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TEMPERATURE DEPENDENCE OF LONGITUDINAL AND TRANSVERSE DIELECTRIC FUNCTIONS OF INHOMOGENEOUS FERMI SYSTEMS IN THE LOCAL DENSITY APPROXIMATION

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The present work is a theoretical study aiming at understanding the role of the electronic temperature on the optical response of simple metal clusters. The electronic temperature dependence of the optical response of simple metal clusters is investigated by many different quantum mechanical theories. The bulk dielectric functions are the most important quantities of a quantum many-electron system. Here we use of bulk dielectric function theory for calculating of longitudinal and transverse dielectric functions in the quantum many-electron system.

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1. Introduction

The electronic temperature dependence of the optical absorption of simple metal clusters is investigated in the framework of bulk dielectric function. In the following atomic units ($\hbar = e = m = 4\pi\varepsilon_0 = 1$) are used unless otherwise specified and k_B is the Boltzman's constant. a_0 =0.529 Å is the Bohr radius and e denotes the absolute electron charge. The quantities with a barre are dimensionless.

2. Theoretical model bulk dielectric functions

A. Longitudinal

The bulk longitudinal dielectric function is one of the most important quantity in discussing the dynamical properties of a quantum many-electron system in a linear regime. It's first use in many-body theory is due to Nozières and Pines [1]. For an infinite system it is well known that in the case of a homogeneous electron gas the retarded dielectric function depends only on the difference between the space coordinates and time.

$$\varepsilon_l(\vec{r}.t;\vec{r}'.t') = \varepsilon_l(\vec{R} = \vec{r} - \vec{r}';\tau = t - t').$$

It is then convenient to work in the momenta and frequencies space leading to

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$$\varepsilon_l(\vec{q},\omega) = \iint \varepsilon_l(\vec{R};\tau) e^{i\vec{q}.\vec{R}} e^{i\omega.\tau} d\vec{R} d\tau \equiv \varepsilon_l'(\vec{q};\omega) + i\varepsilon_l''(\vec{q};\omega) \tag{1}$$

The above quantity characterizes the linear response of the medium to a longitudinal electromagnetic perturbation of wave vector $\vec{q}$ and frequency ω. If the homogeneous electron gas is perturbed by an external potential $V_{ext}(\vec{q};\omega)$ then the total potential inside the system $V_{ext}(\vec{q};\omega)$, is given by

$$V_{tot}(\vec{q};\omega) = \varepsilon_l^{-1}(\vec{q};\omega) V_{ext}(\vec{q},\omega) \tag{2}$$

If for a particular complex frequency $\tilde{\omega}_r$ the total field inside the medium becomes large even for infinitesimally small external potential then the system is in an eigen –resonance. Thus it results from Eq. (2) that the condition for plasma oscillation in the electron gas is given by

$$\varepsilon_l(\vec{q};\tilde{\omega}_r) = 0 \tag{3}$$

These free charge oscillations are characterized by the fact that we can have a field in the system even in the absence of a driving external potential. Since $\varepsilon_l(\vec{q};\tilde{\omega}_r)$ is a complex function the equation (3) has no solutions for real values of $\tilde{\omega}_r$. There are however, approximate solutions by setting the real part equal to zero. Thus, the condition for plasma oscillation (3) is replaced by

$$\varepsilon_l'(\vec{q};\omega_r) = 0 \tag{4}$$

where $\omega_r = \mathrm{Re}[\tilde{\omega}_r]$ is real . In the following the frequency ω is real. One defines also the so – called energy-loss function $-\mathrm{Im}\left[\frac{1}{\varepsilon_l}\right] = \frac{\varepsilon_l''}{\varepsilon_l'^2 + \varepsilon_l''^2}$ which is related to the excitations of the quantum system and to the self-energy [2,3]. In the linearized time-dependent Hartree theory (this approximation is also referred to as the random phase approximation (RPA)) or the self-consistent-field method (SCF) [4], the longitudinal dielectric function of a 3-dimensional homogeneous electron gas reads [5]

