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Temporal phase synchronization in a double electrical discharge plasma

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Temporal dynamics of space charge structures have been extensively investigated both experimentally and by simulation, showing a variety of nonlinear comportments such as bifurcations, chaos, mode locking, synchronization, stochastic resonance, etc. In this paper, we present an alternative analysis of phase synchronization in a system of two coupled van der Pol oscillators modeling the dynamics of charge structures in a double electrical glow discharge plasma. This system was previously demonstrated to model correctly the nonlinear dynamics of this particular experimental.

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1. Introduction

Synchronization is a very important phenomenon in many branches of science and in various natural and artificial systems. In the case of chaotic systems, this problem has received particular attention in the last decade [1-5].

In the classical sense, synchronization means adjustment of the frequencies of periodic oscillators due to weak interaction. Recently, the *phase synchronization* of a chaotic system was described as phase locking with no restriction imposed on amplitudes [1]. If the oscillations are close to harmonic, the phase changes nearly uniformly, while in an arbitrary periodic situation, the dynamics of the phase is affected by the change of the amplitude.

Unlike the *temporal phase* defined as the elapsed time measured from an arbitrary reference divided by the intrinsic period [4], we use the *instantaneous angular frequency* for the description of the time-evolution of the system in phase space trajectories.

Coupled non-linear oscillators can model a large class of complex dynamics observed in different physical systems, as e.g. metal-ferroeloectric-semiconductor heterostructure [6]. In a previous paper [7] we have shown that the harmonic perturbation of a chaotic double electrical discharge plasma can induce transition to ordered states, depending on the parameters of the perturbation.

In the present work, the phenomenon of phase synchronization of a system of two coupled nonlinear oscillators under the influence of an external harmonic driving is investigated experimentally and numerically using a system of two perturbed coupled van der Pol oscillators modeling a double electrical discharge plasma.

2. Experimental procedure

The experiments were performed using an experimental arrangement with two adjacent electrical glow discharges schematically shown in Fig. 1. Additional constructional details about the discharge tubes and apparatus are given elsewhere [8].

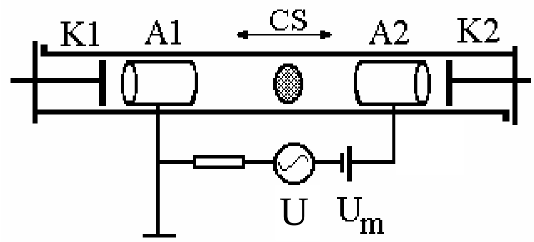


Fig. 1. Sketch of the experimental device.

Our experimental observations were performed on a symmetrical plasma system consisting of two electrical discharges with the anodes facing each other at a distance of 50 centimeters. The two anodes are biased against each other by a dc voltage U_m and optionally by a variable periodic voltage connected in series with the dc source (Fig. 1). A space charge structure (CS), like a double layer one, appears in special conditions, at the contact region between the two discharge plasmas. They are running in the same glass tube (100 cm length, 8 cm inner diameter), in argon flow at low pressure (<1Torr), in the region between the electrodes A_1 and A_2, respectively. The CS oscillations appear when the biasing potential between the anodes A_1 and A_2 exceeds the ionization potential of the working gas. For initial values of the dc biasing voltage a proper frequency g_o of oscillation that depends on the internal conditions is observed. The dynamics of this structure was experimentally investigated and numerically simulated in [8].

In this work, following the phase synchronized phenomenon, we investigate the regular dynamics of CS in a perturbed regime. Such a regime is realized when a small amplitude sinusoidal voltage U_a is superimposed on the dc biasing potential to supplementary bias the inter-anode space of the two discharges. Experimental spectra of the I_{12} current flowing in the discharge A_1A_2 interval were observed for various frequencies of the sinusoidal perturbation in the range 0-100 kHz. Also, time resolved measurements on the local CS plasma parameters correlated with the I_{12} oscillations give the possibility to see the synchronized dynamics of the CS structure with the oscillation phase of the U_a perturbing voltage. An example of space-time evolution of the synchronized CS dynamics is represented in Fig. 2 by time-resolved light intensity distributions (at every 25 μs) along the inter-anode A_1,A_2 interval. These stroboscopic „images” of the A_1A_2 plasma region are correlated with the time evolution of the I_{12} current flowing through the same region.

