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DETERMINATION OF NORMALIZED PROPAGATION CONSTANT FOR OPTICAL WAVEGUIDES BY USING SECOND ORDER VARIATIONAL METHOD

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The second–order variational method is adapted to obtain closer results to the exact values for the normalized propagation constant of the planar graded–index waveguides.

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1. Introduction

The higher–order variational method has been used for approximating energy levels of a physical system (see for example Ref.1). This approach was combined with the finite element method for the study of the generalized anharmonic oscillator in D dimensions [2]. Unlike methods based on finite elements or finite difference, the variational methods yield analytical mode field representations which are defined on the entire plane of the waveguide cross section.

We use here the second-order variational method to obtain an analytical expression for the normalized propagation constant of a planar graded–index waveguide.

2. Second-order variational method

The adimensional form of the wave equation for the planar waveguide is of the form [3].

$$-\frac{d^2\Psi}{dx^2}+V^2[1-\exp(-x)]\Psi=U^2\Psi \quad , \quad x>0 \tag{1}$$

$$-\frac{d^2\Psi}{dx^2}+V^2[1+B]\Psi=U^2\Psi \quad , \quad x<0 \tag{2}$$

$$b=\frac{\left(N_m^2-n_s^2\right)}{\left(n_1^2-n_s^2\right)}=1-\left(\frac{U}{V}\right)^2 , \quad B=\frac{\left(n_s^2-n_c^2\right)}{\left(n_1^2-n_s^2\right)} , \quad N_m=\frac{\beta}{k_0} , \quad V^2=2k_0^2d^2n_s\Delta n , \tag{3}$$

where b is the normalized propagation constant, V is the normalized frequency, β is the propagation constant, k_0 is the free space wave number, d is the effective depth of diffusion, Δn is a measure of the increase in refractive indices, n_s is the substrate value of the refractive index, n_1 is the maximum refractive index of the profile and n_c is the index of the cover region. The approximate solution of the equation (1) can be expressed in terms of Airy functions [3].

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$$\Psi_s(x)=\frac{\sqrt{\xi_s'(0)}Ai[\xi_s(x)]}{\sqrt{\xi_s'(x)}Ai[\xi_s(0)]}\quad ,\quad x<x_2 \tag{4}$$

$$\Psi_d(x)=\frac{\sqrt{\xi_s'(0)}Ai[\xi_d(x)]}{\sqrt{\xi_d'(x)}Ai[\xi_s(0)]}\quad ,\quad x>x_2 \tag{5}$$

where x_2 is the turning point ($b = \exp(-x_2)$) and ξ is an asymptotic variable [3, 4].

$$\xi_s(x)=-\left\{3V\left[\sqrt{\exp(-x)-b}-\sqrt{b}\tan^{-1}[\sqrt{\frac{\exp(-x)}{b}}-1]\right]\right\}^{2/3},\qquad x<x_2 \tag{6}$$

$$\xi_d(x)=\left\{3V\left[-\sqrt{b-\exp(-x)}+\frac{\sqrt{b}}{2}\ln\left(\frac{\sqrt{b}+\sqrt{b-\exp(-x)}}{\sqrt{b}-\sqrt{b-\exp(-x)}}\right)\right]\right\}^{2/3},x>x_2 \tag{7}$$

The field given by Eqs. (4) and (5) satisfies the boundary condition $\psi(\pm\infty)\to 0$ and $\psi(0) = 1$. The solution of the equation (2) satisfies the condition $\psi(0) = 1$

$$\Psi_1=\exp[V\sqrt{b+B}x] \tag{8}$$

The continuity of $\psi'(x)$ at $x = 0$ will yield an eigenvalue equation [3, 4]

$$\frac{V\sqrt{b+B}}{\xi'(0)}=\frac{Ai'[\xi(0)]}{Ai[\xi(0)]}-\frac{\xi''(x)}{2\xi'^2(0)}\ , \tag{9}$$

The value of the normalized propagation constant b for TE_0 mode is calculated by using our second-order variational method [3]

$$b=1-\left(\frac{U}{V}\right)^2;U^2=\frac{-A_1-\sqrt{A_1^{\,2}-4A_2A_3}}{2A_3}\ ,\ \ A_1=(h_{12}+h_{21})f_{12}-h_{11}f_{22}-h_{22}f_{11}\ ,$$

$$A_2=h_{11}h_{22}-h_{12}h_{21}\ ,\ A_3=f_{11}f_{22}-f_{12}^{\ 2},\ \ h_{11}f_{22}+h_{22}f_{11}>(h_{12}+h_{21})f_{12} \tag{10}$$

where, for example (B = 0),

$$h_{11}=\int_{-x_1}^{0}\left[-\Psi_1(b_1)\Psi_1''(b_1)+V^2\Psi_1^2(b_1)\right]dx+\int_{0}^{x_2}\left[-\Psi_2(b_1)\Psi_2''(b_1)+V^2(1-\exp(-x))\Psi_2^2(b_1)\right]dx+$$

