ANALYTICAL SOLUTION OF A PERCOLATION MODEL FOR CHARGE DECAY ON SURFACES

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Recently, a percolation analysis has been applied to describe potential decays on surfaces of corona-charged electrets [Yovcheva *et al.,* J. Phys.: Condens. Matter **16**, 455 (2004)]. In this analysis, a numerical method based on a "black-box" software was used, and this does not allow a transparent treatment of parameters responsible for the decays. We suggest an exact analytical solution of this theoretical problem, within the very same model as the one used by Yovcheva *et al.* The analytical solution provides a much better understanding of the potential decay phenomenon than a "black-box" numerical calculation.

1. Introduction

Recently, a comprehensive experimental study of the time decays of surface potentials after corona-charging were carried out on polypropylene electrets [1]. In this study, the surfaces of the electrets were charged to potentials of several hundreds of Volts, and the discharging was followed for 250 days under various conditions of the surrounding atmosphere. In particular, the humidity of the atmosphere was found to be important for the potential transients [1].

It has been well established that discharging via the surface rather than the bulk can be responsible for potential decays [2]. A theory for such surface discharging has been suggested by Kuzmin and Tairov [3-5]. According to this, water absorption gives rise to the conducting channels on the charged surface, that modify significantly the surface electric properties. As soon as the amount of adsorbed water is sufficient to create a connected path through the surface (a percolation cluster) discharging begins, with the time-dependent surface potential $V(t)$ being proportional to the fraction of the surface area that does not belong to the percolation cluster: $V(t) \propto [1- P(t)]$, where $P(t)$ is the surface density of the percolation cluster at time t. The percolation cluster density is a function of the surface fraction $\theta(t)$ covered by the adsorbate. This quantity has been calculated by Kuzmin and Tairov, using the Kolmogorov theory for the adsorption kinetics [6].

This percolation approach has been recently applied by Yovcheva *et al.* [1], in order to interpret their experimental data on the surface potential decays in various polypropylene electrets. The authors devised a computer program with a non-linear algorithm, based on the Flexyplex software package [1]. Although this package allows one to fit experimental data, it does not provide any transparent insight into the physics of the phenomenon.

In this paper we show that the theoretical model used by Yovcheva *et al.* [1] can be solved purely analytically. Below we present the solution and compare the results with experimental data.

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2. Percolation model for the surface potential

In the percolation model suggested by Kuzmin and Tairov [3-5], the time-dependent surface potential is determined by the equation

$$V(t) = V_0[1 - P(\theta(t))], \tag{1}$$

where V_0 denotes the initial value of the surface potential. The density of the percolation cluster has the form [1,5]

$$P(\theta) = \begin{cases} 0 & if \ 0 \le \theta < \theta_c \\ \sum_{i=0}^{4} a_i(\theta - \theta_c)^i & if \ \theta_c \le \theta < \theta_c^{'} \ , \\ \theta & if \ \theta_c^{'} \le \theta < 1 \end{cases} \tag{2}$$

where $\theta_c = 0.428$, $\theta_c^{'} = 0.533$ and, $a_0 = -1.042 \ 10^{-2}$, $a_1 = 21.26$, $a_2 = -3.959 \ 10^2$, $a_3 = 3.549 \ 10^3$, and $a_4 = -1.178 \ 10^4$ are the parameters of the fourth order polynomial regression [5]. The function $\theta(t)$ is determined by the fraction of the dry surface at the beginning of the discharge, the rate of formation of nuclei, $\beta(t)$, and the time-dependent radius of a nucleus, R. The latter is determined as in [3-5] by the equation

$$R(t',t) = \int_{t'}^{t} \nu(\xi)d\xi, \ \nu(\xi) = \nu_1 e^{-\xi/\tau} + \nu_2, \tag{3}$$

where $\nu(\xi)$ is the growth speed of the nucleus size. The speed is assumed to have a constant component ν_2 and an additional component ν_1, which relaxes with some time constant τ [3-5]. The surface fraction $\theta(t)$ covered by the adsorbate that determines the discharge kinetics is given by the equation [1,3-5].

$$\theta(t) = 1 - q \cdot e^{-\int_{t_0}^{t} \beta(t')\pi R^2(t',t)dt'} = 1 - q \exp\left(-\int_{t_0}^{t} \beta(t')\pi\left(\int_{t'}^{t} \nu(\xi)d\xi\right)^2 dt'\right), \tag{4}$$

where q is the surface fraction that remained free from the adsorbate at the end of the charging process. Equations (1-4) were solved by Yovcheva *et al.* [1] using a numerical software package. This solution cannot be considered as transparent. Moreover, it does not provide the values for such an important parameter as the relaxation time τ for the radius growth rate. Below, we present an analytical solution of these equations for the very same model as that used in [1].

