

# Novel optimization design algorithm for discrete Raman amplifier

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In this letter, a novel optimization design algorithm for discrete Raman amplifiers is proposed. The novel algorithm is based on the nonlinear relation in Raman amplifier and the loop midpoint integration method derived from Newton's iteration and continuation method. The flat gains are obtained through the pump configuration designed by the method in different conditions with different initial seek values and various required performance specification.

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## 1. Introduction

Multi-wavelength pumped fibre Raman amplifiers have recently been recognized as one of the key technologies to realize the ultralong-haul broad-band dense wavelength-division-multiplexing systems due to their distinctive broadband gain and low noise. Broadband gain can be obtained if multiple pumps are employed, and the gain profile can be flattened if the pump configuration is appropriately designed.

Now, there are several different approaches for the pump configuration of Raman amplifier. These methods are simulated annealing algorithm [1], genetic algorithm (GA) [2], neural networks algorithm [3], and the method based on the geometry compensation technique [4]. These methods require a time-consuming process. In this letter, we propose a new algorithm for FRA pump configuration. The novel algorithm is based on the relation between the pump power configuration and the gain profile. The relation is described as the nonlinear function in which the pump powers are the input variables and the gain profile (including average gain and gain flatness) are the output variables. The objective function of the optimization problem is the difference between the nonlinear function and the required gain profile. The optimization problem can be converted into the nonlinear equation problem because of the convergence of iteration array. The nonlinear equation can be solved by Newton's iteration which is improved from conventional Newton's iteration. Continuation method is employed in the algorithm in order to solve the local convergence problem of Newton's iteration, which results in the independent on the starting value. The combination method of Newton's iteration and continuation method is called the loop midpoint integration method.

## 2. Amplifier model

In the steady state, the coupled equation in the multi-wavelength pumped Raman amplifier can be described as [5]:

$$\frac{d}{dz}P_i(z) = \pm P_i(z) \left[ -\alpha_i + \sum_{j=1}^{i-1} g_{ij} \frac{P_j(z)}{K_{ij}A_{ij}} - \sum_{j=i+1}^N g_{ij} \frac{\lambda_j P_j(z)}{\lambda_i K_{ij}A_{ij}} \right] \quad 1 \leq i \leq N \quad (1)$$

Here, the value  $P_i(z)$ ,  $\lambda_i$  and  $\alpha_i$  describe, respectively, the power, wavelength, and the linear attenuation coefficient for the  $i$ th wave.  $g_{ij}$  is the Raman gain coefficient in the fibre between waves  $i$  and  $j$ ,  $A_{ij}$  is the effective area of the fibre, and  $K_{ij}$  is a polarization factor.  $N$  is the total number of pumps and signal channels. The sign  $\pm$  denotes the direction of propagation. This model does not take into consideration spontaneous Raman scattering and Rayleigh backscattering, because these noise do not affect the amplifier gain profile.

The output gain profile can be obtained by solving equation (1) and using the formulas followed.

$$G_i = P_i(L) / P_i(0) \quad i = n+1, n+2, \dots, N \quad (2)$$

$$Flatness = 10 \cdot \log_{10} G_{i_{max}} - 10 \cdot \log_{10} G_{i_{min}} \quad (3)$$

$$AverageGain = \sum_{n+1}^N 10 \cdot \log_{10} G_i / (N - n) \quad (4)$$

Then, the nonlinear function is established based on the relation in the practical amplifier using equation (1)-(4) ( $n$  is the number of pumps channels). The input and output variables are describe as followed,

$$X = [x_1, x_2, x_3, x_4] = [P_1, P_2, P_3, P_4] \quad (5)$$

$$G(X) = [g_1, g_2] = [AverageGain, Flatness] \quad (6)$$

## 3. Optimization algorithm

The optimization problem for gain profile is seeking the appropriate pump configuration in order to flatten the gain and approach to the required gain level. The objective function in the optimization problem is the difference between the nonlinear function expressing the amplifier model and the required gain profile. The optimization process is finding the  $X$  value to minimizing the function  $F(X)$ .

$$F(X) = \text{abs}(G(X) - R), R = [r_1, r_2] \quad (7)$$

Here  $r_1, r_2$  denote the required average gain and gain flatness, respectively.

