Journal of Optoelectronics and Advanced Materials Vol. 7, No. 6, December 2005, p. 3179 - 3183

THE ROLE OF INTERFERENCE IN STANDARD LASER CALORIMETRY

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The standard laser calorimetry is a powerful tool in describing the properties of the sample and of the beam. In view of that, one has to solve the classical heat equation. The present paper is dealing with the situation when the laser beams make up an angle θ and, therefore, in this situation it exists an important interference term. In this paper the thermal fields for different angles θ are simulated.

(Received July 25, 2005; accepted November 24, 2005)

Keywords: Laser beams, Heat equation, Interference, Laser calorimetry

1. Introduction

The standard laser calorimetry is a powerful tool for the description of sample and laser beam. In this paper the thermal effects for different angles between two laser beams are analyzed.

2. Theory

Assuming that only a photothermal action is developing, and that all absorbed energy is transformed into heat, the linear heat flow in solid is fully described by the partial differential equation [1-5]:

$$\frac{\partial^2 T(r,z,\varphi,t)}{\partial r^2}+\frac{1}{r}\frac{\partial T(r,z,\varphi,t)}{\partial r}+\frac{\partial^2 T(r,z,\varphi,t)}{\partial z^2}-\frac{1}{\gamma}\frac{\partial T(r,z,\varphi,t)}{\partial t}=-\frac{S(r,z,\varphi,t)}{k} \quad , \quad (1)$$

where: γ is the thermal diffusivity in cm^2/s, k is the thermal conductivity in W/cm.^{0}K and $S(r,z,\varphi,t)$ is the heat source strength or the rate of heat production per unit volume in J/cm^3.s . In general, one can consider the linear heat transfer approximation q = $h(T-T_0)$ in which q is the heat energy flux in J/cm^2.s, T is the temperature in ^{0}K and h is the heat transfer coefficient in W/cm^2.^{0}K. Also, we will consider only radiation case, with a zero convection, for which $h = h_{rad} = 4\sigma E T_0^3$, where σ = 1.355.10^{-12} W/cm^2. ^{o}K is the Stefan-Boltzmann constant and E is the emissivity of the sample.

Let us consider a parallelepiped sample with dimensions *a*, *b*, and *c* in Cartesian directions x, y and z, respectively. Suppose there are two laser beams of the same wavelength (and, consequently, there is the same absorption coefficient).

For the interfering laser beams the relation is: $\vec{E}=\vec{E}_1+\vec{E}_2$, and consequently:

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$I \sim < \vec{E}^2 > = < (\vec{E}_1 + \vec{E}_2)^2 > = < \vec{E}_1^{\,2} > + < \vec{E}_2^{\,2} > + 2 < \vec{E}_1 \cdot \vec{E}_2 >$, which is equivalent with: $I \sim < \vec{E}^2 > = < (\vec{E}_1 + \vec{E}_2)^2 > = I_1 + I_2 + 2 \cdot E_1 \cdot E_2 \cdot \cos\theta$.

Consider the following relations:

$$S(x,y,z,t) = S_1(x,y,z,t) + S_2(x,y,z,t) + S_{\text{int}}(x,y,z,t).$$

with the heat transfer coefficients: $h_1 = h_2 = h_3 = h_4 = h_5 = h_6 = h$.

In the model the considered angle θ between the two laser beams is small enough, so that we may consider the same boundary conditions for both lasers beams. If a linear heat transfer at the sample surface (the "radiation" boundary condition [1]) is considered then for the first and second laser beam, along x axis the equations are:

$$\left[\frac{\partial K_x}{\partial x} - \frac{h}{K}K_x\right]_{x=-\frac{a}{2}} = 0\ ;\quad \left[\frac{\partial K_x}{\partial x} + \frac{h}{K}K_x\right]_{x=\frac{a}{2}} = 0\ ;\quad \left[\frac{\partial K_y}{\partial y} + \frac{h}{K}K_y\right]_{y=\frac{b}{2}} = 0\ ;$$
$$\left[\frac{\partial K_y}{\partial y} + \frac{h}{K}K_y\right]_{y=-\frac{b}{2}} = 0\ ;\quad \left[\frac{\partial K_z}{\partial z} + \frac{h}{K}K_z\right]_{z=\frac{c}{2}} = 0\ ;\quad \left[\frac{\partial K_z}{\partial z} + \frac{h}{K}K_z\right]_{z=-\frac{c}{2}} = 0 \qquad (2)$$

