SYNCHRONIZATION OF CHAOTIC ELECTRONIC SPROTT'S CIRCUITS

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Many linear and nonlinear control methods have been employed to control chaos. In practical applications the investigators would like to use simple and efficient controllers. The design of the simplest controller is an open problem yet. In this paper we applied Master-Slave Synchronization to the chaotic Sprott's circuits. These circuits may have practical application in secure communications because they can be controlled.

1. Introduction

Over the last three decades, chaos in engineering systems such as nonlinear circuits has gradually moved from being simply a curious phenomenon to one with practical significance and applications. Chaos has been found to be useful or have great potential in many disciplines such as in high-performance circuit design for telecommunication, thorough liquid mixing with low power consumption, collapse prevention of power systems, biomedical engineering applications to the human brain and heart.

In 1963, Lorenz found the first chaotic system, which is a third-order autonomous system with only two multiplication-type quadratic terms but displays very complex dynamical behaviors. In 1999 Chen found another similar but topologically non-equivalent chaotic system, the Chen system being a dual system to the Lorenz system. Now there are some analytical results reported about these chaotic systems, which are called the Lorenz family. In addition many other chaotic system were reported: Duffing, Van der Pol, Rossler, Lu, Chua, Sprott and so on. Chaotic systems have the distinguishing feature of extreme sensitivity to variations of initial conditions. Due to this intrinsic dynamical complexity, chaos was once believed neither controllable nor predictable, characteristics that are very undesirable in engineering. Chaotic attractors contain theoretically an infinite number of unstable periodic orbits (UPOs).

Stabilizing UPOs means to control chaotic systems. So a controlled chaotic system can have a lot of opportunities in terms of predictable periodic behaviors and could be more useful than a nonchaotic one. A reliable control is more than necessary.

The importance of synchronization does not only consist in the practical applications that can be obtained, but also in the many phenomena that be explained by synchronization theory. Many systems can be modelled as oscillators or vibratory systems and those systems show a tendency towards synchronous behaviour.

Since its introduction by Pecora and Carrol in 1990 chaos synchronization has received increasing attention due to great potential applications in many discipline such as nonlinear circuits, chemical reaction, biomedical engineering to the human brain and heart and so on. The state of the art is contained in several books [1-5] and review papers and special issues of some journals. There are known several types of synchronization: mutual, master-slave, weak, strong, phase and generalized. The methods are specific or are borrowed from control theory.

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One of the master-slave synchronization is the OPCL synchronization [6] and has the roots in the OPCL control method[7]. This method gives precise driving for any continuous system in order to reach any desired dynamics. In this paper we apply the OPCL synchronization strategy to the synchronization of two identical Sprott circuits[8]. These circuits contain only resistors, capacitors, diodes and inverting operational amplifiers (Fig. 1). G(x) is a nonlinear function presented in Table 1 and that can be realized practically (Fig. 2).
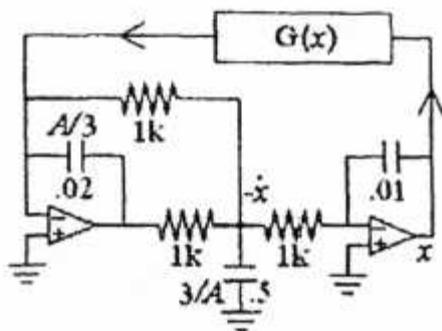
Fig. 1. A general circuit [8].
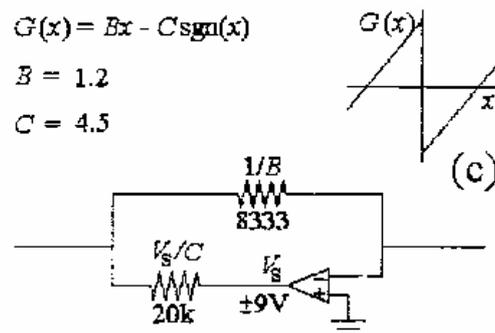
Fig. 2. A nonlinear form for G(x) that produces chaos and circuit to produce it [8].

Table 1 [8].

G(x)	B	G(x)	B
$\pm (B|x| - C)$	1	$Bx(x^2/C - C)$	1.6
$-B\max(x,0) + C$	6	$-Bx(x^2/C - C)$	0.9
$Bx - C\operatorname{sgn}(x)$	1.2	$B[x - 2\tanh(Cx)/C]$	2.2
$-Bx + C\operatorname{sgn}(x)$	1.2	$\pm B\sin(Cx)/C$	2.7
$\pm B(x^2/C - C)$	0.58	$\pm B\cos(Cx)/C$	2.7

2. The OPCL method

The original system

$$\dot{\mathbf{x}} = F(x); \qquad x \in R^n \tag{1}$$

has to be driven with the term:

$$D(t,x,g) = \dot{\mathbf{g}} - F(g) + (A - \partial F/\partial x|_{x=g})(x - g) \tag{2}$$

in order to reach the goal dynamics $g(t) \in R^n$.