Because the minimum value of the objective function is zero and the iteration array in the nonlinear equation has convergence, the optimization problem is converted into the solving of the nonlinear equation followed.

$$F(X) = 0 \quad (8)$$

The nonlinear equation can be solved by Newton's iteration method because of its high convergence. The equation of Newton's iteration to solve (8) as

$$X^{k+1} = X^k - F'(X^k)^{-1} F(X^k), k = 0, 1, \dots, \quad (9)$$

The nonlinear function in equation is the vector-valued function. The conventional Newton's iteration must be expanded to this field. The derivative of  $F(X^k)$  is the Jacobi matrix of the vector-valued function. [6]

$$F(X^k) = \begin{bmatrix} \partial g_1 / \partial x_1 & \partial g_1 / \partial x_2 & \partial g_1 / \partial x_3 & \partial g_1 / \partial x_4 \\ \partial g_2 / \partial x_1 & \partial g_2 / \partial x_2 & \partial g_2 / \partial x_3 & \partial g_2 / \partial x_4 \end{bmatrix} \quad (10)$$

It is not feasible to compute these partial derivatives analytically. Rather, each requires a separate computation of  $G(X)$ , followed by the evaluation as

$$\frac{\partial g_i}{\partial x_j} \approx \frac{g_i(x_1, \dots, x_j + \Delta x_j, \dots) - g_i(x_1, \dots, x_j, \dots)}{\Delta x_j} \quad (11)$$

Here, the value of  $\Delta x_j$  is set as  $x_j / 10^{12}$ . The inverse of  $F'(X^k)$  is calculated by using the Moore-Penrose generalized inverses instead of the common inverses. Thus, Newton's iteration can be used in this nonlinear equation.

Furthermore, for the robustness and the convergence speed, Newton's iteration is improved as

$$X^{k+1} = X^k - [F'(X^k) + \lambda_k A]^{-1} F(X), k = 0, 1, \dots, \quad (12)$$

Where  $A$  is the  $2 \times 4$  ones matrix,  $\lambda_k$  is the damping factor in order to eliminate the singularity of  $F'(X^k)$ . For the appropriate value of  $\lambda_k$ , the recall and

search method is utilized. The search times are limited avoiding the limitless search process.

The method above has the dependence on the initial value of iteration process. Once the initial values are far from the final results, Newton's iteration would have no convergence. In order to get rid of the dependence of the method, continuation method is introduced in the propose algorithm. The optimization process is solving the nonlinear equation  $F(X) = 0$ . [7] [8].

Based on the principle of the continuation method, one defines a homotopy or deformation

$$H : R^4 \times [0, 1] \rightarrow R^2$$

such that  $H(x, 0) = F_0(x), H(x, 1) = F(x)$ ,

where  $F_0 : R^4 \rightarrow R^2$  is a smooth map having known zero points and  $H$  is also smooth. Typically, one may choose a convex homotopy such as

$$H(x, \lambda) = F(x) + (\lambda - 1)F(x^0)$$

$$F_0(x) = F(x) - F(x^0) \quad (13)$$

and attempt to trace an implicitly defined curve  $c(s) \in H^{-1}(0)$  from a starting point  $(x^0, 0)$  to a solution point  $(\bar{x}, 1)$ . If this succeeds, then a zero point  $\bar{x}$  of  $F$  is obtained. So the problem has changed into the solution of the equation followed.

$$H(x, \lambda) = F(x) + (\lambda - 1)F(x^0) = 0 \quad (14)$$

If the curve  $c$  can be parametrized with respect to the parameter  $\lambda$ , then the classical embedding methods can be applied. The basic idea in these methods is explained in the following algorithm for tracing the curve from, say  $\lambda = 0$  to  $\lambda = 1$ . Then, the solution of equation above is the solution of initial problem about differential equations followed.

$$\begin{cases} x'(\lambda) = -[F'(x(\lambda))]^{-1} F(x^0) \\ x(0) = x^0 \end{cases} \quad (15)$$

