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SYNCHRONIZATION AND COMMUNICATION BASED ON SEMICONDUCTOR LASERS WITH PHASE-CONJUGATE FEEDBACK

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Chaotic synchronization characteristics of external cavity semiconductor lasers with phase-conjugate feedback and its application to optical cryptograph communication are investigated numerically. The results indicate that the range of feedback coefficient making the system well synchronized is related to the bias current of the semiconductor laser. The robustness of the synchronization scheme is sensible to the internal parameter mismatch between the transmitter and the receiver. Encoding message with high bit rate can be recovered easily without any filter.

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Keywords: Semiconductor laser, Phase-conjugate feedback, Chaotic synchronization, Cryptograph communication

1. Introduction

Chaotic synchronization characteristics of semiconductor lasers and its potential application to optical cryptograph communication have been widely investigated [1-5]. Most of the chaotic laser systems based on semiconductor lasers employ optical feedback or optical injection [6-10], where optical feedback includes all-optical feedback and optoelectronic feedback. Semiconductor laser with delayed optical feedback has a number of positive Lyapunov exponent, in other words, can produce hyper-chaos dynamics [11-13], which makes message more secret. Because chaos carrier bandwidth generated by optoelectronic feedback lasers is limited by photodetector bandwidth bottleneck, the optical cryptosystem based on all-optical feedback has greater potential application. There exist two types of all-optical feedback. One is the conventional mirror optical feedback (CMOF), where the laser output is coupled into the laser internal cavity by the CMOF and the laser phase changes with the delayed feedback time. Therefore, the dynamic behaviors of laser depend on the precision positioning of the conventional mirror. The other is the phase-conjugate optical feedback (PCOF), which is considerably different from the CMOF. Compared with the CMOF, the PCOF can compensate the feedback phase shift. Furthermore, semiconductor laser subject to PCOF can display richer chaotic dynamics or higher dimension chaos, and the dynamics do not depend on an accurately positioning of the phase-conjugate mirror (PCM). However, we have noticed that the optical cryptosystem based on semiconductor lasers with the PCOF is paid little attention. Based on this consideration, in this paper, chaotic synchronization characteristics of external cavity semiconductor lasers with the PCOF and its application to optical cryptograph communication are investigated in detail.

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2. Theory

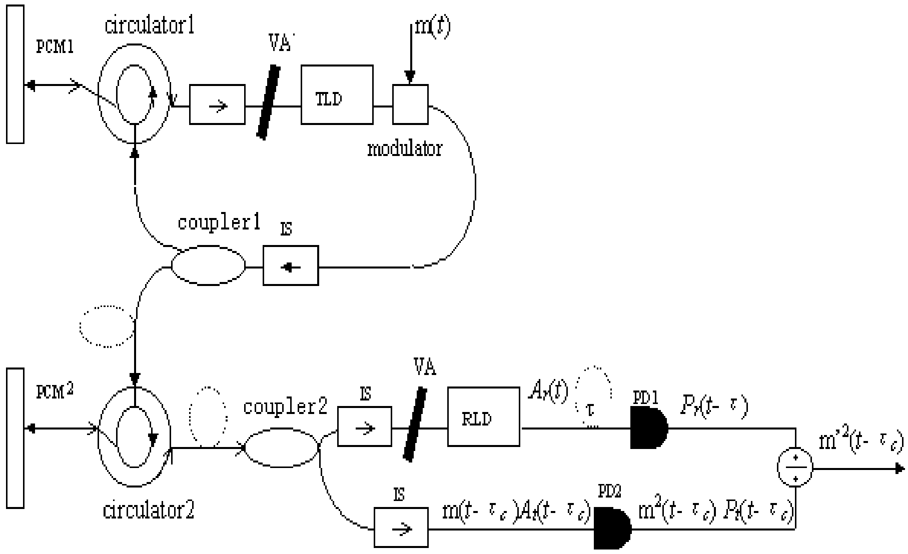

Fig. 1. Schematic diagram of the optical cryptosystem based on semiconductor lasers with PCOF, where PCM, phase conjugation mirror; IS, optical Isolator; PD, photodetector; TLD, the transmitter laser diode; RLD, the receiver laser diode; VA, variable attenuator.

