JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS Vol. 8, No. 5, October 2006, p. 1736 - 1740

The residual stresses of FeBSi-type in an ingot mould

I. AŞTEFĂNOAEI[a], D.RADU[a], H. CHIRIAC[b]
[a]*"Al.I. Cuza" University, Carol I Blvd., 11, 700506, Iasi, Romania*
[b]*National Institute of Research and Development for Technical Physics,47 Mangeron Blvd., 700050 Iasi, Romania*

In this paper we analysed the internal residual stresses that appear during the rapid cooling of a FeBSi-type alloy, lain in the interior channel of an ingot mould whose exterior walls are maintained at the room temperature. The theoretical model presented in the paper emphasizes in a complete and synthetical manner, the spatio-temporal distribution of the induced stresses in a FeBSi-type alloy during its cooling to the room temperature, taking into account both the thermal behaviour of the alloy and the supplementary stresses induced by the ingot-mould, as a result of the difference between the thermal expansion coefficients of the two materials in contact.

(Received September 6, 2006; accepted September 13, 2006)

Keywords: Residual stresses, Ingot-mould, Thermal gradients

1. Introduction

The aim of this paper is to evaluate of the thermal stresses during the cooling of an alloy in the inner vertical channel of an ingot mould, whose exterior walls are maintained at the room temperature, $T_w = 300K$. More precisely, our purpose is to determine the spatio-temporal distribution of the stresses that appear during the rapid cooling of the whole material to the temperature T_w. This model contains two important tasks: (1)The evaluation of the spatio-temporal temperature's distribution during the rapid cooling [1, 2, 3] of the material to the room temperature. For different values of the radius of the inner channel, taking into account the thermal conditions on the alloy-ingot mould interface, we analyzed the spatial and temporal evolution of the temperature;(2)Starting from the spatio-temporal distribution of the temperature, one can obtain the stresses due to both rapid cooling (big thermal gradients) and constraints produced to the alloy by the cooled ingot mould as a result of a difference between the thermal expansion coefficients of the two materials.

The theoretical model presented here is based on the following working hypotheses:(i)the form (the geometry) of the flowing channel of the melted metal/alloy (with a cylindrical symmetry) demands a cylindrical coordinates system $(r,\ \theta,\ z)$, with the z axis pointed vertically downward, along with the ingot mould channel and the r - coordinate along its radial direction; (ii) we consider the length L of the ingot mould flowing channel much bigger than its radius, R_2, $(L >> R_2)$; (iii) the material (made from a FeBSi alloy) has at the initial moment the temperature $T_m = 1200K$ and (iv) we assume that there are no temperature gradients along the ingot mould channel. The characteristics of the alloy are [2, 3]: $c_p = 530\ J/kg\ K$ is the specific heat, $k_1 = 30\ W/mK$ is the thermal conductibility, $\rho_M = 7.2\cdot 10^3\ kg/m^3$ is the mass density, $E_{alloy} = 2\cdot 10^{11} N/m^2$ is the Young's modulus and $\alpha_{alloy} = 8.7\cdot 10^{-6} K^{-1}$ is the thermal expansion coefficient. The characteristic of the ingot mould are [2,3]: $k_2 = 383\ \ W/mK$ is the thermal conductibility, $L = 15cm$ is the length, $\alpha_{Cu} = 17\cdot 10^{-6} K^{-1}$ is the thermal expansion coefficient, $E_{ingot} = 13\cdot 10^{10} N/m^2$ is the Young's modulus, $R_1 = 1mm$ is the radius of the inner vertical channel and $R_2 = 5mm$ is the total radius.