The solution of the heat equation (1) subjected to boundary conditions (2) is [1]:

$$\Delta T(x,y,z,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{o=1}^{\infty} a(\alpha_i,\beta_j,\chi_o) b(\alpha_i,\beta_j,\chi_o,t) K_x(\alpha_i,x) K_y(\beta_j,y) K_z(\chi_o,z), \qquad (3)$$

where

$$a(\alpha_i,\beta_j,\chi_o) = \frac{\alpha I_{ox}}{K C_i C_j C_o}\int_{-\frac{a}{2}}^{+\frac{a}{2}} e^{-\alpha x} K_x(\alpha_i,x)dx \int_{-\frac{b}{2}}^{+\frac{b}{2}}\int_{-\frac{c}{2}}^{+\frac{c}{2}} e^{-2(y^2+z^2)/w_{ox}^2} K_y(\beta_j,y) K_z(\chi_o,z) dydz \qquad (4)$$

$$b(\alpha_i,\beta_j,\chi_o,t) = \frac{1}{\alpha_i^2+\beta_j^2+\chi_o^2}[1 - e^{-\gamma_{ijo}^2 t} - (1 - e^{-\gamma_{ijo}^2 (t-t_o)}) h(t-t_o)] \qquad (5)$$

and

$$\gamma_{ijo}^2 = \gamma(\alpha_i^2+\beta_j^2+\chi_o^2). \qquad (6)$$

Here: $\alpha_i, \beta_j, \chi_o$, are the eigenvalues corresponding to the eigenfunctions: K_x, K_y, K_z,, and C_i are the normalization coefficients. α is the linear absorption coefficient and $h(t-t_0)$ is the step function. Considering the expression of I, the results are:

$$\Delta T(x,y,z,t) = \Delta T_1(x,y,z,t) + \Delta T_2(x,y,z,t) + \Delta T_{\text{int}}(x,y,z,t) \qquad (7)$$

where one can identify, for example:

$$\Delta T_1(x,y,z,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{o=1}^{\infty} a_1(\alpha_i,\beta_j,\chi_o) b(\alpha_i,\beta_j,\chi_o,t) K_x(\alpha_i,x) K_y(\beta_j,y) K_z(\chi_o,z) \qquad (8)$$

where: $a_1(\alpha_i,\beta_j,\chi_o) = \frac{\alpha I_{o1x}}{K C_i C_j C_o}\int_{-\frac{a}{2}}^{+\frac{a}{2}} e^{-\alpha x} K_x(\alpha_i,x)dx \int_{-\frac{b}{2}}^{+\frac{b}{2}}\int_{-\frac{c}{2}}^{+\frac{c}{2}} e^{-2(y^2+z^2)/w_{ox}^2} K_y(\beta_j,y) K_z(\chi_o,z) dydz$

and

$$\Delta T_{\text{int}}(x,y,z,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{o=1}^{\infty} a_{\text{int}}(\alpha_i,\beta_j,\chi_o) b(\alpha_i,\beta_j,\chi_o,t) K_x(\alpha_i,x) K_y(\beta_j,y) K_z(\chi_o,z) \qquad (9)$$

where:

$$a_{\text{int}}(\alpha_i,\beta_j,\chi_o) = \frac{\alpha 2\sqrt{I_{01x} I_{02x}}\cos\theta}{K C_i C_j C_o}\int_{-\frac{a}{2}}^{+\frac{a}{2}} e^{-\alpha x} K_x(\alpha_i,x)dx \int_{-\frac{b}{2}}^{+\frac{b}{2}}\int_{-\frac{c}{2}}^{+\frac{c}{2}} e^{-2(y^2+z^2)/w_{ox}^2} K_y(\beta_j,y) K_z(\chi_o,z) dydz \qquad (10)$$

3. Model and results of the sample under one laser beam irradiation

Various characteristics of dielectrics under one laser beam irradiation have been very well studied in literature. See the case of a ZnSe sample (all characteristics of the material can be found in reference 1).

