

Modeling of viscosity phenomena in models of hysteresis with local memory

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Magnetic viscosity phenomena are analyzed in the framework of various models of hysteresis with local history by using the Monte Carlo technique. Numerical results related to the decay of the magnetization as of function of time as well as to the viscosity coefficient are presented. It is shown that a $\log t$ - type dependence of the average value of the magnetization can be qualitatively predicted even in the framework of simplified models of hysteresis, such as the Jiles-Atherton and Hodgdon models. These models have only local memory and are computationally much more efficient than models with global memory like Preisach-type models.

(Received March 15, 2006; accepted May 18, 2006)

Keywords: Jiles-Atherton model, Hodgdon model, Aftereffect, Magnetic hysteresis, Magnetic viscosity

1. Introduction

The state of a magnetic hysteretic system can change from higher energy metastable states to lower energy metastable states due to thermal energy fluctuations. This phenomenon is responsible for the gradual change of magnetization over time [1] and is usually referred to as magnetic viscosity or aftereffect. While the physical origin of viscosity phenomena is well-known, the mathematical modeling of these phenomena is difficult since it usually requires solving highly nonlinear and history dependent stochastic equations.

There are two different approaches to the analysis of viscosity phenomena. The first approach [2,3] is based on thermal activation models and is usually applied to "noninteracting particle" magnetic systems or bulk materials. The second approach is based on the assumption that the effect of random thermal agitations is equivalent to the effect of a stochastic input superimposed on the input variable. This approach is more general since it does not require any assumption about the nature of the magnetic system. It was first introduced by Neél [1,4] and then developed and applied to the Preisach model by Mayergoyz *et al* [5] who established analytical equations for the decay of the average value of the output of a hysteretic system described by rectangular elementary hysteresis loops. Although very powerful, the technique developed in [5] can be applied only to hysteresis models that are based on superposition of elementary hysteresis loops, such as the classical or generalized Preisach models of hysteresis [6]. It is practically impossible to find analytical equations for the decay of the average value of the output variable in systems described by other models of hysteresis like the Jiles-Atherton model [7] and the Hodgdon model [8].

In this paper we analyze for the first time the aftereffect phenomena by using the Monte Carlo method.

This method has the advantage that it is universal in the sense that it can be applied to describe viscosity phenomena in the framework of any model of hysteresis. The article is structured as follows. The basic idea of the Monte Carlo approach for the analysis of viscosity in magnetic materials is presented in Section II. Numerical results are presented and discussed in Section III, which is followed by Conclusions.

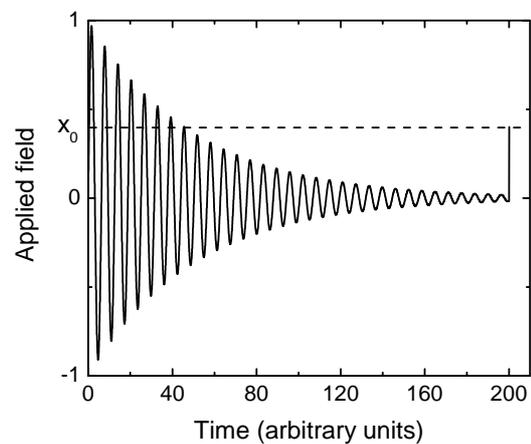


Fig. 1. The initial state of hysteresis is obtained by ac demagnetizing the magnetic material and then applying a magnetic field equal to H_0 .

2. Monte Carlo technique

Consider the following model of hysteresis:

$$M(t) = \hat{\Gamma}H(t), \quad (1)$$

where $M(t)$ is the magnetization, $H(t)$ is the total magnetic field, and $\hat{\Gamma}$ is the hysteresis operator. Following the technique presented in [5], we assume that the total magnetic field can be written as $H(t) = H_0 + h(t)$ where $h(t)$ is a stochastic process with zero expected value, which is superimposed on a deterministic input H_0 . To simplify the problem we further suppose that $h(t)$ is a discrete-time, i.i.d. random variable, and the total applied magnetic field can be rewritten as:

$$H_n = H_0 + h_n, \quad n = 1, 2, 3, \dots \quad (2)$$

where $\langle h_n \rangle = 0$. If the probability density function $\rho(h_n)$ is known, the average value of the magnetization can be computed as follows:

1. First, generate a large number of input processes h_n .
2. Then, solve eq. (1) for each input process, H_n , to compute M_n .
3. Finally, compute $\langle M_n \rangle$, the average value of the output as function of "time" n .