The schematic diagram of the optical cryptosystem based on semiconductor lasers with the PCOF is shown in Fig. 1, where the upper part of the system is the transmitter, and the lower part is the receiver. The transmitter DFB laser diode (TLD) and the receiver DFB laser diode (RLD) are single-mode semiconductor lasers, whose central wavelengths are 1310nm. The optical isolator (IS) ensures the light transmitting unidirectionally. The output of the TLD is modulated by external modulator signal. Chaos carrier modulated with message is coupled to optical fiber channel, and then is equally divided into two beams by coupler 1. One arrives at PCM1, then comes back to circulator1 by PCM1 reflection and is coupled back into the TLD. The other comes to PCM2 by circulator2, then comes back to circulator2 by PCM2 reflection, and finally, is equally separated into two bundles of light by coupler 2, where one is injected into the RLD whose parameter is the same as that of the TLD, the output is delayed and received by the PD1, the other is received directly by PD2. As a result, message can be recovered by the intensity of PD2 divided by the intensity of PD1, and accordingly the system can realize drive-response and unidirectionally coupled chaotic synchronization secure communication.

For simplicity, we assume that the PCOF is provided by a perfect PCM whose response is almost instantaneous, the circulator 1 and the circulator 2 have the same attenuation, the PCM1 and the PCM2 have the same pumped frequency, and the nonlinearity of fiber can be ignored. Then the nonlinear behavior of TLD and RLD can be described by the modified Lang-Kobayashi equations [14-15]. For the TLD

$$\frac{dE_t(t)}{dt} = \frac{1}{2}(G_t - \gamma)(1 + i\beta_c)E_t(t)$$

$$+\frac{k}{T_L} m(t-\tau)E_t^*(t-\tau)\exp[i(2\Delta\omega t + \phi_1)] + F_{E_t}(t) \quad (1)$$

$$\frac{dN_t(t)}{dt} = \frac{I}{q} - N_t(t)\gamma_{et} - V_p G_t |E_t(t)|^2 \quad (2)$$

and for the RLD

$$\frac{dE_r(t)}{dt} = \frac{1}{2}(G_r - \gamma)(1 + i\beta_c)E_r(t) + \frac{k}{T_L}m(t-\tau_c)E_t^{*}(t-\tau_c)\exp[i(2\Delta\omega t + \phi_2)] + F_{E_r}(t) \tag{3}$$

$$\frac{dN_r(t)}{dt} = \frac{I}{q} - N_r(t)\gamma_{er} - V_p G_r \left|E_r(t)\right|^2 \tag{4}$$

with

$$G_{t,r} = \frac{g(N_{t,r} - N_0)}{\sqrt{1 + \left|E_{t,r}(t)\right|^2 / E_{sa}{}^2}} \tag{5}$$

$$\gamma_{et,r} = A_{nr} + \frac{B}{V}N_{t,r}(t) + \frac{C}{V^2}N_{t,r}{}^2(t) \tag{6}$$