Using the Newton method and the parametrized differential method, the solving numerical method is

$$\begin{cases} x^{k+1} = \phi(x^k, \dots, x^{k-l}; h), k=0,1,\dots, N-1, l < k \\ x^{k+1} = x^k - [F'(x^k)]^{-1} F(x^k), k = N, N+1, \dots \end{cases} \quad (16)$$

Different numerical integration methods for equation (16) result in different embedding methods. In this work, midpoint integration method shown in follow is employed. For the simplified selection of integration step length, the loop continuation method is used called loop midpoint integration method.

$$\begin{cases} x^1 = x^0 - \frac{1}{N} [F'(x^0)]^{-1} F(x^0) \\ x^{k+1/2} = x^k + \frac{1}{2} (x^k - x^{k-1}) \\ x^{k+1} = x^k - \frac{1}{N} [F'(x^{k+1/2})]^{-1} F(x^0) \end{cases} \quad k = 1, \dots, N-1 \quad (17)$$

The calculation steps of the total algorithm are:

Step 0 given initial value  $x^0$  (n dimension vector), step count N, loop count M, required accuracy  $\varepsilon_1, \varepsilon_2$  I=0

Step1 calculate  $F(x^0), F'(x^0)$ , also solving the

equation  $F'(x^0)z = F(x^0)$ , get  $z^0 := [F'(x^0)]^{-1} F(x^0)$

Step 2 base on equation (17), solving  $x^1 = x^0 - \frac{1}{N} z^0$ , k=1

Step3 known  $x^{k-1}, x^k$ , based on the formula (17) above

calculate  $x^{k+1} = x^k - \frac{1}{N} z^k$ , where

$$z^k = [F'(x^{k+1/2})]^{-1} F(x^0), k=k+1$$

Step 4 If  $k < N$  then goto Step 3, otherwise goto Step 5.

Step 5  $x^N \Rightarrow x^0$ , I=I+1, k=0,  $z^0 = \eta$

Step6 calculate k times use the formula

$$x^{k+1} = x^k - [F'(x^k)]^{-1} F(x^k), k = N, N+1, \dots$$

$$x^{k+1} = x^k - z^1,$$

where  $z^1 = [F'(x^k)]^{-1} F(x^k), k = k+1$

Step 7 If  $\|Z^1\| < \|Z^0\|$ , then  $z^0 = z^1$  goto step 9, otherwise goto step 8

Step 8  $x^k \Rightarrow x^0$ , if I < M then goto step 1, otherwise goto step 11

Step 9 If  $\|z^1\| < \varepsilon_1$ , then goto step 11, otherwise goto step 10.

Step 10 If  $\|F(x^{k+1})\| < \varepsilon_2$ , then goto step 11, otherwise goto step 6.

Step 11 print  $x^{k+1}, F(x^{k+1}), \|z^1\|, \|F(x^{k+1})\|, k, l$  then end.

To conclude the description above, we can solve the optimization problem using the nonlinear function model and the combination method of continuation method and improved Newton's iteration method.

#### 4. Numerical analysis and discussion

In the optimization of discrete multipump Raman amplifier, the mathematic model  $F(x) = \text{abs}(G(x)-R)$  will be placed in the proposed algorithm.

In the numerical calculation, we employ the real parameter of Raman amplifier and set that the dispersion compensation fibre module (DCF) ( $A_{\text{eff}} = 19 \mu\text{m}^2$ ) with the length of 8.5 km. The Raman gain spectrum and attenuation spectrum of the fibre are shown in figure 1. The amplifier pumped at 4 wavelengths (1428 nm, 1442 nm, 1463 nm and 1493 nm) operates in the 9 THz (~70 nm) WDM system in which there is 89 signal channels with 100-GHz spacing taking the input signal level at -20 dBm/channel. The control parameters are set N=8, M=20.

All the calculations are carried out on a personal computer with a 2.4 GHz Intel Celeron processor and 512M RAM. The calculation engine is MATLAB R2006a.