$$\varepsilon_l(\vec{q};\omega) = 1 - V(q)\chi^0(\vec{q};\omega) \tag{5}$$

where $V(q) = \frac{4\pi e^2}{q^2}$ and $\chi^0(\vec{q};\omega)$ are respectively, the Fourier transform of the coulomb energy potential and the non-interacting retarded density correlation function. The latter is given by

$$\chi^0(\vec{q};\omega) = \lim_{\eta\to 0^+} 2(2\pi\hbar)^{-3} \int \frac{n_F(\vec{p}) - n_F(\vec{p}+\hbar\vec{q})}{\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}+\hbar\vec{q}} - \hbar\omega - i\eta} d^3\vec{p} \tag{6}$$

where $n_F(\vec{p}) = \frac{1}{e^{\beta(e_{\vec{p}}-\mu_F)}+1}$ is the Fermi-Dirac function which depends on $\vec{p}$ only through the electronic energy $\varepsilon_{\vec{p}} = \varepsilon_p = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$, $\beta = \frac{1}{k_B T_e}$ and μ_F is the Fermi chemical potential. In the above formula, the variable η has been originally introduced by Lindhard for causality requirements [6]. The existence of an infinity of poles in the denominator of (6) does not allow us to use numerical techniques to perform the integration. Note also, that a non vanishing value of η can be used in order to take the finite electron lifetime $\tau_e^{-1} = \frac{\eta}{\hbar} \equiv \delta$ into account within the so-called relaxation-time approximation [7,8].

The Fermi chemical potential μ_F is solution of the equation [9]

$$S(-\beta\mu_F, \frac{1}{2}) = \alpha_B \tag{7}$$

with the Sommerfeld's function

$$\mathrm{S}(\overline{\alpha}_F, \rho) = \frac{1}{\Gamma(\rho+1)} \int_0^\infty \frac{z^\rho dz}{e^{(\overline{\alpha}_F + z)} + 1} \tag{8}$$

and

$$\alpha_B \equiv e^{\beta\mu_B} = \frac{1}{2} n_0 \lambda_T^3 \tag{9}$$

where n_0 is the equilibrium particle density, μ_B is the Boltzman chemical and λ_T is the thermal wave length defined by $\lambda_T = \left(\frac{2\pi\hbar^2}{mk_B T_e}\right)^{\frac{1}{2}}$. If $\overline{\alpha}_F = -\beta\mu_F \geq 0$ i.e. $\mu_F \leq 0$ then $\alpha_F \equiv e^{-\overline{\alpha}_F} \leq 1$ and

$$\mathrm{S}\,(\overline{\alpha}_F, \rho) = f_{\rho+1}(\alpha_F) \tag{10}$$

with

$$f_s(\alpha) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\alpha^k}{k^s} = -poly\log(s, -\alpha) \tag{11}$$

The above series is convergent for $\alpha \leq 1$ and $f_s(1) = (1 - 2^{1-s})\xi(s)$ where $\xi(s)$ is the Riemann's zeta function which is defined for $R(s) \succ 1$ and is extended to the rest of the complex plane (except for the point s = 1) by analytic continuation. By using the usual rule (Plemelj formula) $\lim_{\eta\to 0}\left(\frac{1}{z - i\eta}\right) = P\frac{1}{z} + i\pi\delta(z)$ one obtains from Eqs (5) and (6)

$$\varepsilon_l''(\vec{q};\omega) = -2\pi V(q) \int \left[n_F(\vec{p}) - n_F(\vec{p} + \hbar\vec{q})\right] \delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}+\hbar\vec{q}} - \hbar\omega) \frac{d^3\vec{p}}{(2\pi\hbar)^3} \tag{12}$$

The above formula may be rewritten as

$$\varepsilon_l''(\vec{q};\omega) = \frac{V(q)}{4\pi^2\hbar^3} \int [\delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}+\hbar\vec{q}} + \hbar\omega) - \delta(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}+\hbar\vec{q}} - \hbar\omega)] n_F(\vec{p}) d^3\vec{p} \tag{13}$$