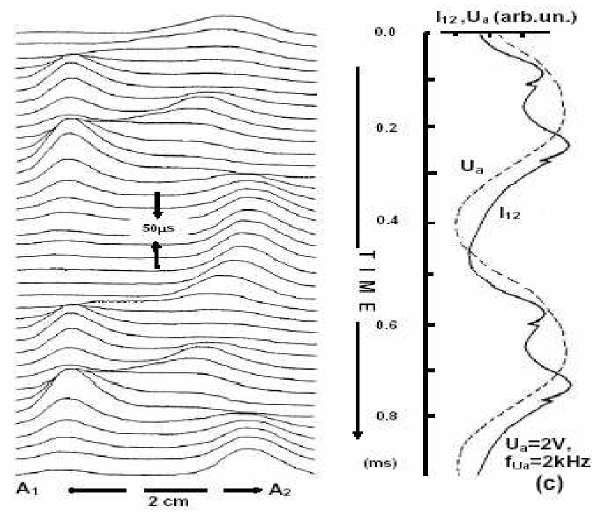


Fig. 2. Spatiotemporal representation of the dynamics of the CS synchronized under the influence of the sinusoidal signal shown as dashed line on the right side.

3. Computer model

The computer model mimics the dynamics of the CS generated in the inter-anode space of the two glow discharges [8, 9].

$$
\begin{cases}
\dot{x}[1] = x[2] + mx[4] \\
\dot{x}[2] = -c\left(x[1]^{2} - 1\right)x[2] - x[1] + e\cos\left(x[5]\right) + m(x[4] + x[3]) \\
\dot{x}[3] = x[4] - mx[2] \\
\dot{x}[4] = -n\left(x[3]^{2} - 1\right)x[4] - x[3] - e\cos\left(x[5]\right) - m(x[2] + x[1]) \\
\dot{x}[5] = g
\end{cases}
$$

Here, x[1] and x[3] stand for the flowing currents in the two discharges while x[2] and x[4] represent voltages. The periodic forcing term, with amplitude e and frequency g, is introduced in a conventional way and corresponds to the ac perturbing supply. The characteristics of the two individually supplied discharges are included in the nonlinearity constants c and n, while the biasing potential U_m is related to the coupling parameter m – taken as theoretical control parameter.

Based on the phenomenology of coupled Van der Pol oscillators some results of a modeling of the perturbed dynamics of the CS structure were presented in [9, 10].

The plots in Fig. 3 show the transition to synchronization induced by a small amplitude sinusoidal signal (upper trace). Middle and lower traces represent the amplitudes of the first (x[1]) and of the second (x[3]) oscillators, respectively. The synchronization process is clearly visible in these time-series and also the robustness of the synchronized state once reached.

On the left side of Fig. 3, the phase portraits for the synchronized state corresponding to the oscillators are represented.

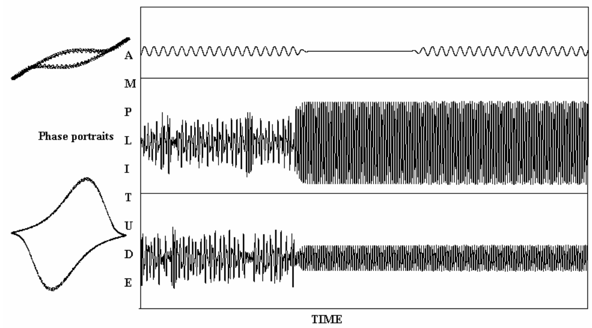
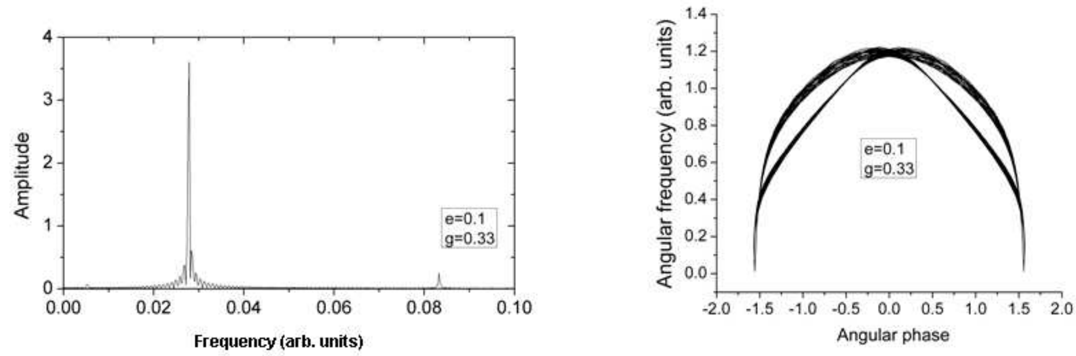