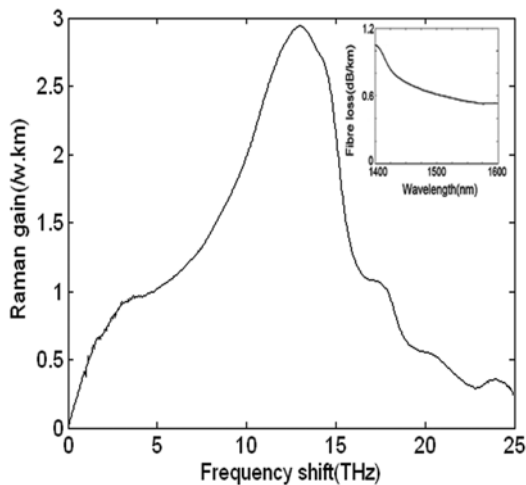


Fig. 1. The Raman gain spectrum  $g_R$  of DCF at pump wavelength  $\lambda_0=1448\text{ nm}$ . Inset shows the linear attenuation spectrum of DCF.

For the feasibility of the proposed algorithm, two schemes have been solved in forward pump and backward pump construction, respectively. Two schemes have different initial pump powers with 100mw, 100mw, 50mw, 50mw and 120mw, 120mw, 120mw, 120mw, respectively. They have required average gain with 12dB. The optimization results in two schemes in different pump construction are shown in Fig. 2.

From Fig. 2, one can see that:

1) The proposed algorithm is independent on the starting seek value, which can solve the optimization problem with the different initial seek valued problem whatever good or poor values.

2) The proposed algorithm is independent on the pump construction, which can obtain the required optimal results at last whatever forward pump or backward pump.

To more clearly understand the merits of the proposed algorithm, the optimization problems for the required gain with 15dB in two pump constructions are taken. Fig. 3 shows that the gain profiles in two pump constructions with the required 15dB gain level.

It is found that, from Fig. 3:

1) The proposed algorithm can also quickly achieve the optimal results in various objective values.

2) With the required gain increase, the optimal gain flatness would be degraded. The optimal gain flatness increases from 0.88dB to 1.02~1.04dB while the gain level increase from 12dB to 15dB,

Base on three groups of the optimization results, the conclusion is achieved that the proposed algorithm has good stability which has the independence on the initial condition, the objective values and the pump construction. This is the robustness aspect of the proposed algorithm.

For the calculation speed and time aspect, the proposed algorithm also has excellent performance. To demonstrate this aspect, the average gain and gain flatness evolutions along loop process are shown in Fig. 4 and Fig. 5 corresponding with the optimization examples in figure 3.

It is found that, from figure 4 and figure 5:

1) The proposed algorithm has fast calculation speed. All optimization samples have good optimal results nearly in 6th loop.

2) The continuation method has good ability of solving the poor starting values from the comparisons of (a) and (b) in forward pump, (c) and (d) in backward pump with different starting values.

To investigate the reason of the fast optimization speed, the iteration times in each optimization cases are listed in table 1. Moreover, the elapsed times of four optimization cases are also listed in Table 1.

From Table 1, one can see that:

1) More iteration process results in more elapsed time.

2) The optimization process in backward pump spend much more time than that in forward pump because of the complicated numerical model of backward pumping amplifier.

3) The continuation method has more effective contribution in the poor starting values which result in less iteration times.

Base on the analysis above, the proposed algorithm has been proved that it has fast optimization speed and good robustness, which can quickly make optimal design for Raman amplifiers. The proposed algorithm has fast optimization speed and good robustness thanks to continuation method and Newton's iteration.