By using spherical coordinates, the angular integration is straightforward and we get

$$\varepsilon_l''(\vec{q};\omega) = \frac{mV(q)}{\pi\hbar^4 q} \int_{p-}^{p+} \frac{pdp}{e^{\beta(\varepsilon_p - \mu_F)} + 1} \tag{14}$$

with $p_\pm = \frac{m}{\hbar q}\left|\varepsilon_q \pm \hbar\omega\right|$ leading to [2]

$$\varepsilon_l''(\vec{q};\omega) = \frac{m^2 V(q)}{2\pi\hbar^4 \beta q} \ln\left[\frac{1+e^{-\beta(\varepsilon_- - \mu_F)}}{1+e^{-\beta(\varepsilon_+ - \mu_F)}}\right] \tag{15}$$

where $\varepsilon_\pm = \dfrac{p_\pm^2}{2m}$.

For $T_e = 0$ one has

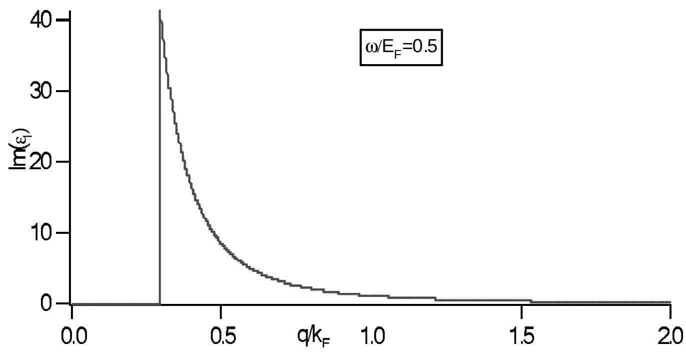
$$\varepsilon_l''(\overline{q};\overline{\omega}) = \frac{\overline{\kappa}\overline{\Upsilon}}{\overline{q}} \begin{cases} \overline{\omega}; \quad 0 \le \overline{\omega} \le 2\overline{q} - \overline{q}^2; \quad \overline{q} \prec 2 \\ 1 - \frac{1}{4}\left[\overline{q} - (\overline{\omega}/\overline{q})\right]^2; \quad 2\overline{q} - \overline{q}^2 \le \overline{\omega} \le 2\overline{q} + \overline{q}^2; \overline{q} \prec 2 \\ 0; \quad \overline{\omega} \succ 2\overline{q} + \overline{q}^2; \quad \overline{q} \prec 2 \\ 1 - \frac{1}{4}\left[\overline{q} - (\overline{\omega}/\overline{q})\right]^2; \quad \overline{q}^2 - 2\overline{q} \le \overline{\omega} \le 2\overline{q} + \overline{q}^2; \quad \overline{q} \succ 2 \end{cases} \tag{16}$$

with the dimensionless quantities : $\overline{q} = \dfrac{q}{k_F}$, $\overline{\omega} = \dfrac{\hbar\omega}{E_F}$, $\overline{\Upsilon} = \dfrac{E_{pot}}{E_{kin}} = (\dfrac{e^2}{r_s})/(\dfrac{\hbar^2 q^2}{2m})$ and $\overline{\kappa} = \dfrac{1}{2\chi}$

with $\chi = \left[\dfrac{4}{9\pi}\right]^{\frac{1}{3}}$.This result is normally found in textbook [2,10].

This result can be also rewritten as (see also Fig. 1 and Fig. 12.9 of [10])

$$\varepsilon_l''(\overline{q};\overline{\omega}) = \frac{\overline{\kappa}\overline{\Upsilon}}{\overline{q}} \begin{cases} 1 - \frac{1}{4}\left[\overline{q} - (\overline{\omega}/\overline{q})\right]^2; \quad -1 + \sqrt{1+\overline{\omega}} \le \overline{q} \le 1 - \sqrt{1-\overline{\omega}}; \quad \overline{\omega} \prec 1 \\ \overline{\omega}; \quad 1 - \sqrt{1-\overline{\omega}} \le \overline{q} \le 1 + \sqrt{1-\overline{\omega}}; \quad \overline{\omega} \prec 1 \\ 1 - \frac{1}{4}\left[\overline{q} - (\overline{\omega}/\overline{q})\right]^2; \quad 1 + \sqrt{1-\overline{\omega}} \le \overline{q} \le 1 + \sqrt{1+\overline{\omega}}; \quad \overline{\omega} \prec 1 \\ 1 - \frac{1}{4}\left[\overline{q} - (\overline{\omega}/\overline{q})\right]^2; \quad -1 + \sqrt{1+\overline{\omega}} \le \overline{q} \le 1 + \sqrt{1+\overline{\omega}}; \quad \overline{\omega} \succ 1 \end{cases} \tag{17}$$



