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Length-scale dependence of vortex dynamics in HTSC superconductors

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The length-scale dependence and critical behaviour of vortex dynamics in high transition temperature superconductors (HTSC) are studied by means of renormalization group and by electrical transport methods. We present some current transport measurements on $Bi_2Sr_2CaCu_2O_8$ single crystals which indicate the need for a better description of the length-scale dependence of the vortex response in layered systems. Due to the weak coupling between the superconducting copper oxide layers and extremely high anisotropy of HTSC materials, the layered XY model is one of the most accepted model which describes the vortex-dominated properties of layered superconductors. The layered XY model can be mapped into the layered sine-Gordon model. We perform a renormalization group analysis of the layered sine-Gordon model and show that the model exhibits a Kosterlitz-Thouless type phase-transition.

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1. Introduction

The vortex dominated properties of high transition temperature superconductors can be considered by several theoretical and experimental techniques. Electrical transport measurement is a powerful experimental method to investigate the length-scale dependence and critical behaviour of vortices in layered systems. HTSC materials usually consist of copper-oxide superconducting planes separated by insulating layers. The CuO_2 planes interact with each other through Josephson coupling. Due to this weak coupling, especially in case of extremely high anisotropy like in $Bi_2Sr_2CaCu_2O_8$ (BSCCO), one of the commonly accepted model is the layered XY model where the weak interlayer coupling is given by a Lawrence-Doniach type term. The topological excitations in superconducting layers are vortex-antivortex pairs which can form vortex loops and rings by Josephson coupling. The layered XY model can be mapped onto the layered sine-Gordon model which is a two-dimensional quantum field theory where two sine-Gordon (SG) models are coupled [1]. The vortex dynamics and critical behaviour in layered superconductors can be considered by mapping out the phase structure of the layered XY and layered SG models.

The critical behaviour and phase structure of the layered XY and the layered SG model are investigated by means of various renormalization group (RG) methods using the dilute gas approximation [1]. One of the most complete and rigorous theoretical RG study of weakly coupled superconducting layers has been done by S.W. Pierson and his co-workers [1] using real-space and momentum-space RG methods. They obtained four regions in the current-temperature phase diagram where different correlation lengths between vortices determine the behaviour of the system.

The purpose of this paper is twofold. On the one hand, we analyse our recent experimental data obtained by secondary voltage measurements on BSCCO single crystals [2] in terms of the above theoretical predictions. We demonstrate that while below T_C, where T_C is the zero field 3D/2D dimensional transition temperature, the theoretical results are supported well by experiments, above T_C they differ from each other. This indicates the need for a better description of length-scale dependence of the vortex dimensionality at 3D/2D crossover.

On the other hand we perform an RG analysis for the layered SG model using the Wegner-Houghton RG method [3] in order to go beyond the dilute gas approximation and to achieve a better description of vortex dynamics.

2. Theoretical model. The layered sine-Gordon model

The vortex dynamics of HTSC materials can be considered in the framework of the anisotropic Ginzburg-Landau (GL) theory of superconductivity. The GL theory was developed by applying a variational method to an assumed expansion of the free-energy in powers of the local density of superconducting electron pairs which serves as a complex order parameter. In case of strong anisotropy, in the absence of fields and assuming that the amplitude of the order parameter is identical in each layer, the total free energy of the system reads as follows

$$F = \sum_n \int d^2 r \left(\frac{1}{2} J_{\text{II}} (\nabla \phi_n)^2 + J_{\perp} [1 - \cos(\phi_n - \phi_{n-1})] \right), \quad (1)$$

where ϕ_n is the phase of the order parameter. In this equation n runs from 1 to the total number of layers. The parameters of the model are $J_{\mathrm{II}} \approx 1/m_{ab}$ and $J_{\perp} \approx 1/m_c$, where m_{ab} and m_c are the effective masses. The last term is related to the Lawrence-Doniach term, in which the coupling between the layers is the Josephson coupling. Expanding the cosine term in Taylor series around zero, one arrives at the layered or quasi two-dimensional XY model [1]

$$H = \frac{1}{2}\sum_n \int d^2 r J_{\mathrm{II}} (\nabla \phi_n)^2 + \frac{1}{2}\sum_n \int d^2 r J_{\perp} (\phi_n - \phi_{n-1})^2 \, . \quad (2)$$

The layered XY model can be mapped onto a coupled sine-Gordon type model, it is called layered SG model, which in case of two layers has the following Lagrangian [4]

$$L = \frac{1}{2}(\partial \phi_1)^2 + \frac{1}{2}(\partial \phi_2)^2 + \frac{1}{2} J (\phi_2 - \phi_1)^2 + u[\cos(\beta\phi_1) + \cos(\beta\phi_2)] , \quad (3)$$

where ϕ_1, ϕ_2 are one-component scalar fields, u is the fugacity parameter of the vortices, $J = J_{\perp}/J_{\mathrm{II}}$ is the coupling between the two SG models and $\beta = J_{\mathrm{II}}$ is related to the temperature. Both are dimensionless coupling constants. For J=0 the layered SG model is reduced to two un-coupled SG models.