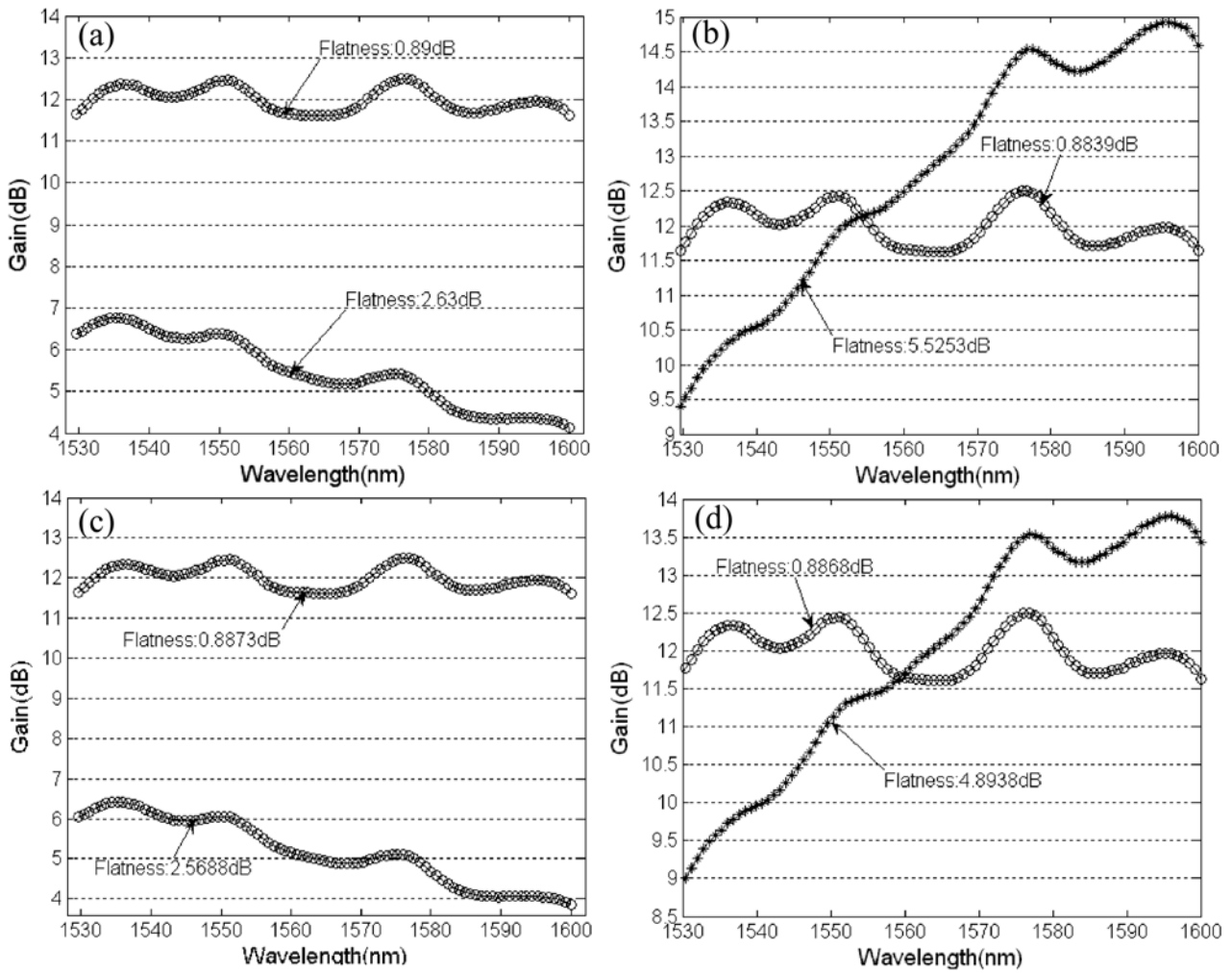


Fig. 2. The optimization results of two schemes in different pump constructions. (a) The gains in scheme one in forward pump. (b) The gains in scheme two in backward pump. (c) The gains in scheme two in forward pump. (d) The gains in scheme two in backward pump.