2. The rapid cooling process of the solidified alloy

2.1. The temperature distribution in the alloy-ingot mould system

Temperature distribution in the alloy. In the following it is analysed the rapid cooling of the solidified metal from the temperature T_g to the room temperature. As it will be shown, from a mathematical point of view, we may consider this as a problem of conduction and thermal transfer with a source, and it implies the determination of the spatio-temporal distribution of the temperature $T_1(r,t)$ of the solidified material, which has in the center of the ingot mould's channel the temperature T_g (T_g=800K is the solidification temperature); the source is distributed on the inner surface of the ingot mould's wall, having the temperature T_w. The heat losses of the alloy due to its forced cooling may be considered as uniformly distributed negative sources. The determination of the temperature $T_1(r,t)$ demands to find the solution of the thermal balance equation for the material submitted to a rapid cooling process:

$$\frac{\partial T_1}{\partial t} = a\left(\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r}\frac{\partial T_1}{\partial r}\right) - b\left(T_1 - T_w\right),\ \text{for}\ 0 \le r \le R_1, \quad (1)$$

where: $a = k_1/(\rho_M c_p)$, $b = (aPL)/V$, $A = \pi R_1^2$, $P = 2\pi R_1$ and L is the ingot mould's length. The general solution of eq.(1) is of the form:

$$T_1(r,t) = T_w + C\, I_0\left(r\sqrt{b/a - m^2}\right)e^{-m^2 at},$$

where C is an integration constant, $I_0\left(r\sqrt{b/a - m^2}\right)$ are modified Bessel functions of the zero order and the constant m will be determined from the thermal conditions on the interface alloy – ingot mould. By imposing the particular conditions: $T_1(r = 0, t = 0) = T_g$ and $\left.\frac{\partial T_1}{\partial r}\right|_{r=0} = 0$ one can obtain the expression for the constant C: $C = T_g - T_w$, which leads us to the expression of the temperature distribution in the material:

$$T_1(r,t) = T_w + (T_g - T_w)I_0\left(r\sqrt{b/a - m^2}\right)e^{-m^2 at}. \quad (2)$$

The temperature distribution in the ingot mould's wall. During the cooling process, the ingot mould's wall receives the heat amount from the cooling alloy. We will consider the temperature distribution into the ingot mould's wall of the form [4]:

$$T_2(r,t) = A_1 \ln(r) + A_2, \quad (3)$$

where A_1 and A_2 are variables depending on the time t: $A_1 \equiv A_1(t)$, $A_2 \equiv A_2(t)$.

Boundary conditions for the alloy – ingot mould interface. In order to determine the final expressions of the temperatures $T_1(r,t)$ and $T_2(r,t)$ we must use the following boundary conditions:

i) **The heat flux from the alloy is received by the ingot-mould**. This heat flux (from the alloy-ingot interface) must be continous. So, for $r = R_1$, we must have:

$$k_1 \left.\frac{\partial T_1}{\partial r}\right|_{r=R_1} = k_2 \left.\frac{\partial T_2}{\partial r}\right|_{r=R_1}, \quad (4)$$

where k_1 and k_2 are the coefficients of thermal conductivity of the alloy and ingot mold, respectively;

ii) **On the alloy-ingot interface ($r = R_1$), the temperatures from the adjacent regions must be equal**:

$$T_1(r = R_1) = T_2(r = R_1), \quad (5)$$

iii) **On the outer surface of the ingot mould, we consider that we have the room temperature, that is**:

$$T_2(r = R_2) = T_w. \quad (6)$$

Using the boundary conditions given by (4) and (6), we obtain the following expressions for A_1 and A_2:

$$A_1(t) = (k_1/k_2)R_1(T_g - T_w)\left(\sqrt{(b/a) - m^2}\right)\times I_1\left(R_1\sqrt{(b/a) - m^2}\right)exp\left(-am^2 t\right), \quad (7)$$

$$A_2(t) = T_w - (k_1/k_2)R_1(T_g - T_w)\left(\sqrt{(b/a) - m^2}\right)\times I_1\left(R_1\sqrt{(b/a) - m^2}\right)\ln(R_2)\,exp\left(-am^2 t\right), \quad (8)$$