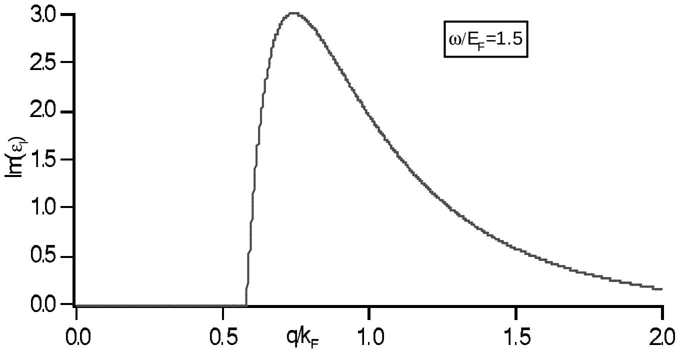


Fig. 1. Imaginary part of the longitudinal bulk dielectric function at $T_e = 0$ and $\frac{r_s}{a_0} = 4$.

The real part of the dielectric function is obtained from the Kramers-Kronig relation

$$\varepsilon_l'(\vec{q};\omega) = 1 + \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\varepsilon_l''(\vec{q};\omega)}{(\xi - \omega)} d\xi \tag{18}$$

where the symbol denotes the principal value of the integral . After simple

calculations from Eq. (15,18) one gets

$$\varepsilon_l'(\vec{q};\omega) = 1 + \frac{mV(q)}{\hbar^2 q} \lambda_T^{-2} (\frac{1}{\sqrt{\pi}}) \left\{ \tilde{\phi}(x_+, \alpha_F) - \tilde{\phi}(x_-, \alpha_F) \right\} \tag{19}$$

with

$$x_\pm = \left(\frac{1}{2}\beta m\right)^{\frac{1}{2}} \left(\frac{\omega}{q} \pm \frac{\hbar q}{2m}\right) \tag{20}$$

and

$$\tilde{\phi}(x, \alpha_F) = \left(\frac{1}{\sqrt{\pi}}\right) P \int_{-\infty}^{+\infty} \frac{\ln(1 + \alpha_F e^{-y^2})}{(x - y)} dy \tag{21}$$

The above principal-value integral may be evaluated numerically by using, for instance, the finite-interval method of Thompson [11]. This method makes use of the even-order derivatives of the function $\ln(1+x)$ which can be obtained by a symbolic-differentiation program.

For $T_e = 0$, the longitudinal dielectric function may be expressed in terms of analytical functions and are known as the Lindhard formulae [6] one has

$$\varepsilon_l(\vec{q};\omega) = \lim_{\delta \to 0^+} \varepsilon_L(\overline{q};\overline{\omega})$$

with

$$\varepsilon_L(\overline{q};\overline{\omega}) = 1 + \frac{\overline{\chi}}{\overline{q}^3} \left\{ \begin{array}{l} 2\overline{q} + \left[1 - \frac{1}{4}(\overline{q} - \frac{\overline{w}}{\overline{q}})^2\right] \ln\left(\frac{\overline{q} - \overline{w}/\overline{q} + 2}{\overline{q} - \overline{w}/\overline{q} - 2}\right) + \\ \left[1 - \frac{1}{4}(\overline{q} + \frac{\overline{w}}{\overline{q}})^2\right] \ln\left(\frac{\overline{q} + \overline{w}/\overline{q} + 2}{\overline{q} + \overline{w}/\overline{q} - 2}\right) \end{array} \right\} \tag{22}$$

with the dimensionless variables, $\overline{w} \equiv \hbar(\omega + i\delta)/E_F$, $\overline{r}_s = \frac{r_s}{a_0}$, where $a_0 = \frac{\hbar}{me^2}$ is Bohr radius, $\overline{\chi} \equiv \frac{1}{\pi}\left[\frac{4}{9\pi}\right]^{\frac{1}{3}}\overline{r}_s = \frac{1}{\pi}\chi\overline{r}_s$.