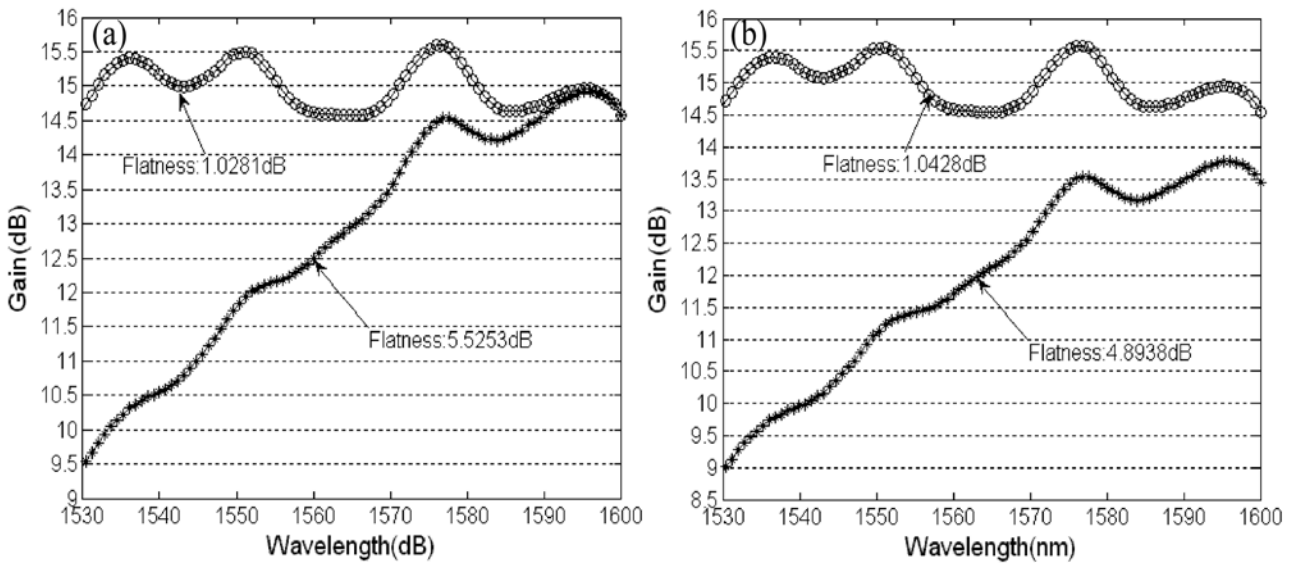


Fig. 3. The optimization results for the required gain 15dB in two pump construction.