The critical behaviour and phase structure of the layered XY and layered SG models have been considered by various RG methods (e.g. anisotropic smooth cutoff momentum space RG analysis, real space RG) using the dilute gas approximation which assumes that the fugacity of the vortex gas (coupling constant u) is small [1]. The common result of the various RG analysis can be summarised as follows. Three characteristic critical currents or temperatures T_{UB}, T_C and T_{C2} were found ($T_{UB}<T_C<T_{C2}$). These results can be applied to explain the experimental current transport properties. Below T_{UB} the current is not strong enough to overcome the Josephson attraction and, so, vortex depairing is not possible. Between T_{UB} and T_C one can have thermally activated depairing of vortex loops. In this temperature range the Josephson coupling is finite, therefore the motion of free vortices in the upper layers is transmitted to the bottom layers and the vortices form vortex lines. The vortex system is three-dimensional, the current distribution is homogeneous. The system undergoes a Kosterlitz-Thouless type phase transition. Above the transition temperature T_C but below T_{C2} individual vortices can be spontaneously created and the layer decoupling begins. The vortex system is not purely three-dimensional and the current density becomes inhomogeneous. Above T_{C2} but below the Ginzburg-Landau transition temperature T_{C0} the layers are completely decoupled, therefore one expects a two-dimensional behaviour. We would like to emphasis that the critical temperature T_C which separates the two phases of the layered system is found to be independent of the number of layers.

3. Results and discussion

3.1. Electrical transport measurements

Electrical transport measurements were performed on $Bi_2Sr_2CaCu_2O_8$ single crystals. The chemical inhomogeneity and surface smoothness of the optically smooth rectangular crystals were measured by microbeam PIXE and atomic force microscopy, respectively. The superconducting quality of the samples was checked by DC and AC magnetization measurements. The mean-field Ginzburg-Landau transition temperature of the samples was 86÷88 K. The sample dimensions were about 1×1.5 mm^2, the thickness was between 8 and 3 μm. Electrical contacts to the surface of the samples were made by bonding 25 μm gold wires with Dupont 6838 silver epoxy. The contact resistance was a few ohms. Two current and potential electrodes were attached to both faces of the crystals. The current was injected into one face of the crystal through the current contacts. This is the primary current I_P. The voltage measured on the opposite face of the crystal where the current was injected is the secondary voltage V_S. In our experiments we measured the secondary voltage as a function of temperature and current density. More detailed description of the experimental arrangement is given in Ref. [2]. In Fig. 1 we show the temperature dependence of the secondary voltage measured on a BSCCO single crystal in zero magnetic field with different transport current. The measurement of secondary voltage gives a possibility to study the vortex dimensionality experimentally at the 3D/2D superconducting phase transition which is assumed to be a length-scale dependent layer decoupling process.

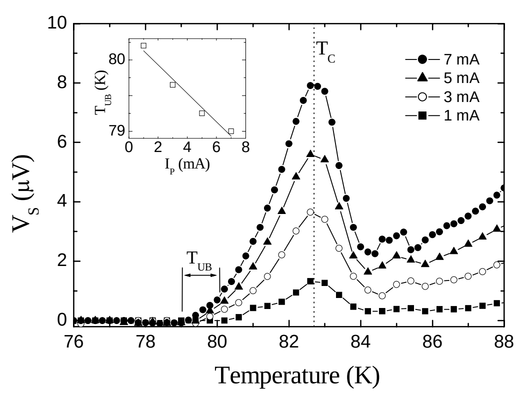

Fig. 1. Temperature dependence of the secondary voltage of a $Bi_2Sr_2CaCu_2O_8$ single crystal measured with different current. T_C and T_{UB} are the critical temperature and unbinding temperature, respectively. The inset shows the current dependence of the unbinding temperature.