Since, the dielectric function depends on $\vec{q}$ only through it's modulus q, the dielectric function expressed in the real space is $\varepsilon_l(R \equiv |\vec{r} - \vec{r}'|;\omega)$ (isotropic space).

Thus, according to (1) one can write

$$\varepsilon_l(R;\omega) = \int \varepsilon_l(q;\omega)e^{-i\vec{q}.\vec{R}}d\vec{q} \tag{23}$$

By using the well known expansion of the plane-wave

$$e^{-i\vec{q}.\vec{R}} = \sum_{lm} 4\pi(-i)^l j_l(qR)Y_{lm}(\hat{q})Y^*_{lm}(\hat{R}) \tag{24}$$

we obtain

$$\varepsilon_l(R;\omega) = 4\pi\int_0^\infty \varepsilon_l(q;\omega)j_0(qR)q^2dq \tag{25}$$

with $j_0(qR) = \frac{\sin(qR)}{qR}$.

The same is true for the response function. Therefore, from (5) we obtain

$$\chi^0(R;\omega) = \frac{4\pi}{R}\int_0^\infty \chi^0(q;\omega)\sin(qR)qdq \tag{26}$$

with

$$\chi^0(q;\omega) = \frac{1-\varepsilon_l(q;\omega)}{V(q)} \tag{27}$$

For spherically symmetric systems we have

$$\chi^0(R;\omega) = \sum_{\lambda m_\lambda} J_\lambda(r,r';\omega)Y^*_{\lambda m_\lambda}(\hat{r})Y_{\lambda m_\lambda}(\hat{r}') \tag{28}$$

with

$$J_\lambda(r,r';\omega) = 2\pi\int_{-1}^{+1}\chi^0(R;\omega)P_\lambda(u)du \tag{29}$$

$$\mathrm{R}=|\vec{r} - \vec{r}'| = \sqrt{r^2 + r'^2 - 2rr'u} \tag{30}$$

The optical response of the valence electrons is treated quantum-mechanically. Within the so-called local density approximation, the induced electronic density $\delta n(\vec{r};\omega)$ is related to $V_{ext}(\vec{r};\omega)$, the Fourier transform (with respect to time) of the external energy potential associated to the electric field of the laser (amplitude E_1), by [12]

$$\delta n(\vec{r};\omega) = \int \chi(|\vec{r}-\vec{r}'|;\omega;n_0(\vec{r}))V_{ext}(\vec{r}';\omega)d\vec{r}' \tag{31}$$

where $\chi(|\vec{r}-\vec{r}'|\,;\omega;n_0(\vec{r}))$ is the retarded density correlation function or the dynamic response function of a system of uniform density n_0. The local Wigner-Seitz radius is defined as $r_s(r) = \left[\dfrac{3}{4\pi n_0(r)}\right]^{\frac{1}{3}}$.

In the dipole approximation ($R \prec\prec \lambda$ *where* λ is the wavelength of the incident radiation) the external energy potential is given by (if an electrical field E_1 is applied to the cluster in the z direction it causes a perturbation energy)

$$V_{ext}(\vec{r}';\omega) \equiv v_{ext}(r';\omega)Y_{10}(\hat{r}') = eE_1 z' \tag{32}$$

$$z' = r'\cos\theta = r'\frac{\sqrt{4\pi}}{\sqrt{3}}y_{10}(\hat{r}') \tag{33}$$

$$v_{ext}(r') \equiv r'\frac{\sqrt{4\pi}}{\sqrt{3}} \tag{34}$$

From the frequency-dependent dipole polarizability defined by

$$\alpha(\omega) = \frac{1}{E_1^2}\int \delta n(\vec{r};\omega)V_{ext}(\vec{r};\omega)d\vec{r}$$
$$= \frac{1}{E_1^2}\int_0^\infty\int_0^\infty J_1(r,r';\omega)v_{ext}(r;\omega)v_{ext}(r';\omega)r^2 r'^2 drdr' \tag{35}$$