and from conditions (5) we obtain the constant m as the solution of equation:

$$I_0\left(R_1\sqrt{(b/a) - m^2}\right) = \left(k_1 R_1/k_2\right)\left(\sqrt{(b/a) - m^2}\right)\times I_1\left(R_1\sqrt{(b/a) - m^2}\right)\ln\left(R_1/R_2\right), \quad (9)$$

where $I_1\left(R_1\sqrt{b/a - m^2}\right)$ are modified Bessel functions of the first order. In order to determine the stresses which appear during the cooling process of the alloy placed into the ingot mould, we will consider the temperature distribution in the alloy (2), where the constant m is given by the solution of eq. (9).

2.2. The internal stresses which appear during the forced cooling process

In this section, our purpose is to analyze the stresses which appear both due to the thermal gradients and from the constraints produced on the alloy by the cooled ingot mould's wall as a result of the difference between the thermal expansion coefficients of the two materials in contact. The radial temperature gradients lead to the appearance of some displacements, both in alloy(u_r^{alloy} and u_z^{alloy}) and in the wall of the ingot (u_r^{ingot} and u_z^{ingot}). These displacements satisfy the differential displacements' equation. In cylindrical coordinates these equations reads [5]:

$$\textbf{1)}\ \frac{d}{dr}\left[\frac{1}{r}\frac{d(u_r^{alloy} r)}{dr}\right] = \frac{1+\mu}{1-\mu}\alpha_{alloy}\frac{dT_1(r,t)}{dr},$$

$$\textbf{2)}\ \frac{du_z^{alloy}}{dz} = const., \quad (10)$$

for alloy and

$$\textbf{1)}\ \frac{d}{dr}\left[\frac{1}{r}\frac{d(u_r^{ingot}r)}{dr}\right]=0\,,\quad \textbf{2)}\ \frac{du_z^{ingot}}{dz}=const.,\quad(11)$$

for the ingot mould's wall. In the above relations α_{alloy} is the alloy's thermal expansion coefficients and μ is the Poisson's coefficient. It is assumed that the values of Poisson's coefficient for alloy and ingot mould are the same: $\mu_{alloy}=\mu_{ingot}=\mu=1/3$ [6]. As it will be shown in section 3, the thermal gradients into the ingot mould's wall are small, compared to the thermal gradients in the alloy; this allows us to further consider, in the calculation of the stresses, only the constriction/dilatation effects of the ingot mould over the alloy, due to the different cooling of the two materials in contact. The solutions of eqs. (10) and (11) (representing both the radial and the axial displacements in the alloy: u_r^{alloy} and u_z^{alloy} and respectively, the radial and axial displacements in the ingot mould: u_r^{ingot} and u_z^{ingot}) lead us [6] to the following expressions of the stresses for the alloy and for the ingot mould's wall:

$$\begin{aligned}
\sigma_{rr}^{alloy}(r,t)&=E_{alloy}C_1-\frac{E_{alloy}\alpha_{alloy}}{r^2(1-\mu)}\int_0^r rT_1(r,t)dr\,,\\
\sigma_{zz}^{alloy}(r,t)&=E_{alloy}C_1-\left(\alpha_{alloy}E_{alloy}T_1(r,t)\right)\big/(1-\mu)\qquad(12)\\
\sigma_{\theta\theta}^{alloy}(r,t)&=E_{alloy}C_1+\frac{E_{alloy}\alpha_{alloy}}{r^2(1-\mu)}\int_0^r rT_1(r,t)dr-\\
&\quad\left(\alpha_{alloy}E_{alloy}T_1(r,t)\right)\big/(1-\mu)\,,
\end{aligned}$$