where the subscripts t and r denote the transmitter and receiver, respectively, k is the feedback parameter, $E(t)$ is the slowly varying complex amplitude of the intra-cavity optical field, $\gamma = v_g(a_m + a_{int})$ is the photon delay rate, v_g is the group velocity, a_m is the facet loss, a_{int} is the internal loss, γ_e is the carrier recombination rate, $g = a\Gamma v_g/V$ is the gain coefficient, Γ is the confinement factor, V is the active layer volume of the semiconductor laser, a is the gain constant, $\Delta\omega(=\omega_0 - \omega_p)$ is the frequency mismatch between the solitary semiconductor laser and the laser used to pump the PCM, $T_L(=2L/v_g$, L is the length of the semiconductor laser) is the round trip time in the laser diode, τ is the delay feedback time, τ_c is the propagation time from the TLD to the RLD, I is the bias current, q is the electron charge, β_c is the line-width enhancement factor, $N_0 = n_0 V$ is the carrier number at transparency (n_0 is the corresponding carrier density), $V_p = V/\Gamma$ is the laser mode volume, E_{sa} is the saturation optical field, A_{nr} is the nonradiative recombination rate, B is the radiative recombination coefficient, C is the Auger recombination coefficient, the parameters Φ_1 and Φ_2 are constant phase shift and chosen arbitrarily. The Langevin noise source $F_E(t)$ is resulted from spontaneous emission, under the Markoffian approximation, the autocorrelation of $F_E(t)$ can be written as

$$\left\langle F_{E_{t,r}}(t) F_{E_{t,r}}{}^{*}(t') \right\rangle = R_{spt,r}\delta(t - t') \tag{7}$$

where $\delta(t-t')$ is the Dirac delta function, R_{sp} is the spontaneous emission rate. In this paper, the effect of the spontaneous emission has been neglected since it degrades slightly the synchronization quality [16]. The pulse of encoding message is supposed to be super-Gaussian shape, which can be expressed as $\{1 + A_m \exp[-(t/t_0)^{2M}/2]\}$, where t_0 characterizes the pulse width, A_m is the modulation amplitude, the parameter M controls the degree of edge sharpness. The pulse code period $T_B = 2(2\ln 2)^{1/2M} t_0/r$, r is the duty factor. Encoding message $m(t)$ is the random non-return-to-zero code composed of super-Gaussian pulses.

3. Results and discussion

3. 1 Effects of external parameters on the chaotic synchronization

Whether the message is easily recovered or not depends on the chaotic synchronization quality of the transmitter and receiver, so we firstly analyze the effects of external parameters on the synchronization stability without message. When the transmitter has the same parameters as the receiver, in the system exists lag chaotic synchronization, which is similar to the optical cryptograph based on the CMOF [17]. We assume $E(t)=A(t)\exp[i(\Delta\omega t+\phi(t))]$, where $A(t)$ and $\phi(t)$ denote the amplitude and the phase of the optical field, respectively, the sufficient conditions for the synchronization are as follows:

$$A_t(t-\Delta t)=A_r(t) \tag{8}$$

$$\phi_r(t)=\phi_t(t-\Delta t)+\Delta\omega\Delta t+\phi_2-\phi_1 \tag{9}$$

where $\Delta t=\tau_c-\tau$. When the laser operates at the steady state, $A_t(t)=A_r(t)=A_s$, $\phi_t(t)=\phi_{st}$, $\phi_r(t)=\phi_{sr}$ and $N_t(t)=N_r(t)=N_s$, one can obtain the following steady state equations from Eqs. (1)-(4)

$$\frac{1}{2}(G_s-\gamma)+\frac{k}{T_L}\cos(\psi_s)=0 \tag{10}$$

$$\frac{1}{2}\beta_c(G_s-\gamma)-\Delta\omega-\frac{k}{T_L}\sin(\psi_s)=0 \tag{11}$$

$$\frac{I}{q}-N_s\gamma_{es}-G_sA_s^2V_p=0 \tag{12}$$

where $\psi_s=\phi_{st}-\Delta\omega\tau-\phi_1=\phi_{sr}-\Delta\omega\tau_c-\phi_2$, G_s and γ_{es} denote the steady-state gain and the steady state carrier recombination rate, respectively.