one obtains the dipole absorption cross-section [13]

$$\sigma(\omega) = \frac{4\pi\omega}{c}\mathrm{Im}[\alpha(\omega)] \tag{36}$$

one needs

$$\mathrm{Im}[\chi^0(R;\omega)] = \frac{4\pi}{R}\int_0^\infty \mathrm{Im}[\chi^0(q;\omega)]\sin(qR)dq \tag{37}$$

with

$$\mathrm{Im}[\chi^0(R;\omega)] = -\frac{\varepsilon_l''(\overline{q};\overline{\omega})}{V(q)} \tag{38}$$

B. Transverse

The transverse dielectric function of an electron gas in quantum mechanical treatment of electrons was obtained by Lindhard.

$$\varepsilon_t(\vec{q},\omega)=1+\frac{2\omega_p^2}{\omega^2}\sum_n\frac{f(E_n)}{N}\left\{\left(q_n^2-\frac{(\vec{q}.\vec{q}_n)^2}{q^2}\right)\left(\frac{1}{q^2+2\vec{q}.\vec{q}_n-\frac{2m\omega}{\hbar}}+\frac{1}{q^2-2\vec{q}.\vec{q}_n+\frac{2m\omega}{\hbar}}\right)-\frac{1}{2}\right\} \quad (39)$$

This result is found in [6].

Which $E_n=\frac{\hbar^2 q_n^2}{2m}$ is the electron energy, $\omega_p=\frac{4\pi e^2 n_0}{m}$ is the classical resonance frequency of the electron gas and n_0 is the equilibrium particle density.

The Eq. (39) after some calculations can be rewritten as:

$$\varepsilon_t(\vec{q},\omega)=\lim_{z\to 0^+}2(2\pi\hbar)^{-3}\int\frac{\left(\vec{p}^2-\frac{(\hbar\vec{q}.\vec{p})^2}{\hbar^2q^2}\right)(n_F(\vec{p})-n_F(\vec{p}+\hbar\vec{q}))}{E_{\vec{p}}-E_{\vec{p}+\hbar\vec{q}}-\hbar\omega-iz}d^3\vec{p} \quad (40)$$

By using the usual rule (Plemelj formula) one obtains imaginary part of the transverse dielectric function from Eq.(40)

$$\varepsilon_t''(\vec{q},\omega)=\left(\frac{2\pi e}{m\omega}\right)^2\int\frac{d^3\vec{p}}{(2\pi\hbar)^3}\left[\vec{p}^2-\frac{(\vec{p}.(\hbar\vec{q}))^2}{\hbar^2q^2}\right]n_F(\vec{p})\left\{\delta\left(\varepsilon_{\vec{p}}-\varepsilon_{\vec{p}+\hbar\vec{q}}+\hbar\omega\right)-\delta\left(\varepsilon_{\vec{p}}-\varepsilon_{\vec{p}+\hbar\vec{q}}-\hbar\omega\right)\right\} \quad (41)$$

The above formula may be written as two parts:

$$\varepsilon_t''(\vec{q},\omega)=\left(\frac{2\pi e}{m\omega}\right)^2\left[\varepsilon_{1t}''(\vec{q},\omega)+\varepsilon_{2t}''(\vec{q},\omega)\right] \quad (42)$$

which

$$\varepsilon_{1t}''(\vec{q},\omega)=\int\frac{d^3\vec{p}}{(2\pi\hbar)^3}\vec{p}^2n_F(\vec{p})\left\{\delta\left(\varepsilon_{\vec{p}}-\varepsilon_{\vec{p}+\hbar\vec{q}}+\hbar\omega\right)-\delta\left(\varepsilon_{\vec{p}}-\varepsilon_{\vec{p}+\hbar\vec{q}}-\hbar\omega\right)\right\}$$