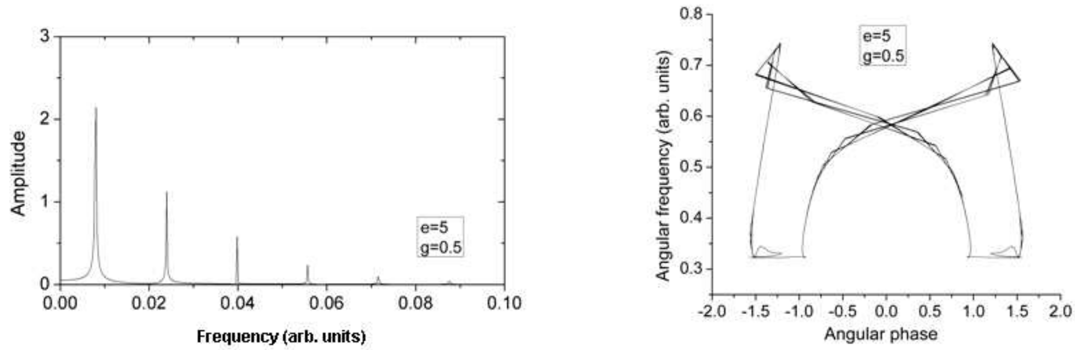
Fig. 3. Transition from chaos to periodic oscillations (synchronization) induced by a small amplitude sinusoidal signal. This situation corresponds to the spectrum in Fig. 4a.

The projection of the limit cycle attractor onto the plane corresponding to one of the oscillators (e.g. x[1]-x[2] plane) is usually a complicated curve. The representative point on this trajectory has a velocity that changes from one region to another. Taking a reference point in the neighborhood of the “center” of this cycle we can define the angular phase simply by the angle:

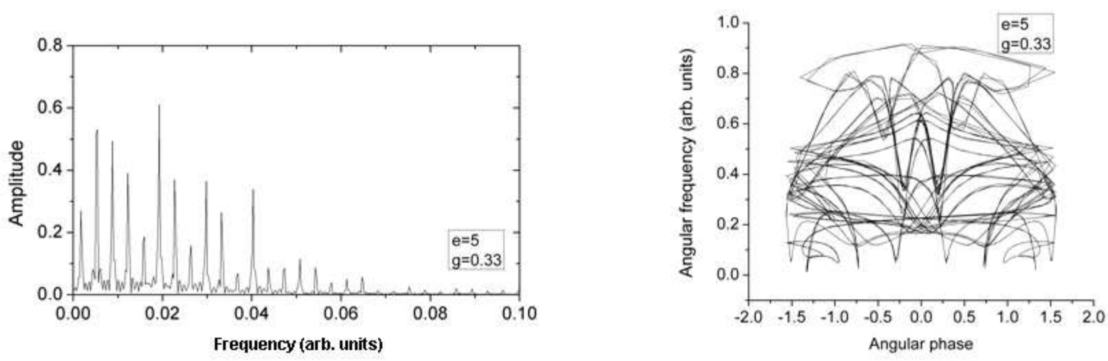
$$\varphi = \arctan\left(x[2]/x[1]\right)$$



(a)



(b)



(c)

Fig. 4. Spectra of x[1] and the corresponding „instantaneous angular frequency" versus angular phase diagrams for three synchronization regimes: (a) oscillation on the intrinsic frequency of the system for e=0.1; g=0.33; (b) frequency locking on the driving signal for e=5; g=0.5; (c) quasi-periodic superposition for e=5; g=0.33.