Pecora and Carroll [18-19] have given a necessary condition for the synchronization, where all the conditional Lyapunov exponents (CLEs) associated with the delay errors differential equations must be negative. To compute the CLEs, we assume that steady-state value A_s, ϕ_{st}, ϕ_{sr} and N_s are perturbed by small amounts $\delta A(t)$, $\delta\phi(t)$, $N(t)$. Under the small signal analysis, we can get the delay errors differential equations from Eqs. (1)-(6) as follows:

$$\frac{d}{dt}\begin{vmatrix}\delta A_t(t-\Delta t)-\delta A_r(t)\\ \delta\phi_t(t-\Delta t)-\delta\phi_r(t)\\ \delta N_t(t-\Delta t)-\delta N_r(t)\end{vmatrix}=\begin{vmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{vmatrix}\begin{vmatrix}\delta A_t(t-\Delta t)-\delta A_r(t)\\ \delta\phi_t(t-\Delta t)-\delta\phi_r(t)\\ \delta N_t(t-\Delta t)-\delta N_r(t)\end{vmatrix} \tag{13}$$

with

$$A_{11}=\frac{1}{2}G_AA_s+\frac{1}{2}G_s-\frac{\gamma}{2},\quad A_{12}=-\frac{k}{T_L}A_s\sin(\psi_s),\quad A_{13}=\frac{1}{2}A_sG_N \tag{13a}$$

$$A_{21}=\frac{1}{2}\beta_cG_A+\frac{k}{T_LA_s}\sin(\psi_s),\quad A_{22}=-\frac{k}{T_L}\cos(\psi_s),\quad A_{23}=\frac{1}{2}\beta_cG_N \tag{13b}$$

$$A_{31}=-(V_pA_s^2G_A+2V_pG_sA_s),\quad A_{32}=0,$$

$$A_{33}=-(A_{nr}+\frac{2B}{V}N_s+\frac{3C}{V^2}N_s^2+V_pG_NA_s^2) \tag{13c}$$

$$G_N = \frac{\partial G_t}{\partial N}\bigg|_{A_t(t)=A_s, N_t(t)=N_s}, \quad G_A = \frac{\partial G_t}{\partial A}\bigg|_{A_t(t)=A_s, N_t(t)=N_s} \tag{13d}$$

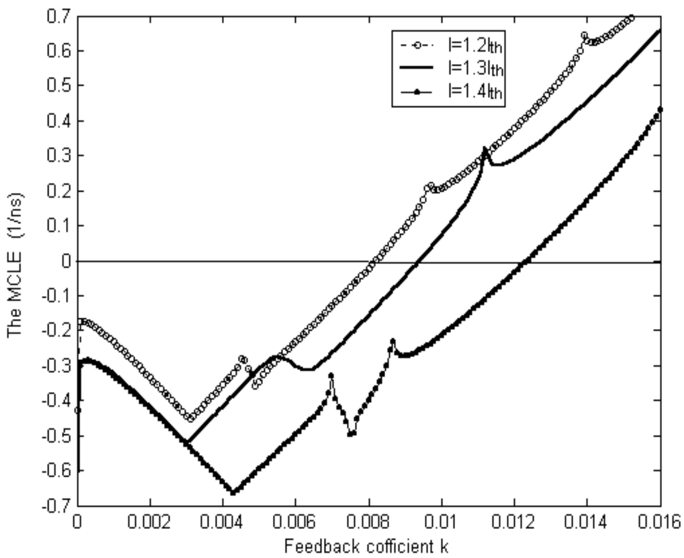

Fig. 2. MCLE versus *k* for different bias currents.

