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MODELING AND CHARACTERIZATION BY MEANS OF THE MAGNETIC DISPERSION

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In 1978, we pointed out firstly the frequency power law dependence of the viscous friction coefficient corresponding to the magnetization walls oscillations for the low frequency part (of interest in Electronics) of the magnetization permeability dispersion of some spinelic ferrimagnetic materials. A similar dependence was found approx. concomitantly for the quality factor corresponding to ultra-sounds propagation in certain media. The present work analyzes several phenomenological models of the magnetic dispersion and points out the common "denominator" of the oscillations of the mesoscopic domains corresponding to the magnetization walls and to the elastic media grains, respectively. The obtained results allow rather simple and accurate descriptions of the magnetic dispersion of ferrimagnetic materials, in the frequency domain of major interest for Electronics.

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1. Introduction

Starting from the basic works concerning the experimental methods of magnetism [1]-[4], the procedures used to evaluate the parameters of the permeability dispersion of some radiofrequency spinelic ferrimagnetic materials were studied in detail in frame of work [5]. The compatibility of the main theoretical models of the permeability dispersion relative to the existing experimental data [6]-[10] was studied [10], the most simple and accurate phenomenological model corresponding to a frequency power law dependence of the viscous friction coefficient. Similar frequency power laws dependencies were pointed out also for the quality factor corresponding to the ultrasounds propagation through some materials [11]-[19].

2. Modeling the permeability dispersion

The basic theoretical model [20a] of the permeability dispersion is rather intricate. Because the electrical conductivity of the ferrimagnetic materials is very weak, the corresponding eddy currents can be neglected, even at microscopic level; for this reason, the theory of the rotational and translational magnetic susceptibility dispersion for such materials [20b]-[20d] allows simpler descriptions of this phenomenon. Taking into account that all these descriptions (as well as the classical theory of the ferromagnetic resonance) lead to some resonance and relaxation phenomena, it results that:

a) the most simple (phenomenological) model of the (ferrimagnetic) permeability dispersion is that of the **forced oscillations of the magnetization domains walls, under the action of an external harmonic magnetic field:**

$$m\ddot{\xi} + r\dot{\xi} + k\xi = A \cdot H_0 \cdot e^{i\omega t} \quad . \tag{1}$$

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Assuming a frequency independent friction coefficient r, there are obtained the frequency dependencies of the real magnetic susceptibility χ and of the tangent of magnetic losses $\tan\delta_m$:

$$\frac{1}{\chi(f)} = \frac{1}{\chi_o}\left[1 - \frac{f^2}{f_0^2} + \frac{f^2/f_{rel}^2}{1 - f^2/f_0^2}\right], \qquad \tan\delta_m(f) = \frac{f_0^2}{f_0^2 - f^2}\cdot\frac{f}{f_{rel}}, \tag{2}$$

where χ_o is the static (at null frequency) magnetic susceptibility, while f_0, f_{rel} are the resonance frequency and the relaxation frequency, respectively:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}, \qquad f_{rel} = \frac{k}{2\pi\cdot r}. \tag{3}$$

Taking into account that the phenomenological model of forced oscillations of the magnetization domains walls does not agree too well – for frequency independent viscous friction coefficients r - with the experimental results (concerning the frequency dependence of the tangent of magnetic losses, especially), some new theoretical models were proposed:

b) **the Sandy-Green's model [21]**, according to whom the frequency dependence of the complex magnetic susceptibility is described by the function:

$$\chi' = \frac{f_s(f_0 + i\cdot f_{rel})}{(f_0 + i\cdot f_{rel})^2 - f^2}, \tag{4}$$

represented by the straightline:

$$\frac{\mathrm{Re}(1/\chi')}{f_0/f_s} + \frac{\mathrm{Im}(1/\chi')}{f_{rel}/f_s} = 1 \tag{5}$$

in the "inverse" Cole-Cole diagram: $\mathrm{Im}(1/\chi') = f[\mathrm{Re}(1/\chi')]$, while f_s $(< f_0)$ is a characteristic frequency specific to the studied magnetic material,

c) **the Mikami's model [22]**, which combines the classical resonance-relaxation description (first term) with the typical Cole-Cole correlation [23] (second term of the following expression):

with a classical resonance-relaxation description:

$$\chi' = \frac{\chi_{oL}}{\left(1 - f^2/f_0^2\right) + i\cdot f/f_{rel}} + \frac{\chi_{oC}}{1 + (if/f_1)^{\beta}}, \tag{6}$$

d) **the Naito's model [24]**, which utilizes frequency power series in order to describe the frequency dependence of the imaginary part μ" of the complex magnetic permeability:

e)

$$\mu''(f) = \mu''_{max}\cdot\sum_{n=1}^{N} a_{2n-1}\left(f/f_{\mu''_{max}}\right)^{2n-1}\cdot\left[1 + \sum_{n=1}^{N} a_{2n}\left(f/f_{\mu''_{max}}\right)^{2n}\right]^{-1}. \tag{7}$$

Taking into account that while the classical resonance-relaxation model uses 3 parameters (χ_o, f_0, f_{rel}), the Mikami's model [22] uses 6 such parameters (χ_{oL}, f_0, f_{rel}, χ_{oC}, f_1, and β), and the Naito's model [24] uses $2N+2$ $(N \geq 2)$ such parameters (μ''_{max}, $f_{\mu''_{max}}$ and $a_1, a_2, \ldots a_{2N}$) in order to describe the permeability dispersion, it results that – despite the rather good descriptions of the permeability dispersion ensured by the Mikami's and the Naito's model, resp. – it seems that the physical explanation of the experimental data could be provided by a simpler model (using less physical parameters). Starting from this finding, the numerical analysis of the existing experimental data concerning the permeability dispersion of some industrial spinelic ferrimagnetic materials led us to the assumption that:

f) **the frequency dependence of the friction coefficient of the oscillations of the magnetic domains walls could be described by a frequency power law:** $r \propto f^n$**.**

The corresponding frequency dependencies of the permeability and of the tangent of magnetic losses are [7]:

$$\chi(f) = \chi_o \frac{f_0^2 - f^2}{f_0^2\left[\left(f^2/f_0^2 - 1\right)^2 + (f/f_{rel})^{2(n+1)}\right]} \quad , \quad \tan\delta_m(f) = \frac{f_0^2}{f_0^2 - f^2}\cdot\left(\frac{f}{f_{rel}}\right)^{n+1} , \tag{8}$$

which agrees with the experimental data (in the limits of the existing errors).

3. Modeling the dispersion of elastic waves

The detailed study of the attenuated elastic waves pointed out that: a) the most general equation of these waves is [25], [26] the Christensen's one [27], which involves (as a particular case) the Zener's equation [28], as well as the Müller's description [17], b) with convenient choices of the corresponding effective parameters, the Christensen's equation reduces to the Zener's one, and – in following – this (Zener's) equation reduces [29] to the classical Maxwell's type equation:

$$\rho\cdot\ddot{u} + R\cdot\dot{u} = E\cdot u'' \quad . \tag{9}$$

One finds so that – despite of its similitude to the basic equation (1) of the forced oscillations of the magnetization domains walls – the attenuated waves differential equation (9) differs because – due to the extinction factor of the attenuated waves integrated equation: $u(x,t) = u_o \cdot e^{-sx}\cdot\cos(\omega t - kx)$, the restoring force term $E\cdot u''$ involves also a dissipative component. In fact, the complex wave equation: $\rho\cdot\ddot{\xi} + R\cdot\dot{\xi} = E\cdot\xi''$, leads – for harmonic excitations: $\xi(t) = \xi_o \cdot e^{i\omega t}$, to the equivalent form:

$$\rho\cdot\ddot{\xi} = \overline{E}\cdot\xi'' ,$$

where the complex stiffness (elastic modulus) is:

$$\overline{E} = E' + i\cdot E'' = E\cdot\cos^2\delta_e\left(1 + i\cdot\tan\delta_e\right),$$

the tangent of the elastic (mechanical) losses and *the corresponding quality factor* Q_e being defined as:

$$\tan\delta_e = \frac{R}{\rho\omega} = \frac{1}{Q_e} \quad . \tag{10}$$

After many years when the quality factor of elastic waves (in the seismic frequency band, especially) was considered constant, the works [11]-[16] converged towards a power-law dependence of Q_e on frequency: $Q_e \propto f^\gamma$, with: $0 < \gamma < 0.5$. The synthesis of these obtained experimental results was accomplished by G. Müller, whose work [17] remains the basic one in this field. Later confirmations of the Müller's model were given by Carcione [30], Szabo [31] and Blanch [25].