Let us suppose that the sample is parallelepiped and the dimensions are: a = 10 mm, b = 4mm and c = 4 mm. The present paper is aimed to analyze just one aspect (regarding the one laser beam irradiation case): the temperature field values as a function of the distance along laser beam propagation (which is supposed to be Gaussian and centered on the sample surface). Fig. 1 presents the temperature field when x = 0. This corresponds to the middle of the sample. Let have the case of a cw CO_2 laser beam, with total power 10 W, operating in the TEM_{00}. Fig. 2 presents the temperature field when x = 5 mm. This corresponds to the sample surface. It is obvious that temperatures are lower because the influence of the heat transfer coefficient is higher at the sample surface.

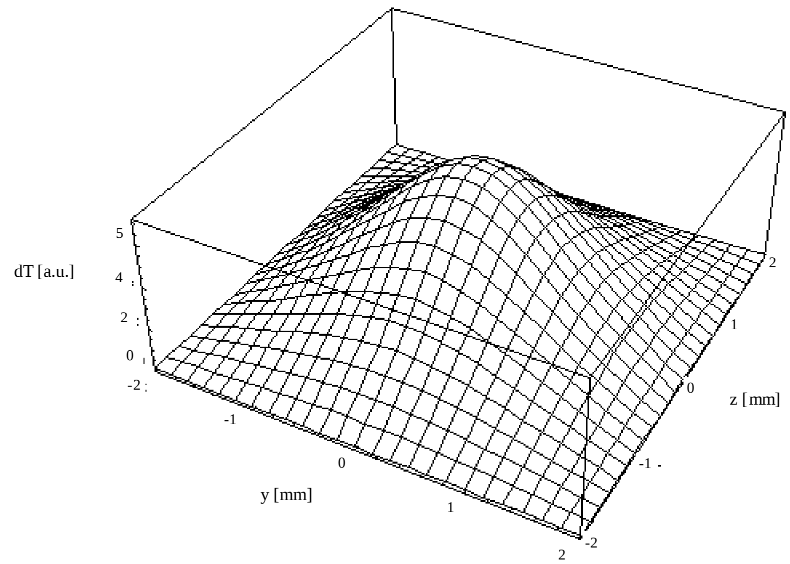

Fig. 1. The temperature field in the plane x = 0, during a 100 s irradiation with a 10 W CO_2 laser beam.

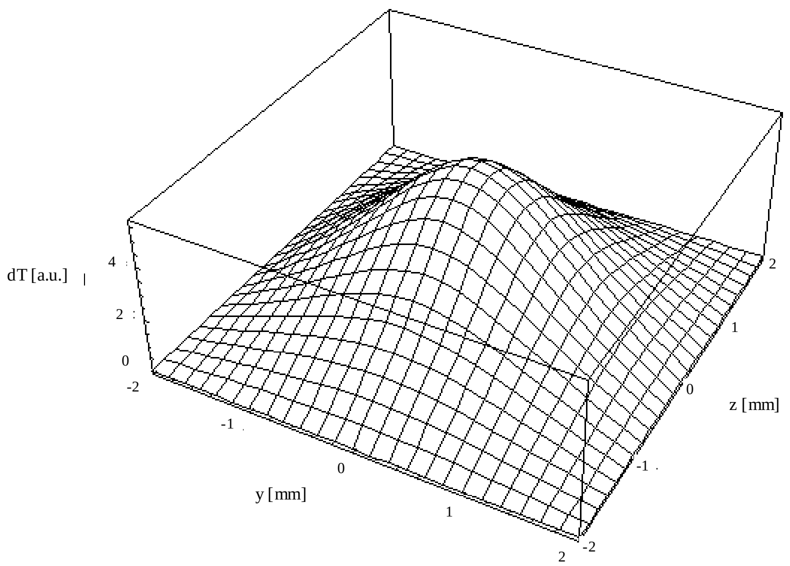

Fig. 2. The temperature field in the plane x = 5mm, during a 100 s irradiation with a 10 W CO_2 laser beam.

4. Model and results of the sample under two laser beams irradiation

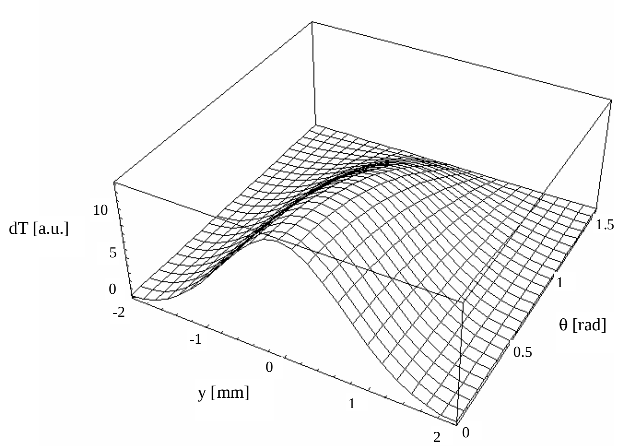

Fig. 3. The "interference" temperature variation as a function of interference angle.