The CLEs can be obtained by solving Eqs. (10)-(13) and using Gram-Schmidt Ortho-normalization (GSR) method [20], the largest value among the CLEs is defined as the maximum conditional Lyapunov exponent (MCLE). The transmitter and receiver can realize synchronization only when the MCLE is negative, otherwise the system is desynchronized. Fig. 2 shows the plot of the MCLE versus the feedback coefficient *k* for different bias currents. The data used in calculation are: β_c=6, V=105 μm^3, L=350 μm, a=2.3×10^{-20}m^2, n_0=1.2×10^{24}m^{-3}, E_s=1.6619×10^{11} m$^{-3/2}$, Γ=0.29, a_m=29 cm^{-1}, a_{int}=20 cm^{-1}, n_g=3.8, A_{nr}=1.0×10^{8}s^{-1}, B=1.2×10^{-16} m^3/s, C=3.5×10^{-41} m^6/s, $\Delta\omega$=0, ϕ_1=0, ϕ_2=0, τ=2ns, τ_c=10ns. As seen from the Fig. 2, for the bias current I is fixed at 1.2I_{th} (the threshold current I_{th} is about 15.1 mA), the MCLE is negative for the feedback coefficient *k* varing from 0 to 0.0082, that is to say, the transmitter and the receiver can achieve stability synchronization; If k>0.0082, the transmitter and the receiver will loss the stability synchronization. With the increase of the bias current, the range of feedback coefficient *k* making the system well synchronized will increase. So the range of the feedback coefficient *k* making the system well synchronized can be widened by increasing the bias current *I* of the LD.

3. 2 Effects of internal parameter mismatch on the quality of synchronization

The lag chaotic synchronization that is satisfactory to Eqs. (8)-(9), is named as the entire chaotic synchronization, which can be realized if the parameters of the TLD and the RLD are identical. Though the laser external parameter *k* and *I* can be controlled easily, the laser internal parameter can't be controlled accurately. So it is important to investigate the effect of internal parameter mismatch between the TLD and the RLD on the quality of synchronization. The correlation coefficient ρ is a critical index to measure the synchronization quality, which is defined as [17]:

$$\rho = \frac{\left\langle \left[P_t(t-\Delta t) - \left\langle P_t(t-\Delta t) \right\rangle \right] \left[P_r(t) - \left\langle P_r(t) \right\rangle \right] \right\rangle}{\left\{ \left\langle \left[P_t(t-\Delta t) - \left\langle P_t(t-\Delta t) \right\rangle \right]^2 \right\rangle \left\langle \left[P_r(t) - \left\langle P_r(t) \right\rangle \right]^2 \right\rangle \right\}^{1/2}} \tag{14}$$

where 〈〉 denotes the time average, P is the optical power. The internal parameters of the RLD are supposed to be relatively changed while that of the TLD is fixed, and the mismatch of internal parameters can be defined as

$$\Delta g = (g_r - g_t)/g_t \qquad \Delta\gamma = (\gamma_r - \gamma_t)/\gamma_t \tag{14a}$$

$$\Delta A_{nr} = (A_{nrr} - A_{nrt})/A_{nrt} \quad \Delta\beta_c = (\beta_{cr} - \beta_{ct})/\beta_{ct} \tag{14b}$$

