriptions of some magnetic dispersion parameters seem to be related to some fractal-type structures. In order to clarify the relation between the frequency power laws and the fractal structures, the basic features of the harmonic and subharmonic generation on fractal structures have to be studied in detail.

References

[1] A. A. Gukhman, Introduction to the Theory of Similarity, Academic Press, New York, 1965.
[2] G. I. Barenblatt, Dimensional Analysis, Gordon and Breach, New York, 1987.
[3] A. W. Date, International Journal Heat and Mass Transfer **17**, 845, (1974).
[4] D. Iordache, Selected Works of Numerical Physics, Printech Publishing House, Bucharest, 2000, vol.1, p. 210.
[5] A. N. Kolmogorov, C.R. Acad. Sci. URSS **31**, 538, (1941) (translated in S. K. Friedlander, L. Topper, eds., Turbulence Classic Papers on Statistical Theory, Interscience Publ., New York, 1961); b) A. N. Kolmogorov, J. Fluid Mechanics **13**, 82, (1962).
[6] a) B. B. Mandelbrot, J. Fluid Mechanics **72**, 401, (1975); b) B. B. Mandelbrot, Géométrie fractale de la turbulence, Comptes Rendus (Paris) **282A**, 119, (1976).
[7] a) B. B. Mandelbrot, The fractal geometry of nature, W. H. Freeman, New York, 1982; b) B. B. Mandelbrot, Multifractals: $1/f$ noise, Springer, New York, 1992.
[8] B. B. Mandelbrot, D. E. Passoja, Nature **308**, 721, (1984).
[9] a) A. Carpinteri, B. Chiaia, Materials and Structures, **28**, 435, 1995; b) A. Carpinteri, B. Chiaia, Chaos, Solitons and Fractals **8**(2) 135, (1997).
[10] J. F. Gouyet, Physique et structures fractales, Masson, Paris – Milan – Barcelone, 1992.
[11] R. Richner, S. Müller, M. Bärtschi, R. Kötz, A. Wokaun, New Materials for Electrochemical System **5**(3) 297, (2002).
[12] F. Gassmann, R. Kötz, A. Wokaun, Europhysics News **34**(5) 176, (2003).
[13] a) *** Siferit- und Sirufer – Materials, Siemens & Halske, Ausgabe 1965; b) ibid., Ausgabe 1975/1976.
[14] *** Soft Magnetic Ferrite Products, Elektroimpex, Budapest, 1965.
[15] a) *** Ferrites Catalogue, Institute for Scientific Research and Technological Engineering for Electronics, Bucharest, 1992; b) ibid., Aferro Ferrites S.A., Bucharest, 1998.
[16] M. T. Johnson, A. Noodermeer, M. M. E. Severin, W. A. M. Meeuwissen, Journal of Magnetism and Magnetic Materials **116**, 169, (1992).
[17] N. N. Scholtz, K. A. Piskarev, Ferrimagnetic materials for radio-frequencies (in Russian), Publishing House Energhia, Moscow, 1966.
[18] a) D. Iordache, L. Burileanu, L. Daniello, V. Iordache, Proc. 1st National Conference of the Romanian Society of Magnetism and Magnetic Materials, Bucharest, 1990, p.65; b) D. Iordache, Vl. Iancu, V. Iordache, Proc. 3rd International Workshop "Materials for Electrotechnics", Bucharest, 2001, vol.1, p.53.
[19] I. Mikami, Jap. J. Appl. Phys. **12**(5) 678, (1973).
[20] a) I. Naito, Trans. IECE **59C,** 297, (1976); b) J. L. Snoek, Physica **14**, 297, (1948).
[21] D. Iordache, D. McClure, Selected Works of Computer Aided Applied Sciences, Printech Publishing House, Bucharest, 2002, vol.2, p. 351.
[22] Cl. Zener, Physica, **15**(1-2) 111, 1949.
[23] D. L. Anderson, J. B. Minster, Geophys. J. R. Astron. Soc., **58**, 431, (1979).
[24] S. A. Sipkin, T. H. Jordan, Bull. Seism. Soc. Am. **69**(4) 1055, (1979).
[25] D. Iordache, L. Daniello, **Ş**t. Pu**şcă**, V. Iordache, Proc. 4th International Workshop "Materials for Electrical Engineering", Bucharest, May 2004.
[26] L. Botvina, L. Ju. Fradkin, B. Bridge, Nondestr. Test. Eval. **12**, 103, (1995).
[27] J. R. Whinnery, H. W. Jamieson, Proc. IRE, **32**, 98, February 1944.
[28] C. Chiroiu, P. P. Delsanto, M. Scalerandi, V. Chiroiu, T. Sireteanu, J. Phys. D: Appl. Phys. **34**, 1579, (2001).