The drived system

$$\dot{\mathbf{x}} = F(x) + D(t,x,g) \tag{3}$$

assures the convergence $x(t) \to g(t)$, for $\| x(0)\text{-}g(0) \|$ small enough. A is a constant matrix with negative real part eigenvalues. So, any two oscillators can be synchronized: a Lorenz system can be driven to oscillate like a Rossler one or the inverse, but the driving term can be large and/or complicated.

Let's consider the master system:

$$\dot{\mathbf{X}} = F(X); \qquad X \in R^n \tag{4}$$

then the slave system:

$$\dot{x} = F(x) + (A - \partial F/\partial x|_{x=X})(x - X) + \ldots \tag{5}$$

assures $x(t) \to X(t)$ for any $\parallel x(0)-X(0) \parallel$ small enough. The last terms in Eq. (5) are the couplings and they are a particular case of Eq. (2) with $g \equiv X$. An important disadvantage of this general method is that the coupling term could be complicated and hard to be implemented in practical/engineering applications. A careful choice of A gives good results. In a recent work [9] we applied the OPCL method to obtain synchronization of two identical systems for Sprott's collection [10].

3. Results and discussion

The chaotic electronic Sprott's circuits can be described by the equation:

$$\ddot{x} + A\ddot{x} + x = G(x) \tag{6}$$

or

$$
\begin{aligned}
dX_1/dt &= X_2 \\
dX_2/dt &= X_3 \qquad \text{with } X_1 = x \\
dX_3/dt &= -AX_3 - X_2 + G(x)
\end{aligned} \tag{7}
$$

To synchronize two chaotic Sprott's circuits we can choose **A** from (5) as follows:

$$
\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & -1 & -A \end{pmatrix} \tag{8}
$$

The Routh-Hurwitz conditions (for characteristic equation (8) is to have negative real part eigenvalues) are:

$$a_1 > 0; \quad a_1 a_2 - a_3 > 0; \quad a_3 > 0$$

The Routh-Hurwitz conditions give:

$$a \in (-0.6; 0) \quad \text{for } A = 0.6 \tag{9}$$

Let be the circuit from Fig. 2. This means $G(x) = Bx - C \operatorname{sgn}(x)$ and the system is a chaotic one (Fig. 3).
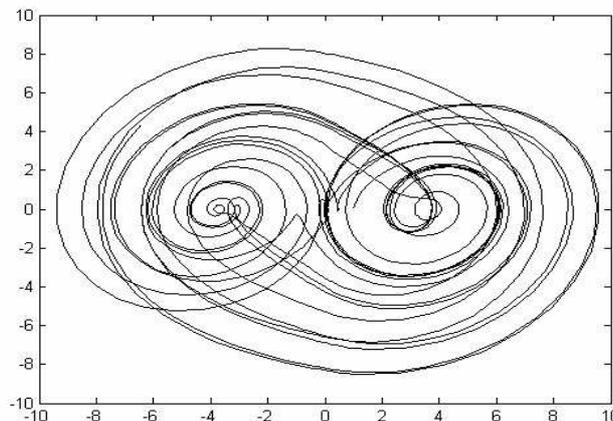
Fig. 3. The strange attractor for Sprott's system with $G(x) = Bx - C \operatorname{sgn}(x)$ ($X_1(0)=1$; $X_2(0)=0.1$; $X_3(0)=0.01$).

The slave system (of the master system (7)) with G(x)= $Bx - C\operatorname{sgn}(x)$ and a=-0.5 is:

$$dx_1 / dt = x_2$$
$$dx_2 / dt = x_3 \qquad\qquad\qquad \text{if } X_1 > 0$$
$$dx_3 / dt = -0.6x_3 - x_2 + 1.2 * x_1 - 4.5 - 1.7 * (x_1 - X_1)$$

else

$$dx_3 / dt = -0.6x_3 - x_2 + 1.2 * x_1 + 4.5 - 1.7 * (x_1 - X_1) \qquad\qquad (10)$$