$$\Delta C = (C_r - C_t)/C_t \qquad \Delta B = (B_r - B_t)/B_t \tag{14c}$$

$$\Delta N_0 = (N_{0t} - N_{0r})/N_{0t} \tag{14d}$$

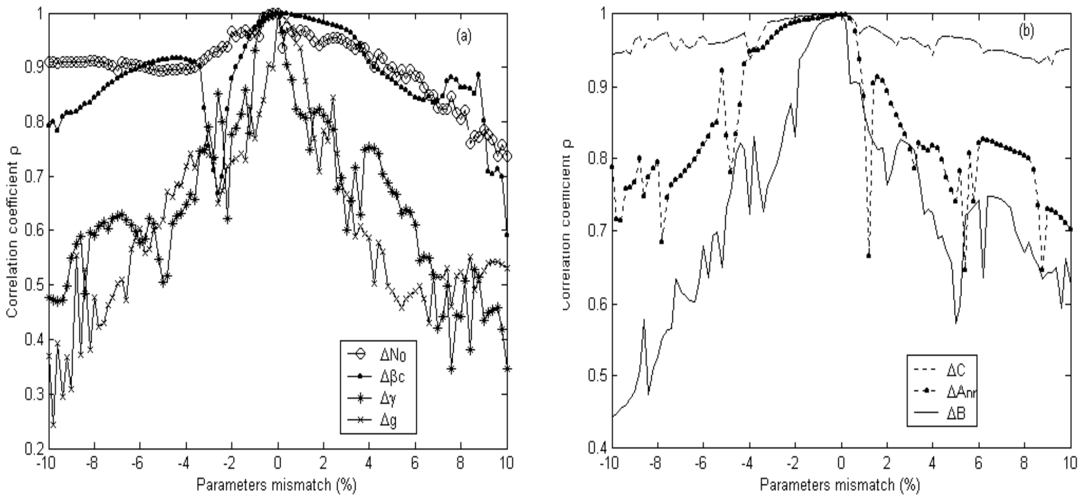

Fig. 3. Correlation coefficient ρ as a function of parameter mismatches.

The correlation coefficient ρ as a function of the internal parameter mismatch is shown in Fig. 3, where the parameter mismatch described in Fig. 3(b) will lead to the parameter γ_e mismatch. From this diagram, it can be seen that, the effects of the mismatch of γ and g on the quality of synchronization are violent, and the effects of the mismatch of the parameters C and N_0 is relatively slight.