3. Numerical results

The technique described in the previous section has been numerically implemented and used to compute the dependence of the output of a magnetic system as a function of time. The probability distribution function of the applied magnetic field is chosen normal with standard deviation σ and mean H_0 :

$$\rho(H) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(H - H_0)^2}{2\sigma^2}\right). \quad (3)$$

An example of such a randomly generated magnetic field is presented in Fig. 2. The initial state of the magnetic material is obtained as follows (see Fig. 1). First, the sample is demagnetized by using a decreasing alternative magnetic field that has a relatively high initial magnitude that is slowly decreased to zero. After that, a constant magnetic field H_0 is applied and the magnetization is measured as a function of time. Unless otherwise mentioned, the applied field H_0 is equal to the coercive field of the material.

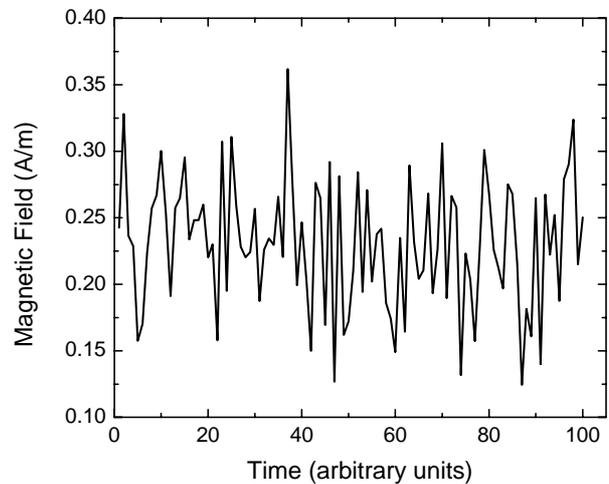


Fig. 2 Example of magnetic field generated randomly with normal distribution centered around the average value $H_0 = 0.24$ A/m.

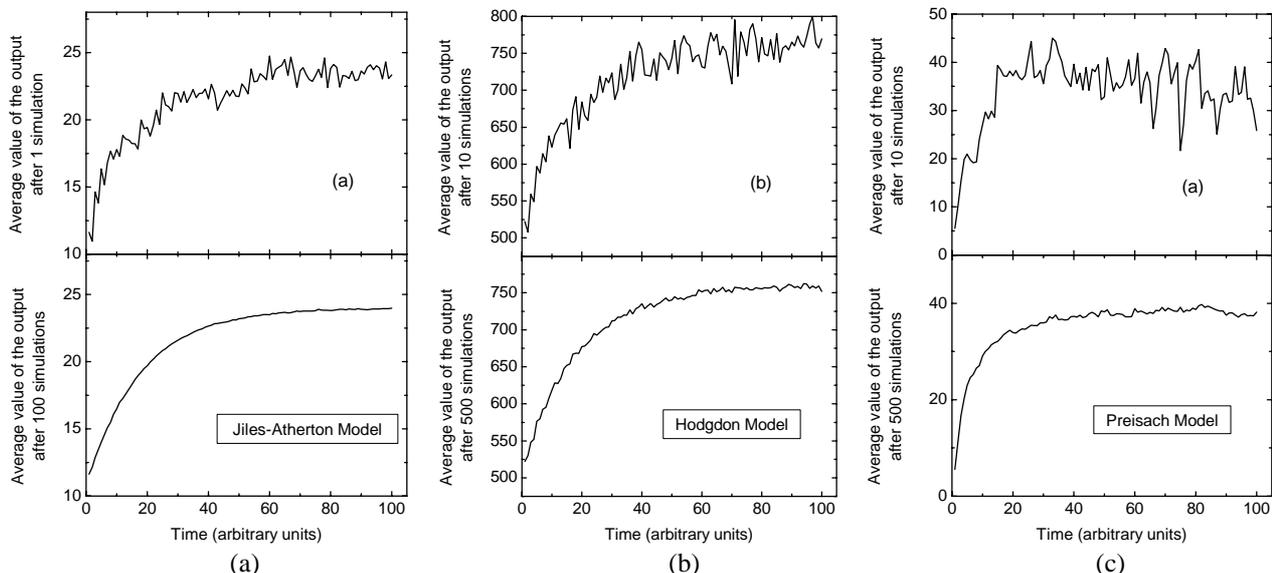


Fig. 3. Gradual change of the magnetization as a function of time computed in the framework of the Jiles-Atherton model (a), Hodgdon model (b), and generalized Preisach model (c).