$$+\int_{x_2}^{x_4}\left[-\Psi_3(b_1)\Psi_3''(b_1)+V^2(1-\exp(-x))\Psi_3^2(b_1)\right]dx,\ f_{11}=\int_{-x_1}^{0}\left[\Psi_1^2(b_1)\right]dx+\int_{0}^{x_2}\left[\Psi_2^2(b_1)\right]dx+\int_{x_2}^{x_4}\left[\Psi_3^2(b_1)\right]dx$$

$$f_{12}=f_{21}=\int_{-x_1}^{0}\Psi_1(b_1)\Psi_1(b_2)dx+\int_{0}^{x_2}\Psi_2(b_1)\Psi_2(b_2)dx+\int_{x_2}^{x_3}\Psi_2(b_2)\Psi_3(b_1)dx+\int_{x_3}^{x_4}\Psi_3(b_1)\Psi_3(b_2)dx$$

In practical applications ($x_1 \rightarrow \infty$, $x_4 \rightarrow \infty$), $x_2 = -\ln(b_1)$ and $x_3 = -\ln(b_2)$ are the turning points, b_1, b_2 are the solutions of the eigenvalue equation (9), $\psi_1(b_1)$, $\psi_1(b_2)$ are defined on the interval $[-x_1,0]$, $\psi_2(b_1)$ is defined on the interval $[0,x_2]$, $\psi_2(b_2)$ is defined on the interval $[0,x_3]$, $\psi_3(b_1)$ is defined on the interval $[x_2,x_4]$, and $\psi_3(b_2)$ is defined on the interval $[x_3,x_4]$. If we set $h_{22} = h_{12} = h_{21} = 0$, $f_{12} = 0$, $f_{22} = 1$, we obtain the result for the first–order variational problem $b = h_{11}/f_{11}$.

3. Numerical results and conclusions

We consider the scalar-wave equation of a planar waveguide with $n_s = n_c = 1.55$ ($B = 0$), $n_1 = 1.57$, $\lambda_0 = 0.5145$ μm, $d = 1$ μm ($V = 3.05061$). Values of the normalized parameter b for TE_0-mode of inhomogeneous planar optical waveguide in the second–order variational method, the first–order variational method, the eigenvalue equation (9) and the WKB method are respectively: 0.382390028; 0.382389932; 0.382906; 0.236862, while the exact value is 0.382390095. The refractive index profile and the electromagnetic field as a function of the distance for TE_0 and TE_1 modes are shown in Fig. 1.

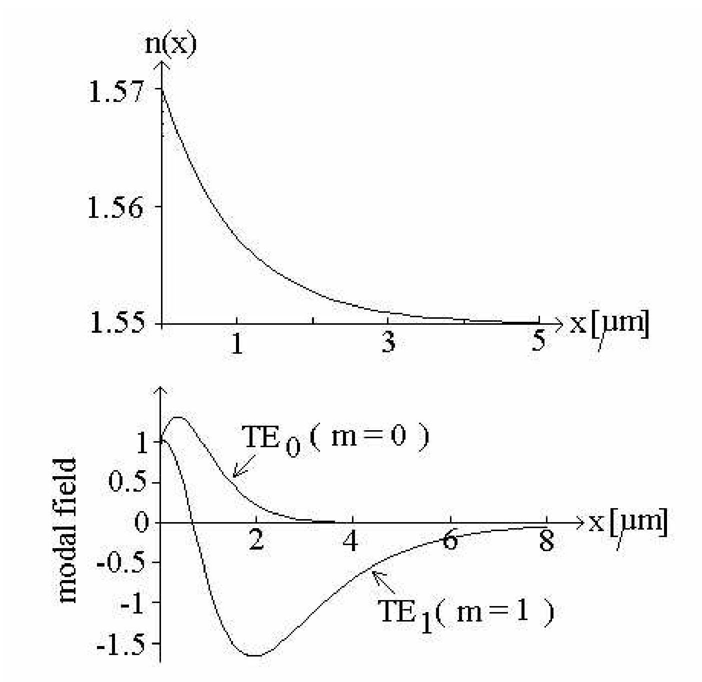

Fig. 1. The refractive index profile and the electromagnetic field as a function of the distance for TE_0 and TE_1 modes.

The finite element and variational methods have been used to determine the propagation constants in a titanium indiffused lithium niobate waveguide with the reconstructed refractive index profile from the near field measurements [5]. Also, the finite element method has been used to determine the propagation constants in an optical waveguide obtained in glass by double ion exchange (Ag^+) [6].

In conclusion, we have obtained closer results to the exact value of the normalized propagation constant for the TE_0 mode of the planar graded-index waveguide by using only the second–order variational method.

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