3. 3 Chaotic cryptograph communication scheme

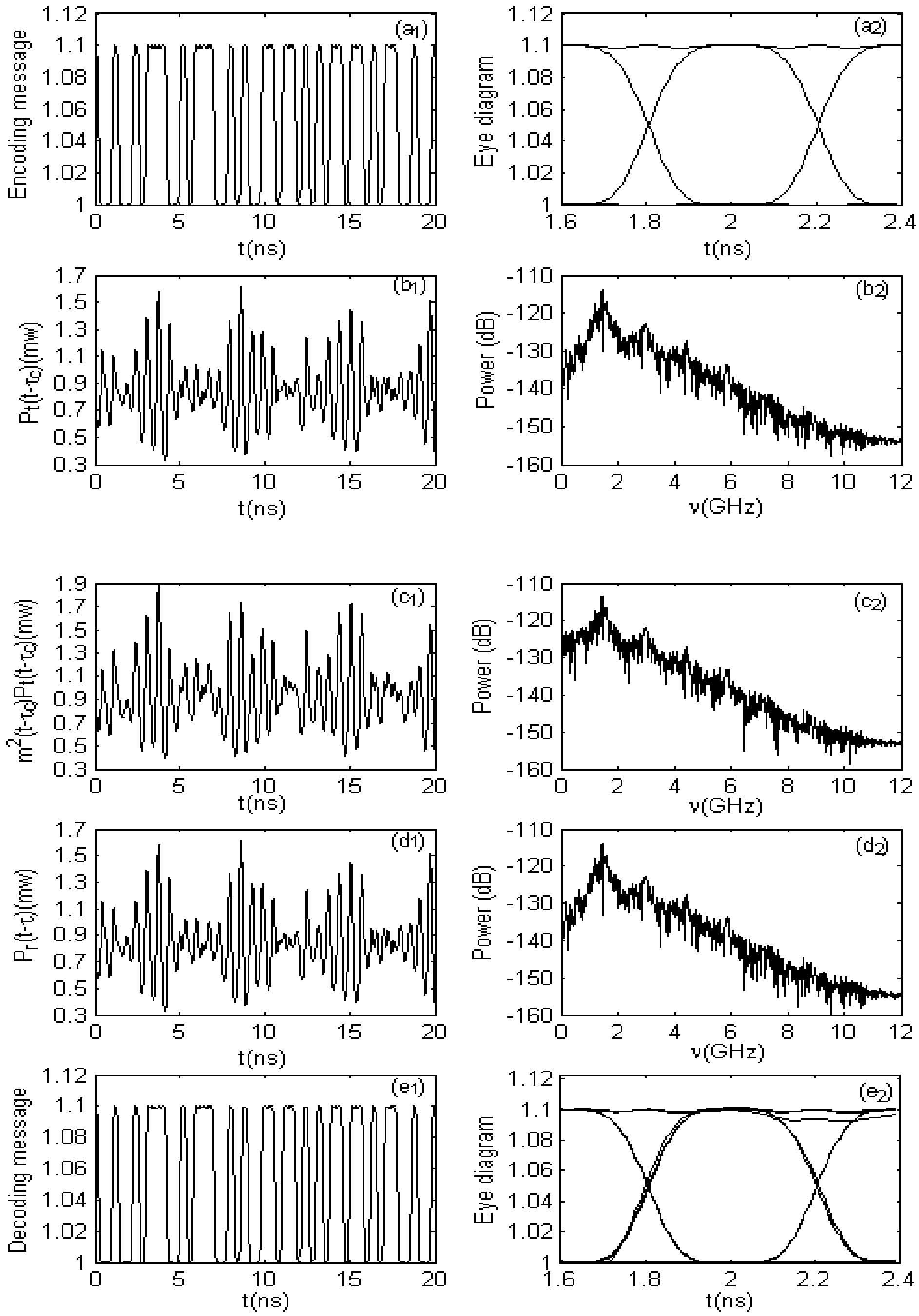

Fig. 4. Dynamical evolution of the optical cryptosystem, where (b_2), (c_2), and (d_2) are the power spectrum corresponding to (b_1), (c_1), and (d_1), respectively.

The optical cryptosystem communication based on semiconductor lasers with PCOF is designed as the Fig. 1, and the message can be decoded by

$$m' = m(t-\tau_c)\sqrt{\frac{P_t(t-\tau_c)}{P_r(t-\tau)}} \tag{15}$$

where *m'* is decoding message. Fig. 4 shows dynamical evolution of the optical cryptosystem. Here, a nonreturn-to-zero pseudo-random bit sequence is employed as the message, and the data used in calculations are: k=0.0075, $I=1.2I_{th}$, M=2, A_m=0.1, r=1, T_B=0.4ns (the bit rate of message is 2.5GHz). From Figs. (b_1) and (d_1), it can be seen that the perfect lag time (Δt=8ns) synchronization between TLD and RLD can be achieved. As shown in Figs. (b_2) and (d_2), the range of the corresponding power spectrum is 0 ~8 GHz or so, and the maximum spectrum peak is at 1.5382GHz. And the time trace of decoding message and the corresponding eye diagram are displayed in Figs. (e_1) and (e_2), respectively, which are almost identical with that of encoding message displayed in Figs. (a_1) and (a_2), respectively. In addition, from Figs. (c_1) and (c_2), it can be seen that the message encoded on the output of the TLD is entirely masked by chaotic signal, and is difficult to be eavesdropped. The above analysis indicates that the system is not only well synchronized but also high security. Moreover, the message can be recovered easily without any filter. The reason is as follows: Firstly, the message can be not only coupled into the RLD, but also injected into the TLD, which maintains the symmetry of the transmitter and the receiver; Secondly, the phase of encoding message does not break the phase-matching condition of the well-synchronized system; finally, the encoding message is directly taken part in changing the chaotic dynamic of the RLD and TLD, which enhances security in the optical cryptosystem.

4. Conclusion

In this paper, we have numerically investigated chaotic synchronization characteristics of external cavity semiconductor lasers with PCOF and its application to optical cryptograph communication. The results show that, the range of feedback coefficient making the system well synchronized is related to the bias current of the semiconductor laser, and the range of the feedback coefficient k making the system well synchronized can be widened by increasing the bias current of the semiconductor laser. The robustness of the synchronization scheme under different internal parameter mismatch between the transmitter and the receiver has been specified. Through using symmetry configuration, the encoding message with high bit rate can be recovered easily without any filter.

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References

[1] Y. Liu, H. F. Chen, J. M. Liu, P. Davis, T. Aida, Communation using synchronization of optical-feedback-induced chaos in semiconductor lasers, IEEE Trans. Circuits Syst. I **48**(12), 1484-1490 (2001).