In 2D superconducting layers the phase fluctuation of the order parameter generates vortex-antivortex pairs as topological excitations. The phase transition in an isolated 2D superconducting layer, where the vortex-antivortex pairs are bound below the phase transition temperature and are unbound above it, is described by the Kosterlitz-

Thouless theory [5]. In high-temperature superconductors, the high transition temperature, short coherence length and layered structure make the phase fluctuation of the order parameter dominant over the other fluctuations in the transition temperature range, but the coupling between superconducting layers modifies the 2D Kosterlitz-Thouless picture. The interaction between superconducting layers leads to vortex-antivortex interaction different from the 2D case. E.g. the 3D phase appears, where the coupling between neighbouring layers arranges the thermally excited pancake vortices into 3D flux lines. These 3D flux lines can form vortex loops as correlated vortex-antivortex line pairs. They are called thermally activated vortex loops, because they are the result of a combined effect of thermally activated vortex excitation and interlayer vortex coupling.

The 3D character modifies the structure of the phase transition between the bound and unbound states: a narrow 3D window appears around the phase transition temperature T_C and a nonzero critical current also appears [1]. Above the phase transition temperature the behaviour of vortices becomes 3D/2D due to decoupling of the superconducting layers (3D/2D phase transition). This layer decoupling is a length-scale-dependent process: the layers become decoupled at length scales larger than an interlayer screening length, while for lengths below this scale they remain coupled. In the 3D regime below T_C the electrical transport behaviour is dominated by vortex loops, above T_C it is dominated by vortex lines and pancake vortices.

In zero applied magnetic field the secondary voltage originates from thermally activated vortex loop unbinding. At low temperatures where V_S is zero, the thermally excited 3D flux lines form vortex loops which are 'pinned' to the crystal. With increasing temperature, the transport current splits these vortex loops into free vortex-antivortex line pairs. The temperature where this splitting starts is the unbinding temperature T_{UB}. Above the unbinding temperature the free vortices move in the sample like 3D vortex lines due to the Lorentz force, producing the same voltage drop on the primary and secondary side of the crystal. This 3D character of the vortex lines remains up to T_C where the secondary voltage has a local maximum. With increasing temperature the 3D character of flux motion disappears and V_S decreases, but another local maximum of V_S can be found as the temperature approaches the mean-field Ginzburg-Landau transition temperature. Consequently the temperature dependence of the secondary voltage has two peaks with a higher and a lower amplitude. This experimental result contradicts the predictions of the theoretical RG analysis [1] above T_C which indicates the need for a better description of the vortex dynamics in layered systems.

This double peak structure of secondary voltage can be explained by the motion of different types of vortex lines. In zero applied magnetic field free vortex lines can be produced in two ways. First, they can be the result of vortex-antivortex depairing of thermally activated vortex loops due to the Lorentz force of the transport current. In this case the number of free vortex lines and the unbinding temperature depend on the transport current density. Secondly, free vortex lines can be spontaneously created by thermal activation, mainly above T_C. While the maximum of V_S increases, T_{UB} decreases with the increase of current density, as it can be seen in Fig. 1.

3.2 Renormalization group analysis

In order to map out the phase structure of the layered system, we perform an RG analysis for the layered SG model by means of the differential RG approach in momentum space where the blocking transformations are realized by successive elimination of the field fluctuations according to their decreasing momentum in infinitesimal steps. The high-frequency modes are integrated out above the moving momentum cut-off *k* and the physical effects of the eliminated modes are encoded in the scale-dependence of the coupling constants. The elimination of the modes above the moving scale *k* is complete in Wegner's and Houghton's method (WH-RG) [4] because of the sharp momentum cut-off. The WH method provides a functional RG equation for the blocked action. In order to solve the WH-RG equation, one has to project it to a particular functional subspace. Therefore, one generally assumes that the blocked action contains only local interactions, expands it in powers of the gradient of the field, and truncates this expansion at a given order, for technical reasons. Here we restrict ourselves to the leading order of the gradient expansion, i.e. to the local-potential approximation (LPA). The WH-RG equation in LPA has been derived for two interacting scalar fields in Ref. [4] and can be written as

$$(2+k\partial_k)V_k(\phi_1,\phi_2) = -\frac{1}{4\pi}\ln\left[(1+V_k^{11})(1+V_k^{22})-(V_k^{12})^2\right], \quad (4)$$

where $V_k(\phi_1,\phi_2)$ is the dimensionless blocked potential, k is the moving momentum cut-off, $V_k^{ij}(\phi_1,\phi_2)$ is the derivative of the potential with respect to the fields ϕ_i,ϕ_j. The dimensionless blocked action for the layered SG model reads as

$$S_k = \int d^2x\left[\frac{1}{2}(\partial_\mu\phi_1)^2+\frac{1}{2}(\partial_\mu\phi_1)^2+V_k(\phi_1,\phi_2)\right], \quad (5)$$

where the dimensionless blocked potential is

$$V_k(\phi_1,\phi_2) = \frac{1}{2}J(k)[\phi_2-\phi_1]^2+u(k)[\cos(\beta\phi_1)+\cos(\beta\phi_2)]. \quad (6)$$