The above results - obtained mainly for the propagation of the seismic waves in rocks - were extended for another frequency fields and different materials by Choudhury et al. [18] (for the ultrasonic attenuation on the microstructure of crystallized glass-ceramics), Botvina et al. [19] (different power laws in the non-destructive evaluation), etc.

4. Results and discussion

For a frequency power law corresponding to the viscous friction coefficient of the magnetization domains walls oscillations, we have:

$$\mathrm{Im}(1/\chi') = \frac{1}{\chi_o}(f_o / f_{rel})^{n+1} \cdot [1 - \chi_o \cdot \mathrm{Re}(1/\chi')]^{(n+1)/2} \ , \tag{11}$$

and:

$$\frac{d^2[\mathrm{Im}(1/\chi')]}{d[\mathrm{Re}(1/\chi')]^2} = \chi_o \frac{n^2-1}{4}\left(\frac{f_o}{f_{rel}}\right)^{n+1} \cdot \left[1 - \chi_o \mathrm{Re}\left(\frac{1}{\chi'}\right)\right]^{\frac{n-3}{2}} \ . \tag{12}$$

Taking into account that the plot $\mathrm{Im}(1/\chi') = f[\mathrm{Re}(1/\chi')]$ corresponding to the experimental data presents a positive curvature, it follows that the phenomenological model [8] agrees with the experimental data for values $n > 1$ of the frequency power law $r \propto f^n$, while the classical model of forced oscillations of magnetization walls ($n = 0$) is in disagreement with the experimental data.

The detailed study of the mesoscopic oscillations of the: (i) magnetic domains walls, which leads to the permeability dispersion, and of the: (ii) material grains, which leads to the elastic dispersion, points our that:

a) their basic features (of the forced oscillations of the magnetization domains walls and of the elastic grains, resp.) are somewhat similar, but they differ because – unlike the restoring force of the magnetic walls which is purely elastic, the restoring force corresponding to the attenuated elastic waves involves also a dissipative component,

b) taking into account the relation (18), it results that the (wave) viscous friction coefficient R obeys also a frequency power law:

$$R = \rho\omega \cdot \tan\delta_e \propto f^{1-\gamma}; \tag{13}$$

it results that *the "common denominator" of the studied mesoscopic oscillations is the frequency power dependence both of the magnetic walls r and of the elastic wave R viscous friction coefficients;*

c) due to the asymmetry of the differential equations of the mesoscopic oscillations of the magnetic domains walls and of the material (medium) grains, resp., the common basic feature: the same frequency power law of the viscous friction coefficient leads to different frequency dependencies of the losses tangents of the: (i) attenuated elastic waves: $\tan\delta_e \propto f^{-\gamma}$, (ii) radio-frequency ferrimagnetic materials (the considerably more intricate frequency dependence given by the equation (10) of our theoretical model [8]). For this reason, the Müller's model [17] is not compatible relative to the existing experimental data for the permeability dispersion. Particularly, the plot $\ln(\tan\delta) = F(\ln f)$ corresponding to the experimental data presents a positive curvature [6] (p. 269), [9].

5. Conclusions

The main findings of this work are the following:

1) it is sufficient to assume a frequency power law dependence of the viscous friction coefficient of the classical phenomenological model (of the forced oscillations of the magnetization domains walls) in order to obtain a compatible model [8] relative to the experimental data concerning the permeability dispersion of the studied spinelic ferrimagnetic materials; this model [8] requires only 4 material parameters, while the other compatible models need 6 material parameters (the Mikami's model [22]) and $2N+2$ ($N \geq 2$) parameters, the Naito's model [24],

2) the classical theoretical models of the permeability dispersion and of the attenuated elastic waves, resp. are somewhat similar, but they differ because the restoring force term $E \cdot u''$ corresponding to the attenuated waves oscillations of the mesoscopic grains involves also a dissipative component,

3) the "common denominator" of the models [8] (permeability dispersion) and [17] (attenuated elastic waves in rocks, especially) consists in the frequency power law dependence of the viscous friction coefficients.

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