Numerical results are given in Fig.4 for (X_1, x_1).
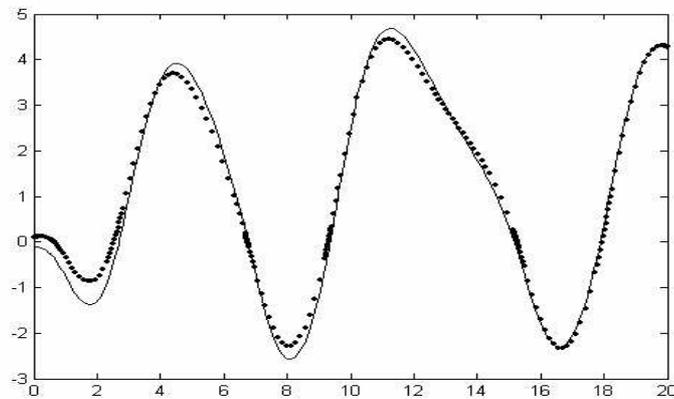
Fig. 4. Numerical results (X_1, x_1) for Sprott's circuit with G(x)= $Bx - C\operatorname{sgn}(x)$ ($X_1(0)=X_2(0)= X_3(0)=0.1$; $x_1(0) = x_2(0)= x_3(0)=-0.1$).

2. Let be the Sprott's system when G(x)= $B[x - 2\tanh(Cx)/C]$, (Fig.5)
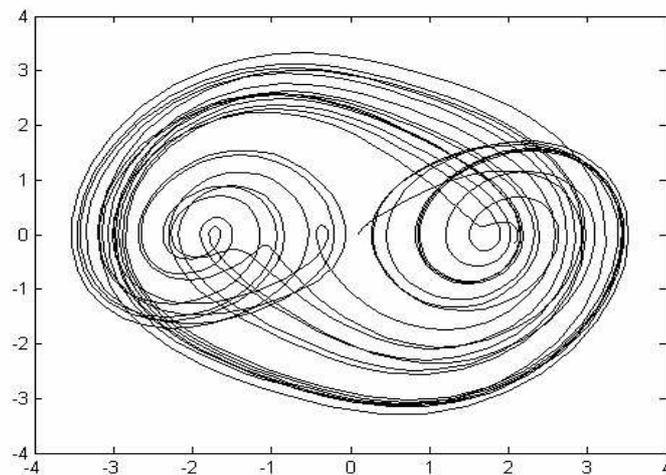
Fig. 5. The attractor for Sprottt's circuit with G(x)= $B[x - 2\tanh(Cx)/C]$
($X_1(0)=0.1$; $X_2(0)=0.01$; $X_3(0)=0.001$).

The slave system (of the master system (7)) with a=-0.5 and G(x)= $B[x - 2\tanh(Cx)/C]$ is

$$dx_1/dt = x_2$$

$$dx_2/dt = x_3$$

$$dx_3/dt = -0.6x_3 - x_2 - 2.2*(x_1 - 2*\tanh x_1) + (-0.5 + 2.2(1 - 2/\cosh^2 X_1))*(x_1 - X_1)$$

Numerical results are given in Fig.6
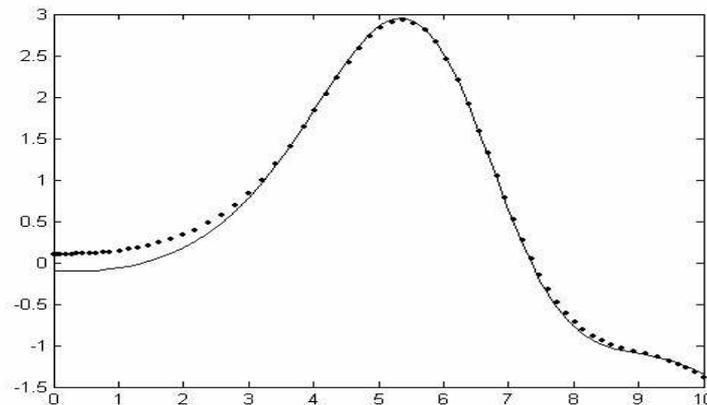
Fig. 6. Numerical results (X_1, x_1) for Sprott's circuit with G(x)= $B\left[x - 2\tanh(Cx)/C\right]$
($X_1(0)$=0.1; $X_2(0)$=0.01; $X_3(0)$=0.001; $x_1(0)$ =-0.1; $x_2(0)$=0.01; $x_3(0)$=-0.001).

4. Conclusions

The chaotic Sprott's circuits may have practical application in secure communications because they can be controlled. In addition they are very well suited for detailed quantitative testing of chaotic properties. The transient time until synchronization depends on initial conditions of two systems and on the values of negative part of eigenvalues. As we can see from Fig. 4 and Fig. 6, the fastest synchronization is for G(x)= $B\left[x - 2\tanh(Cx)/C\right]$.

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