Notice, that all the scale-dependence is encoded in the coupling constants *J(k)* and *u(k)*. It is also important to note that the period length β has no scale-dependence due to the usage of LPA [4]. Inserting the ansatz (6) into the WH-RG equation (4) and separating the periodic and non-periodic part, the following differential equation is obtained for the coupling *J*

$$(2+k\partial_k)J(k)=0 \quad (7)$$

with the solution $J(k)=J_\Lambda\, k^{-2}/\Lambda^{-2}$, where J_Λ is the initial value given at the high-energy (ultra-violet, UV) cut-off Λ. Therefore, the coupling J is a relevant (increasing) parameter if the momentum cut-off goes to zero ($k \to 0$). In order to obtain RG equation for the coupling *u*, one has to expand the periodic part of the WH-RG equation in Fourier series and read off the differential equation for *u* which can be solved numerically. It is possible to obtain analytic solutions for *u* using various approximations. Linearizing the logarithm in the WH-RG equation (4) around the Gaussian fixed point ($V_{FP} \equiv 0$) the following RG equation is obtained

$$(2 + k\partial_k)u(k) = \frac{1}{4\pi}\beta^2 u(k) \qquad (8)$$

with the solution

$$u(k) = u_\Lambda \left(\frac{k}{\Lambda}\right)^{-2+\frac{\beta^2}{4\pi}}, \qquad (9)$$

where u_Λ is the initial value at the UV cut-off Λ. When $k \to 0$, for $\beta^2 > \beta_C^2 = 8\pi$ the coupling *u(k)* runs to zero and for $\beta^2 < \beta_C^2$ the coupling *u(k)* becomes infinitely large. The system undergoes a phase transition with the critical value $\beta_C^2 = 8\pi$. In this approximation scheme, which is equivalent to the dilute gas approximation used in Ref. [1], the critical value is independent of the coupling *J*. Therefore, this phase transition is equivalent to the well-known Kosterlitz-Thouless phase transition of the SG model (the layered model with one layer).

Better approximation can be achieved by incorporating the effect coming from the presence of the coupling *J*. Since the interaction term which describes the coupling between the two SG models can be considered as a mass term, the approximation is called mass-corrected UV approximation. The mass-corrected UV RG for *u* is discussed in Ref. [4] and reads as

$$(2 + k\partial_k)u(k) = \frac{1}{4\pi}\beta^2 \frac{1 + J_\Lambda k^{-2}}{1 + 2J_\Lambda k^{-2}} u(k), \qquad (10)$$

where J_Λ is constant. The solution is read as

$$u(k) = u_\Lambda \left(\frac{k}{\Lambda}\right)^{-2+\frac{\beta^2}{8\pi}} \left(\frac{k^2 + 2J_\Lambda}{\Lambda^2 + 2J_\Lambda}\right)^{\frac{\beta^2}{16\pi}}. \qquad (11)$$

The mass-corrected UV RG predicts two phases for the layered SG model, if $\beta^2 > \beta_C^2 = 16\pi$ the coupling constant *u(k)* is an irrelevant (decreasing) parameter, and if $\beta^2 < \beta_C^2 = 16\pi$ it becomes a relevant (increasing) coupling. The importance of this result is the modification of the critical value from $\beta_C^2 = 8\pi$ to $\beta_C^2 = 16\pi$ which separates the two phases of the model. The numerical solution of the full WH-RG equation (4) which goes beyond the approximations is in progress and the first results fully support this conjecture. Therefore, our RG approach which goes beyond the dilute gas approximation, predicts the modification of the critical temperature of the vortex system. In case of one layer the critical value is $\beta_C^2 = 8\pi$, and for the layered model in case of two layers $\beta_C^2 = 16\pi$. This result might suggest the dependence of the transition temperature on the number of layers.

4. Conclusions

In this paper we analysed the length-scale dependence and critical behaviour of the vortex dynamics in HTSC materials. We present current transport measurements on BSCCO single crystals which indicate the need for a better description of length-scale dependence of the 3D/2D crossover in vortex dimensionality. Due to the weak Josephson coupling between the copper oxide superconducting planes, a high temperature superconductor has a strongly anisotropic layered structure. The critical behaviour of the vortices in layered systems can be studied by means of RG analysis of the layered XY or layered SG models. Here, we considered the phase structure of the layered SG model using the Wegner-Houghton RG approach. This allows us to go beyond the dilute gas approximation [1] and to achieve a better description of vortex dynamics in HTSC materials. Our results suggest the dependence of the critical temperature which separates the two phases of the model on the number of layers.

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