$$\begin{aligned}
\sigma_{rr}^{ingot}(r,t)&=E_{ingot}C_2-\left(E_{ingot}C_2\right)\big/\left[(1+\mu)r^2\right],\\
\sigma_{\theta\theta}^{ingot}(r,t)&=E_{ingot}C_2+\left(E_{ingot}C_3\right)\big/\left[(1+\mu)r^2\right],\qquad(13)\\
\sigma_{zz}^{ingot}(r,t)&=2E_{ingot}C_2
\end{aligned}$$

where C_1, C_2 and C_3 are three integration constants which will be calculated from the equilibrium conditions and from the condition which considers the different thermal behaviour of the two materials in contact. The resultant strain due to the cooling of two materials with different thermal expansion coefficients which are in contact during the entire cooling process is given by the relation: $\varepsilon=\varepsilon_{alloy}-\varepsilon_{ingot}=(\alpha_{alloy}-\alpha_{ingot})\varDelta T$. In our case, for the alloy we have $\varepsilon_{alloy}=\alpha_{alloy}\varDelta T$ and for the ingot's wall, $\varepsilon_{ingot}=\alpha_{ingot}\varDelta T$, where ε_{alloy} and ε_{ingot} are the strains due to the thermal contraction in the alloy and ingot respectivelly and α_{ingot} is the thermal expansion coefficient of the ingot. In this case, $\varDelta T$ is the difference between the T_g and the room temperature, T_w. In order to determine $\sigma_{rr}^{alloy}(r,t)$, $\sigma_{\theta\theta}^{alloy}(r,t)$ and $\sigma_{zz}^{alloy}(r,t)$ we must find the values of the constants C_1, C_2 and C_3. For the alloy – ingot mould interface we impose the following conditions:

1) the strains that appear in this process are due only to the difference between the thermal expansion coefficients of the alloy and ingot:

$$u_r^{alloy}(r=R_1)-u_r^{ingot}(r=R_1)=\varepsilon R_1\,;\qquad(14)$$

2) the equilibrium conditions at the alloy – ingot mould interface:

$$\sigma_{rr}^{alloy}(r=R_1,t)=\sigma_{rr}^{ingot}(r=R_1,t)\ \text{and}\qquad(15)$$

3) on the exterior surface of the ingot mould $(r=R_2)$: $\sigma_{rr}^{ingot}(r=R_2,t)=0$ .From these three particular conditions, one can obtain the numerical values for the constants C_1, C_2 and C_3 which allow us to determine the stresses (12) in the alloy, during the cooling.

3. Results and discussion

For the given above characterinstics of the FeBSi-type alloy and of an ingot mould with copper wall [2], we calculated the constants A_1 and A_2, and we also determined the constant m by solving eq. (9). In the following, we will consider an ingot mould with the inner channel's radius $R_1=1mm$ and the thickness of the wall ;thus, we will calculate the radial temperature distribution at different moments. Fig. 1 shows the shape of the radial distribution of the temperature, both in the transversal section of the alloy (the red curves) and in the ingot mould's wall (the blue curves), at three different moments, *i.e.*: $t_2=0.2\mu s$ and respectively. As this figure shows, there is a difference in the temperature between the center of the channel and its inner surface; this difference is bigger in the first stages of the cooling process and it becomes less significant at the end of the process.

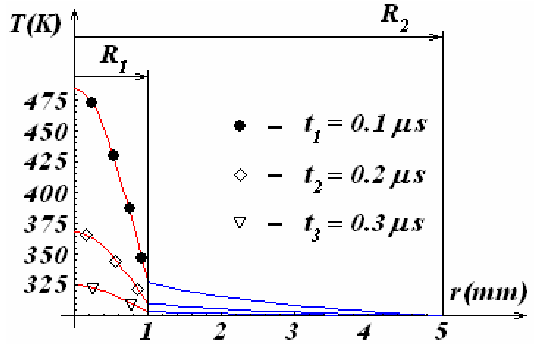


Fig. 1. The radial distribution of the temperature in the mould's transversal section.