$$=2\frac{me^2}{\hbar^4q\omega^2}\left\{\left[\mu_\beta\ln\left[\frac{e^{+\beta v}}{1+e^{+\beta v}}\right]+\frac{1}{2}v^2-v\frac{\ln(1+e^{+\beta v})}{\beta}-\frac{poly\log(2,-e^{+\beta v})}{\beta^2}\right]_{-v}^{+v}\right\} \quad (43)$$

and

$$\varepsilon_{2t}''(\vec{q},\omega)=\int\frac{d^3\vec{p}}{(2\pi\hbar)^3}\frac{(\vec{p}.(\hbar\vec{q}))^2}{\hbar^2q^2}n_F(\vec{p})\left\{\delta\left(\varepsilon_{\vec{p}}-\varepsilon_{\vec{p}+\hbar\vec{q}}+\hbar\omega\right)-\delta\left(\varepsilon_{\vec{p}}-\varepsilon_{\vec{p}+\hbar\vec{q}}-\hbar\omega\right)\right\}$$

$$=-\frac{e^2m^2}{\hbar^4q\omega^2}\left\{\left[\frac{(v+\mu_F)}{\beta}\ln\left(1+e^{-\beta v}\right)\right]_{v_-}^{v_+}\right\} \quad (44)$$

by using equations (42) , (43) , (44) one obtains:

$$\varepsilon_t''(\vec{q},\omega)=\frac{-2me^2}{\hbar^4q\omega^2}\left[\frac{1}{2}v^2+\frac{poly\log(2,-e^{+\beta v})}{\beta^2}\right]_{v_-}^{v_+} \quad (45)$$

with

$$v_{\pm} = \frac{p_{\pm}^2}{2m} - \mu_F \tag{46}$$

$$p_{\pm} = \frac{m}{\hbar q}\left|E_q \pm \hbar\omega\right| \tag{47}$$

$$poly\log(2,z) = -\frac{1}{2}\log^2(-z) - \frac{\pi^2}{6} - \sum_{k=1}^{\infty}\frac{1}{k^2 z^k} \qquad |z| \succ 1$$

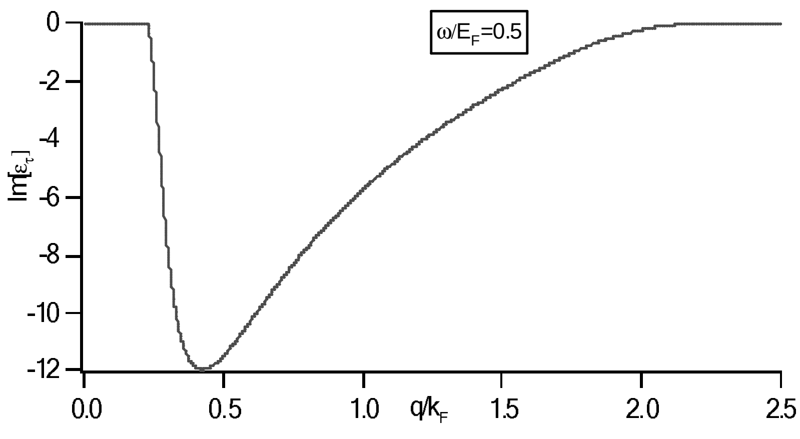
$$poly\log(a,z) = \sum_{n=1}^{\infty}\frac{z^n}{n^a} \qquad |z| \prec 1$$