[2] A. Sanchez-Diaz, C. R. Mirasso, P. Colet, P. Garcia-Femandez, Encoded Gbit/s digital communication with synchronized chaotic semiconductor lasers, IEEE J. Quantum Electron. **35**(3), 292-297 (1998).

[3] K. Kusumoto, J. Ohtsubo, 1.5-GHz message transmission based on synchronization of chaosin semiconductor lasers, Opt. Lett. **27**(12), 989-991 (2002).

[4] J. M. Liu, H. F. Chen, S. Tang, Synchronized chaotic optical communications at high bit rates. IEEE J. Quantum Electron. **38**(9), 1184-1196 (2002).

[5] S. Tang, J. M. Liu, Message encoding-decoding at 2.5 Gbits/s through synchronization of chaotic pulseing semiconductor lasers, Opt. Lett. **26**(23), 1843-1845 (2001).

[6] V. Annovazzi-Lodi, S. Donati, A. Scire, Synchronization of chaotic injected-laser systems and its application to optical cryptography, IEEE J. Quantum Electron. **32**(6), 953-959 (1996).

[7] S. Sivaprakasam, K. A. Shore, Message encoding and decoding using chaotic external-cavity

diode lasers, IEEE J. Quantum Electron. **36**(1), 35-39 (2000).
[8] H. Fujino, J. Ohtsubo, Experimental synchronization of chaotic oscillation in external-cavity semiconductor lasers, Opt. Lett. **25**(9), 625-627 (2000).
[9] H. D. I. Abarbanel, M. B. Kennel, L. Illing, S. Tang, H. F. Chen, J. M. Liu. Synchronization and communication using semiconductor lasers with optoelectronic feedback, IEEE J. Quantum Electron. **37**(10), 1301-1311 (2001).
[10] S. Tang, J. M. Liu, Chaos synchronization in semiconductor lasers with optoelectronic feedback, IEEE J. Quantum Electron. **39**(6), 708-715 (2003)
[11] V. Ahlers, U. Parlitz, W. Lauterborn, Hyperchaotic dynamics and synchronization of external-cavity seimconductor lasers, Phys. Rev. E **58**(6), 7208-7213 (1998).
[12] J. P. Goedgebuer, L. Larger, H. Porte. Optical cryptosystem based on synchronization of hyperchaos generated by a delayed feedback tunable laser diode. Phys. Rev. Lett. **80**(9), 2249-2252 (1998).
[13] I. Fischer, O. Hess, W. Elsaber, E. Gobel, high-dimension chaotic dynamics of external cavity semiconductor laser, Phys. Rev. Lett. **73**(16), 2188-2191 (1994).
[14] A. Murakami, J. Ohtsubo, Y. Liu, Stability analysis of semiconductor laser with phase-conjugate feedback, IEEE J. Quantum Electron. **33**(10), 1825-1831 (1997).
[15] G. R. Gray, D. Huang, G. P. Agrawal, Chaotic dynamics of semiconductor lasers with phase-conjugate feedback, Phys. Rev. A **49**(3), 2096-2105 (1994).
[16] R. Vicente, T. Perez, C. R. Mirasso, Open-versus closed-loop performance of synchronized chaotic external-cavity semiconductor lasers, IEEE J. Quantum Electron. **38**(9), 1197-1203 (2002).
[17] A. Locquet, C. Masollet, C. R. Mirasso, Synchronization regimes of optical feedback induced chaos in unidirectionally coupled semiconductor lasers, Phys. Rev. E 65, 056205-1-056205-12 (2002).
[18] L M. Pecora, T. L. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. **64**(8), 812-824 (1990).
[19] T. L. Carroll, L. M. Pecora, Synchronizing chaotic circuits. IEEE Trans. Circuits Syst. **38**(4), 453-456 (1991).
[20] A. Wolf, J. B. Swift, H. L. Swinney, J. A. Vastano, Determining Lyapunov exponents from a time series. Physica D **16**, 285-317 (1985).