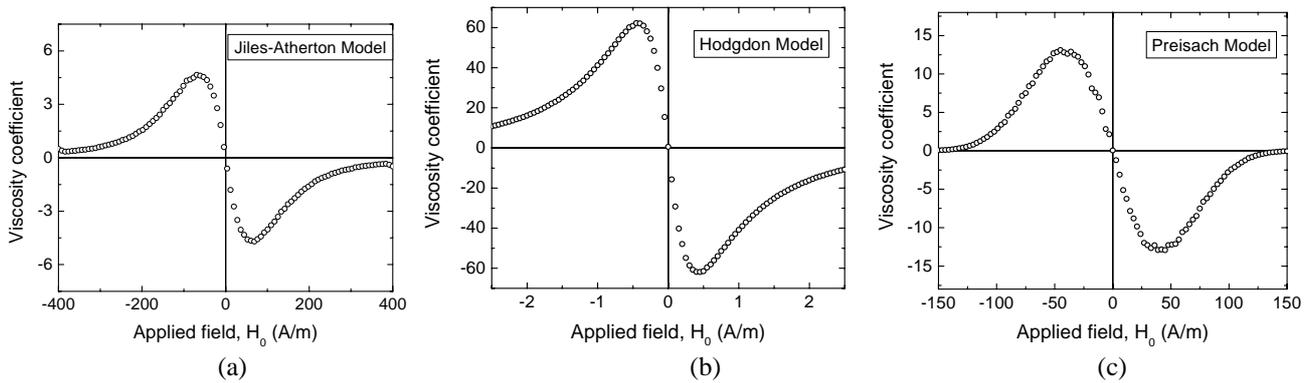


Fig. 4. Viscosity coefficient as a function of the mean value of the applied field (H_0) computed in the framework of the Jiles-Atherton model (a), and the Hodgdon model (b), and the generalized Preisach model (c).

Fig. 3 exhibit the change of the magnetization as function of time computed by using the Jiles-Atherton [7], Hodgdon [8], and generalized Preisach [6] models. In these simulation experiments the magnetic susceptibility was integrated numerically by using standard quadratures. In the case of the Jiles-Atherton model we have used the following set of parameters: $a = 41$, $k = 39$, $c = 0.85$, $M_s = 10^5$, and $\alpha = 0$. In the case of the Hodgdon model we have used $\alpha = 1$, $A_1 = 0.374$, $A_2 = 7.5 \times 10^{-7}$, $A_3 = -0.769$, $A_4 = 4.7 \times 10^{-8}$, $B_{cl} = 2 \times 10^6$, and all rate dependent parameters of the model were neglected. It is remarkable that, although both the Jiles-Atherton and the Hodgdon models are a fairly simple models that display only local memory, they predicts approximately correctly the $\log t$ - dependence of the magnetization, which is in good qualitative agreement with the published experimental data on magnetic viscosity [10]. It is apparent from these simulations that the computational burden is somewhat heavier in the case of the Hodgdon model than in the case of the Jiles-Atherton model. The asymptotic value of the magnetization can be computed by using a larger number of Monte Carlo simulations as compared to the Jiles-Atherton model. It is also important to observe that the Jiles-Atherton and Hodgdon models predict results that are in good qualitative agreement with those obtained by using the more elaborate generalized Preisach model [see Fig. 2(c)].

The viscosity coefficient (also known as the aftereffect decay coefficient) is defined as:

$$S = \frac{d\langle M(t) \rangle}{d(\log t)} \quad (1)$$

and is plotted in Fig. 4(a)-(c) as a function of applied field H_0 at which the gradual change in the magnetization is observed. These results are in very good agreement with the theoretical and experimental results obtained by Mayergoyz *et al* in [9]. The symmetry of the viscosity coefficient plots on these figures can be interpreted on the

basis of the symmetry of the magnetization processes: the total magnetization is equal to zero in the a.c. demagnetized state and a state with the same absolute value of the magnetization should be obtained if we apply a positive or a negative magnetic field.

4. Conclusions

Monte Carlo simulations provide a powerful tool for the analysis of aftereffect phenomena in magnetic materials. By using Monte Carlo simulations one can compute the gradual change of the magnetization as a function of time in the framework of any model of hysteresis such as the Jiles-Atherton, Hodgdon, and Preisach models, etc. It is shown that a $\log t$ - type dependence of the magnetization can be qualitatively predicted even in the framework of models with local history, which are computationally much more efficient than models with global history, such as the Preisach model.

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