For the stable limit cycle dynamics, we propose the instantaneous angular frequency ω for the description of the time-evolution of the system in phase space trajectories. We compute ω as:

$$\omega \cong \frac{\sqrt{\Delta x[1]^2 + \Delta x[2]^2}}{\Delta t\sqrt{\left(x[1]^2 + x[2]^2\right)}}$$

Here, Δt represents the computation time step, $\Delta x[1], \Delta x[2]$ are the changes of the two variables in this interval and the denominator represents the distance from the chosen center (x[1]=0, x[2]=0) to the middle of the respective trajectory segment. This way of computing ω is a more general method than by numerical differentiation of $\theta(t)$.

The instantaneous angular frequency versus angular phase diagram represents the polar coordinates phase space $(\dot{\theta}, \theta)$ corresponding to the cartesian phase space (x[1],x[2]) shown on the left of Fig. 2.

4. Discussion

The main result of this work is presented in Fig. 4. Plot (a) represents side by side the spectrum of the synchronized system in the presence of a small amplitude harmonic perturbation and the corresponding instantaneous angular frequency versus angular phase diagram. The synchronization consists in the transition of the system from a chaotic state to a periodical one as shown in Fig. 3. The synchronized oscillation is taking place on an intrinsic frequency characteristic of the unperturbed system for a slightly different value of the control parameter *m*, the velocity of the characteristic point on the phase space trajectory. This information cannot be obtained from other types of processing e.g. by Cartesian coordinates phase space portraits.

Plots (b) show the same type of representation for frequency locking induced by high amplitude of the driving signal, for a frequency ratio:

$$\frac{g_0}{g} = 5.4$$

The plots in Fig. 4(c) indicate the quasi-periodic dynamics induced by the same high value amplitude of the driving signal but for an incommensurable frequency ratio of

$$\frac{g_0}{g} = 3.861622...$$

The work is still in progress and we are looking for getting other experimental time-scale information on the basis of the results of this analysis.

5. Conclusions

The phenomenon of phase synchronization of a system of two coupled nonlinear oscillators under the influence of an external harmonic driving has been investigated experimentally and numerically using a system of two perturbed coupled van der Pol oscillators modeling a double electrical discharge plasma.

The synchronization consists in the transition of the system from a chaotic state to a periodical one as shown in Fig. 3. The synchronized oscillation is taking place on an intrinsic frequency characteristic of the unperturbed system for a slightly different value of the control parameter *m*, the velocity of the characteristic point on the phase space trajectory.

Acknowledgements

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References

[1] A. Pikovsky, M. G. Rosenblum, G. V. Osipov, J. Kurths, Physica D, **104**, 219 (1997).
[2] C. P. Cristescu, C. Stan, D. Alexandroaei, Phys. Rev. E **70,** 016613 (2004).
[3] C. Stan, C. P. Cristescu, D. Alexandroaei, Proc. of XXVI ICPIG, J. Meichsner, D. Loffhagen, H. E. Wagner (eds), vol.4, Contr. Paper, p. 185-186, Greifswald, Germany, (2003).
[4] S. A. Oprisan, C. C. Canavier, Neural Computation **14**, 1027 (2002).
[5] U. Parlitz, L. Junge, L. Kocarev, Phys. Rev. E **54**, 6253 (1996).
[6] B. Mereu, M. Alexe, M. Diestelhorst, C. P. Cristescu, C. Stan, J. Optoelectron. Adv. Mater. 7(2), 691 (2005).
[7] C. Stan, C. P. Cristescu, D. Alexandroaei, J. Optoelectron. Adv. Mater. 7(5), 2451 (2005).
[8] C. Stan, D. Alexandroaei, C. P. Cristescu, Romanian Reports in Physics, **54**(1-5), 185 (2002).
[9] C. Stan, C. P. Cristescu, D. Alexandroaei, Contrib. Plasma Phys. **42**, 81 (2002).
[10] C. Stan, C. P. Cristescu, D. Alexandroaei, Phys. Rev. E **66**, 016602 (2002).

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