Fig. 3, is a plot of: $\Delta T_{\text{int}}(x, y, z, t)$, as a function of interference angle θ. We have considered two laser beams of the same power (10 W) and the same irradiation time (100 s). One observes showed that $\Delta T_{\text{int}}(x, y, z, t)$ has the maximum value when θ is equal to 0. The temperature interference term has a value almost equal with the sum: $\Delta T_1(x, y, z, t) + \Delta T_2(x, y, z, t)$. In other words, one can say, that the interference term is the dominant one. For θ with values close to $\pi/2$, the interference term is no longer the dominant term ; and for $\theta = \pi/2$ we have $\Delta T_{\text{int}}(x, y, z, t) = 0$. For plotting the interference temperature for values close to $\pi/2$, we have taken , for practical reasons, a<<b and a<< c (in order to preserve the boundary conditions (2)). For those interested in the superposition of two or three laser beams, without taking into account the interference, it is recommended the reference 6. In Fig. 4 is shown a plot of the thermal field when two laser beams, which do not interfere are involved.

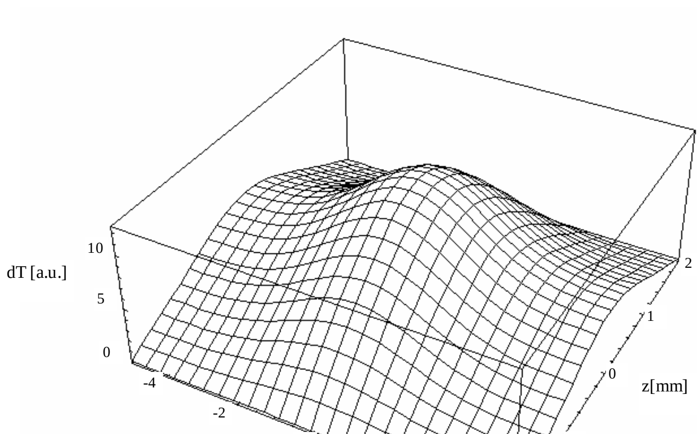

Fig. 4. The temperature variation as a function of Cartesian coordinates when the interference.

5. Conclusions

Some of the first questions that must be answered before estimating whether a laser is good or not for a particular application are: how much power is required?; how long must this power be applied?; etc..

In many situations the answers to these questions can be obtained by performing simple calculations based on classical heat equation. The purpose of the present paper is to find out some solutions to the classical heat equation, when the solid sample is under two laser beams irradiation. Solutions to the heat equation can only be obtained in analytical form when one makes a variety of approximations concerning the spatial and time dependence of the incident laser and the geometry of the sample. As the description of the boundary conditions becomes more and more accurate in terms of the heat source, the interaction laser-solid becomes more and more rigorous, analytical solutions can no longer be obtained and the expression for $T(x,y,z,t)$ can be obtained only numerically. The present paper, show that in the case of a solid under one and two beams the heat equation is able to make predictions of $T(x,y,z,t)$.

Our model can be applied especially for the cases when laser-solid interaction is expressed by a low absorption coefficient which involves low temperature variation and, therefore, one may suppose there is no temperature dependence of the optical (R,α) and thermal (k,γ) parameters.

Our study indicates that for a sample under one or two laser beam irradiation, the heat equation has an exact analytical solution, even when the interference is considered. This solution is not a simple summing-up of solutions from two one-dimensional heat equations, because $T_1(x,y,z,t), T_2(x,y,z,t)$ are coupled via boundary conditions (2).

Moreover, one should take into account the interference phenomenon. Our model can be easily generalized for the cases when: $h_1 \neq h_2 \neq h_3 \neq h_4 \neq h_5 \neq h_6$. The case of lacking interference, the thermal fields in solids under multiple laser irradiation is given in reference 6.

The model could be applied to any laser-solid system whose interaction can be described by Beer law [7-8].

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