3. Analytical solution and comparison with experiment

It was assumed in [1] for simplicity that the nuclei creation rate is constant: $\beta(t) \equiv \beta_0$ and the samples are charged at a moment $t_0 = 0$. In such a case one can rewrite Eq. (4) in the form

$$\theta(t) = 1 - q \cdot e^{I(t)} \tag{5}$$

with

$$I(t) = -\beta_0 \pi \int_0^t \left(\int_{t'}^t v(\xi) d\xi \right)^2 dt' = -\beta_0 \pi \int_0^t \left(\int_{t'}^t v_1 e^{-\xi/\tau} + v_2 d\xi \right)^2 dt'. \qquad (6)$$

After a straightforward integration one obtains

$$I(t) = -\beta_0 \pi \left[v_1^2 \tau^2 \left(t e^{-2t/\tau} \right) + \frac{3}{2} v_1^2 \tau^3 \left(e^{-2t/\tau} \right) - v_1 v_2 \tau \left(t^2 e^{-t/\tau} \right) + \right.$$

$$\left. + 2 v_1 v_2 \tau^3 \left(e^{-t/\tau} \right) + \frac{1}{3} v_2^2 \left(t^3 \right) + 2 v_1 v_2 \tau^2 (t) + \left(\frac{1}{2} v_1^2 - 2 v_1 v_2 \right) \tau^3 \right] \qquad (7)$$

Equations (1), (2), (5), and (7) provide an analytical description for the time-dependent surface potential.

Two sets of samples were studied experimentally in [1]. In Fig. 1, a comparison of the above analytical result and the experimental data are shown, for one sample taken for each set. The values of the material parameters used in the calculations are given in the inserts on the figure.

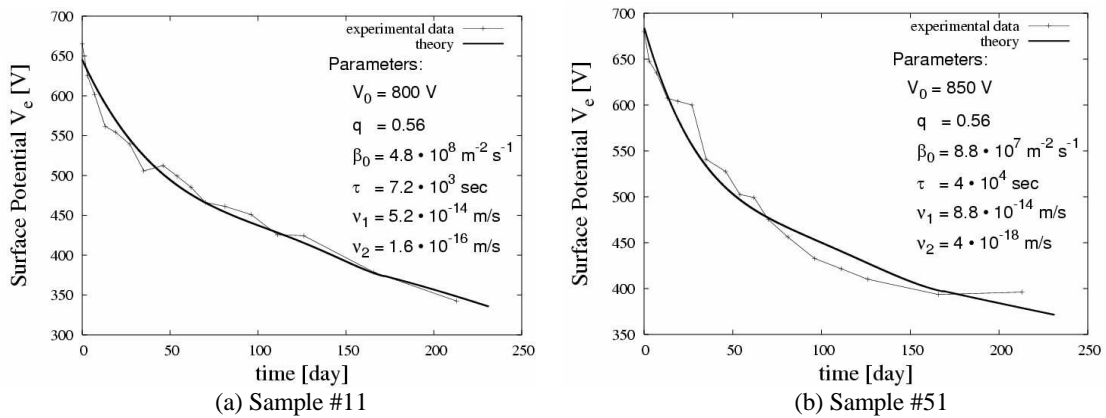(a) Sample #11   (b) Sample #51

Fig. 1. Comparison of analytical results and experiment (from [1]). Broken line with points: Experimental data. Solid line: Analytical results described in Eqs. (1), (2), (5), and (7).

4. Summary

The really good agreement between the theoretical results and the experimental data shows that it is indeed possible to describe the surface discharge kinetics in the electrets using percolation theory, as suggested in [3-5]. Moreover, one can solve the problem analytically without non-transparent software packages.

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References

[1] T. A. Yovcheva, G. A. Mekishev, A. T. Marinov, J. Phys.: Condens. Matter **16**, 455 (2004).

[2] P. G. Kurmazova, G. A. Mekishev, Europhys. Lett. **19**, 481 (1992).

[3] Y. I. Kuzmin, V. N. Tairov, Sov. Phys. Tech. Phys. **29**, 575 (1984).

[4] Y. I. Kuzmin, Proc. 10th Int. Symp. on Electrets (Athens, 22-24 Sep.) Eds A. Konsta, A. Vassilikou-Dova, K. Vartzeli-Nikaki, p.55 (1999).

[5] Y. I. Kuzmin, Ph.D. Thesis, LETI, St. Petersburg, 1984 (in Russian).

[6] A. N. Kolmogorov, Izv. Akad. Nauk SSSR, Ser. Math. **3**, 355 (1937).