One can observe that in the first moments of the alloy's cooling, the amount of heat delivered is bigger; this leads to bigger temperature gradients. For different values of t, the radial distribution of temperature in the ingot mould's wall evolves as follows: after the time $t_1 = 0.1\mu s$, the difference of temperature between the inner wall of the ingot mould and the exterior one is $\Delta T_{R_1R_2} = 27K$, while for $t_3 = 0.3\mu s$, this difference is much smaller $\left(\Delta T_{R_1R_2} = 4K\right)$. Fig. 2 shows the radial distribution of temperature in the ingot mould's section (both in the alloy's transversal section and in the ingot mould's wall) at a time $t_1 = 0.1\mu s$, for three values of the radius of the inner channel: $R_1^{'} = 1mm$, $R_1^{''} = 3mm$ and $R_1^{'''} = 4\,mm$ respectively.

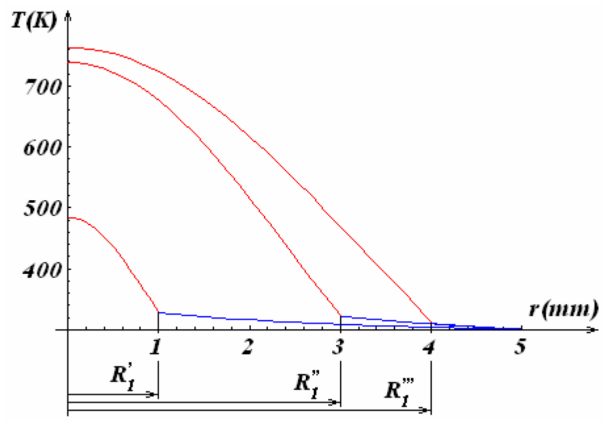
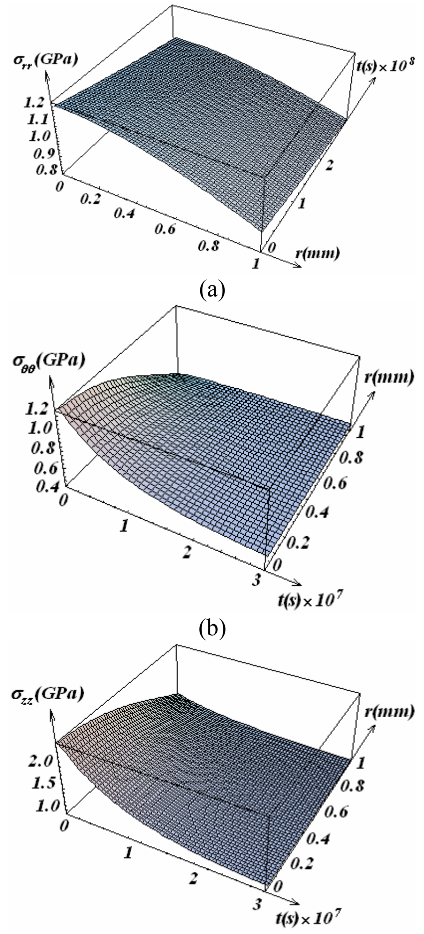


Fig. 2. The radial distribution of the temperature in the ingot mould's ingot section after t_1= 0.1 μs, for three different values of the inner channel's radius.

As one might have expected, after $t_1 = 0.1\mu s$, the temperature in the center of the inner channel of the ingot mould, having the smaller radius ($R_1^{'} = 1mm$) and the thickness of the bigger wall ($g' = 4\,mm$) is smaller than the temperature in the center of the channel having the bigger radius ($R_1^{'''} = 4\,mm$) and the smaller thickness of the wall ($g''' = 1\,mm$); this one will cool slower. One can also notice that, as the thickness of the ingot mould's wall becomes smaller, the temperature difference between the interior surface and the exterior one, $\Delta T_{R_1R_2}$,becomes smaller, *i.e.*, for $R_1^{'} = 1mm$ ($g' = 4\,mm$), $\Delta T_{R_1R_2} = 27K$, while for $R_1^{'''} = 4\,mm$ ($g''' = 1\,mm$), $\Delta T_{R_1R_2} = 10K$ .As for the stresses which appear during the cooling process, by calculating the constants C_1, C_2 and C_3 from the equilibrium conditions and replacing them in eq. (12), one can obtain their explicit form. Fig. 3 represents the graphic spatio-temporal distribution of the radial (a), azimuthal (b) and axial (c) stresses into the alloy.