For $T_e = 0$ one has

$$\varepsilon_t''(\overline{q},\overline{\omega}) = \frac{\overline{\kappa}\,\overline{Y}}{\overline{q}}\begin{cases} -2\dfrac{\overline{q}^2}{\overline{\omega}}\left(1-\dfrac{1}{4}\left(\dfrac{\overline{\omega}^2}{\overline{q}^2}+\overline{q}^2\right)\right) & \overline{q} \prec 1+\sqrt{1-\overline{\omega}} \quad , \quad \overline{\omega} \prec 1 \\ -\dfrac{\overline{q}^2}{\overline{\omega}^2}\left(1-\dfrac{1}{4}\left(\dfrac{\overline{\omega}}{\overline{q}}-\overline{q}\right)^2\right)^2 & \overline{q} \succ 1+\sqrt{1-\overline{\omega}} \quad , \quad \overline{\omega} \prec 1 \\ -\dfrac{\overline{q}^2}{\overline{\omega}^2}\left(1-\dfrac{1}{4}\left(\dfrac{\overline{\omega}}{\overline{q}}-\overline{q}\right)^2\right)^2 & \sqrt{1+\overline{\omega}}-1 \prec \overline{q} \prec 1+\sqrt{1+\overline{\omega}} \quad , \quad \overline{\omega} \prec 1 \end{cases} \tag{48}$$

with the dimensionless quantities $\overline{q}, \overline{\omega}, \overline{Y}, \overline{\kappa}$ which are defined in Eq.(16).

This result is found from [6] by simple calculations (see Fig. 2).



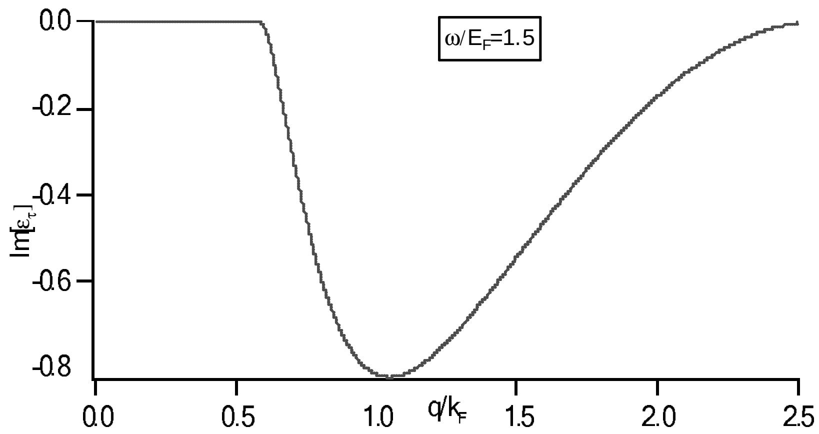


Fig. 2. Imaginary part of the transverse bulk dielectric function at $T_e = 0$ and $\frac{r_s}{a_0} = 4$.

3. Conclusion

In the present work we calculated by analytical and numerical methods the imaginary and real parts of longitudinal dielectric function and imaginary part of transverse function for a quantum many electron system. These results can be used in the study of the electron temperature effects on the optical response of metal clusters. Also we could calculate analytically the relation between the imaginary parts of response function and longitudinal dielectric function.

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References

[1] P. Nozieres, D. Pines, Nuovo Cimento **9**, 470 (1958).
[2] G. D. Mahan, Many-particle physics (Plenum Press, 1981).
[3] P. M. Echenique et al., Chem. Phys. **251**, 1 (2000).
[4] H. Ehrenreich, M. H. Cohen, Phys. Rev. **115**, 786 (1959).
[5] It is possible to include exchange-correlation by using $\varepsilon(\vec{q};\omega) = 1 - V(q)[1 - G(\vec{q};\omega)]\chi^0(\vec{q};\omega)$ where $G(\vec{q};\omega)$ is the so-called local-field factor.
[6] J. Lindhard, Dan. Math. Phys. Medd. **28**(8), (1954).
[7] K. L. Kliewer, R. Fuchs, Phys. Rev. **181**, 552 (1969).
[8] N. D. Mermin, Phys. Rev. B **1**, 2363 (1970).
[9] R. C. Tolman, The principles of statistical Mechanics (Dover Publications, Inc., New York, 1979).
[10] A. L. Fetter, J. D. Walecka, Quantum theory of Many-body Systems (McGraw-Hill, New-York, 1971).
[11] W. J. Thompson, Comput. Phys. **12**, 94 (1998).
[12] S. Chakravarty, M. B. Fogel, W. Kohn, Phys. Rev. Lett. **43**, 775 (1979).
[13] G. D. Mahan, Local Density Theory of polarizability (Plenum Press 1990).