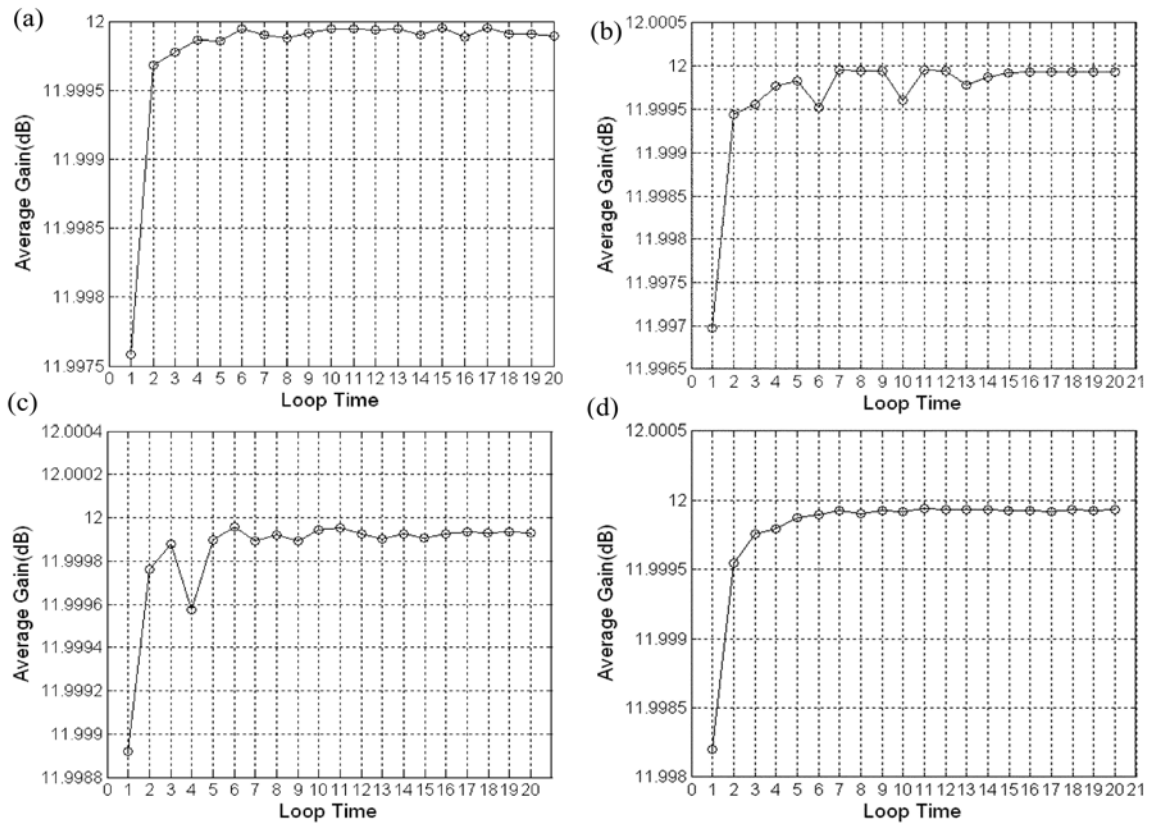


Fig. 4. The average gain evolution along loop process, where (a) for scheme one in forward pump, (b) for scheme two in forward pump, (c) for scheme one in backward pump, (d) for scheme two in backward pump.

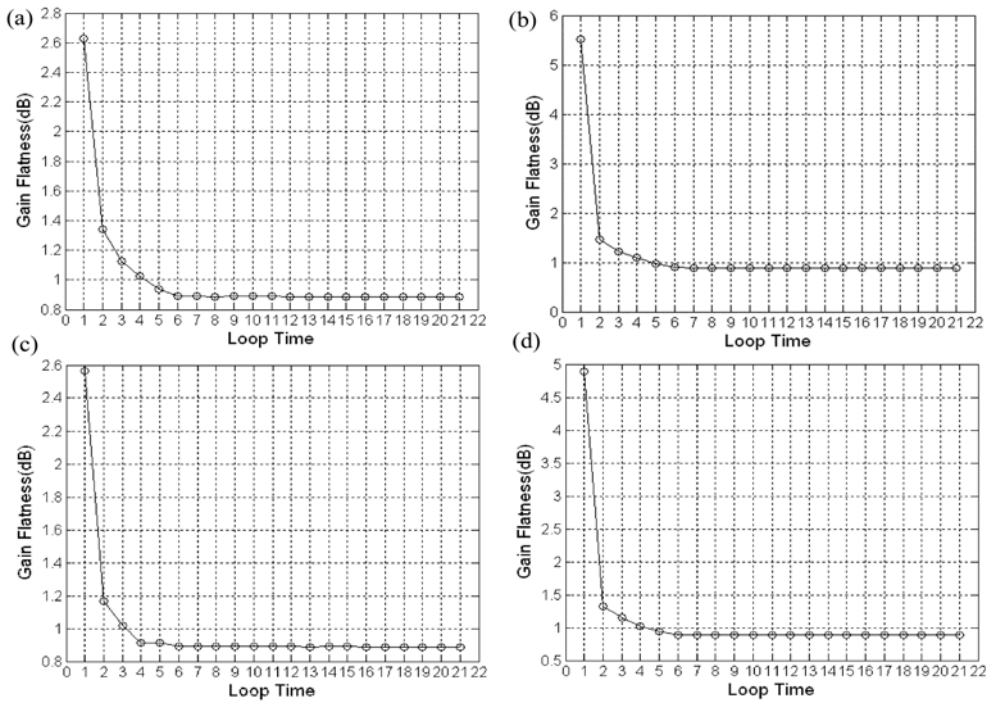


Fig. 5. The gain flatness evolution along loop process, where (a) for scheme one in forward pump, (b) for scheme two in forward pump, (c) for scheme one in backward pump, (d) for scheme two in backward pump.

Table 1. The calculation time and iteration times of optimization process.

Optimization Cases	1 in forward	2 in forward	1 in backward	2 in backward
Iteration Times	53	49	65	44
Elapsed Time	104.49s	90.58s	287.69s	250.01s

## 5. Conclusion

In a conclusion, an efficient method of optimization design for multi-wavelength pump fibre Raman amplifier is proposed. The main feature of this method is that the optimization problem is modelled to a nonlinear equation and the optimization problem is converted into the solution of the nonlinear equation solved by the combination of continuation and improved Newton's iteration called loop midpoint integration method.

Further studies are presently being pursued to increase calculation speed through improving iteration method and the numerical model of amplifiers, and add the optimization of pump number and wavelength.

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