(a)

(b)

(c)

Fig. 3. The spatio-temporal distribution of the stresses into the alloy: (a) radial stresses, (b) azimuthal stresses, (c) axial stresses.

One can observe that the magnitude order of these stresses is approximately $10^9\,Pa$; as a common feature, they are all positive (tensile) and tend to a saturation value, corresponding to the room temperature, T_w. The axial stresses are approximately two times bigger than the radial ones. Thus, the axial stresses reach the saturation flat interval more quickly than the others. The stresses' drop to a relatively constant value, in the temperature range $(800 \rightarrow 300)\,K$, shows us that the transformed (solidified) material has got a much more regular structure.Furthermore, one can notice that, the smaller the radius of the inner channel of the ingot mould, the bigger the stresses induced into the alloy.

4. Conclusions

The theoretical model described in this paper presents in a synthetical manner the spatio-temporal distribution of the stresses induced in an alloy during its cooling to the room temperature, considering both the thermal behaviour of the alloy and the supplementary stresses induced by the ingot mould's wall, due to the different cooling of the two materials in contact. We thus analyzed and determined the residual stresses that appear during the cooling process of a FeBSi-type alloy, placed in an ingot mould whose walls are maintained at a constant temperature, T_w. The determination of these stresses implies the stray of the thermal behaviour of this type of alloy. For this reason, we first have determined the spatio-temporal distribution of temperature into the inner channel of the ingot mould, as well as in its walls. We then determined the expressions of the indiced stresses in the alloy, considering both the thermal gradients that appear during the cooling process, and the different thermal behaviour of the two materials (FeBSi alloy - Copper) in contact. As for the alloy in the interior channel of the ingot mould, one can notice that, as the time counted from the beginning of the cooling is bigger, the temperature difference between the center of the channel and its inner surface becomes smaller. The alloy reaches the room temperature $T_w = 300\ K$ in approximately $t = 0.3 \mu s$ from the moment of the solidification of the material. As the radius of the inner channel of the ingot mould gets bigger, the temperature at the center of the inner channel of the ingot mould decreases more slowly. Moreover, as the thickness of the ingot mould's wall becomes smaller, the temperature difference between the interior surface and the exterior one, $\Delta T_{R_1 R_2}$, becomes smaller. Regarding the stresses, we deduced that these depend on the radius of the inner channel, and they are positive (tensile). Their order of magnitude is approximately $10^9 Pa$; as a common feature, all these stresses tend to a saturation value, according to the room temperature, T_w. The decrease of the stresses (their relaxation) up to a relatively constant value in the range $(800 \to 300) K$, shows us that the transformed (solidified) material has got a much more regular structure.

References

[1] H. A. Davies, Proc. 3rd Int. Conf. Rapidly Quenched Metals, RQ 3 Conference, Brighton, (1978).

[2] N. H. Pryds, J. H. Hattel, Modelling Simul. Mater. Sci. Eng. **5**, 451 (1997).

[3] E. Vogt, Int. J. Rapid Solidification **3**,131,(1987).

[4] P. J. Schneider, Conduction Heat Transfer, Addison Wesley Publishing Company Inc.(1955).

[5] S. G. Lekhnitskii, Theory of Elasticity of an Anisotropic Body, Mir Publishers Moscow (1980).

[6] B. A. Bosley, J. H. Weiner, Theory of Thermal Stresses, Wiley, New York, 1960.

---

*Corresponding author: iordana@uaic.ro