

A theoretical approach of the heat transfer in nanofluids

S. H. MOHORIANU, M. AGOP^a

National Institute of Research & Development for Technical Physics, 47 Mangeron Blvd., 700050, Iasi, Romania

^a“Gh. Asachi” Technical University, Department of Physics, 64 Mangeron Blvd., 700029, Iasi, Romania

Using a reaction-diffusion type equations for the thermal transfer, the dynamics of the nano-particle/liquid interface is analyzed. It results, by a finite differences method, thermal breather, thermal breather pair and thermal cluster type solutions. Through self-organizing of the interface, the thermal transfer in the nano-fluids increases. The model was verified by means of experimental data too.

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1. Introduction

Nanofluid is a new kind of heat transfer medium, containing nanoparticles which are uniformly and stably distributed in a base fluid. These distributed nanoparticles with high thermal conductivity greatly enhance the thermal conductivity of the nanofluid [1-5]. Currently, the origin of such remarkable increases in the thermal conductivity of nanofluids eludes theoretical understanding. Koblinski *et al.* [1], Eastman *et al.* [2], Wang *et al.* [3], Xue [4] and Patel *et al.* [5] suggested various potential mechanisms for the thermal conductivity enhancement such as: the Brownian motion, liquid layering, nanoparticle clustering etc. In the present paper we expose an new mechanism capable of explaining the experimentally observed enhanced thermal conductivity of nanofluid.

2. Fractal structure of the heat transfer process in nanofluids

Usually, the thermal transfer in the nanoparticle/liquid (nP/L) interface is obtained by numerical integration of the thermal equations [2], [6]:

$$\partial_t T_l = k_l (\rho_l c_l)^{-1} \partial_{xx} T_l, \quad l = nP, L$$

with adequate initial and boundary conditions. In these relations T_l is the temperature of the nanoparticle and liquid phases respectively, k_l the thermal conductivity coefficient, ρ_l the density and c_l the specific heat. It results: (i) the maximum value of temperature field at a given moment is localised in the nP/L interface; (ii) the thermal transfer between the nanoparticle and fluid is nonlinear; (iii) the nano-particles fluid system has the self-organizing ability, but does not explain it. The last conclusion is important for our model.

In the model, the dynamics of the (nP/L) interface will be described by the reaction-diffusion type equations for thermal transfer [6-8]:

$$\partial_t T_1 = a_1 \Delta T_1 + c_1 T_2, \quad \partial_t T_2 = a_2 \Delta T_2 + c_2 T_1 \quad (1a, b)$$

where a_1, a_2 are the thermal diffusion coefficients and c_1, c_2 the source factors.

For a plane symmetry and through an adequate normalization of the parameters from (1a,b), i.e.

$$\tau = \omega t, \quad \xi = kx, \quad \eta = ky, \quad \alpha_1 = a_1 k^2 / \omega, \quad \alpha_2 = a_2 k^2 / \omega, \quad (2a-i) \\ \beta_1 = c_1 / \omega, \quad \beta_2 = c_2 / \omega, \quad \phi_1 = T_1 / T_0, \quad \phi_2 = T_2 / T_0$$

where (ω, k, T_0) have the usual significance from [1-3], the equations (1a,b) becomes:

$$\partial_\tau \phi_1 = \alpha_1 (\partial_\xi^2 + \partial_\eta^2) \phi_1 + \beta_1 \phi_2, \quad \partial_\tau \phi_2 = \alpha_2 (\partial_\xi^2 + \partial_\eta^2) \phi_2 + \beta_2 \phi_1 \quad (3a,b)$$

In the general case β_1, β_2 coefficients are functions of ϕ_1, ϕ_2 . In other words, the (3a,b) system is nonlinear and admits nonlinear solutions [9]. To these non-linear solutions we can associate certain physical structures. For example, the breathers (two-dimensional dark solitons) with magnetic domains, the kink with the magnetic flux quanta [10], etc.

We shall solve (3a,b) equations using a finite differences method [11]. For $\alpha_1 = \alpha_2 = 1/3, \beta_1 = -0.2, \beta_2 = 2, 0 \leq \xi \leq 100, 0 \leq \eta \leq 100, \tau = 1.25, 1.5$ and the initial and boundary gaussian conditions, the numerical solutions, i.e. the equal thermal “diffusion” curves, are presented in Figs. 1a, b. It results thermal breather. For $\alpha_1 = \alpha_2 = 1/3, \beta_1 = -1.8, \beta_2 = 1.9, 0 \leq \xi \leq 100, 0 \leq \eta \leq 100, \tau = 0.75, 1.25$ and the initial and boundary gaussian conditions the numerical solutions are presented in Figs. 2a,b. There results thermal breather pairs for decreased time sequences (see Fig. 2a) and thermal clusters for increased time sequences, respectively (see Fig. 2b).

In our opinion, and according to ref. [1], the numerical solutions from Figs. (1a,b) and Figs. (2a,b) are sequences of the heat thermal transfer in nP/L interface. Through self-organizing (correlation in amplitude and phase of the thermal breathers – for details see [12]), one gets first thermal breather pairs and then the thermal clusters.

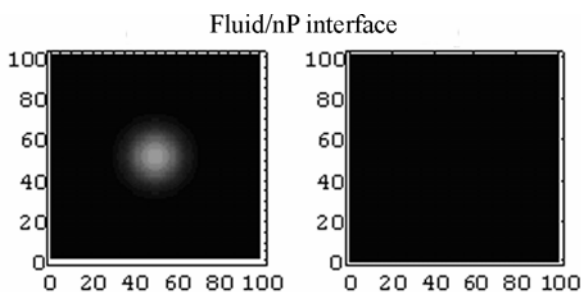


Fig. 1a (Thermal breather)

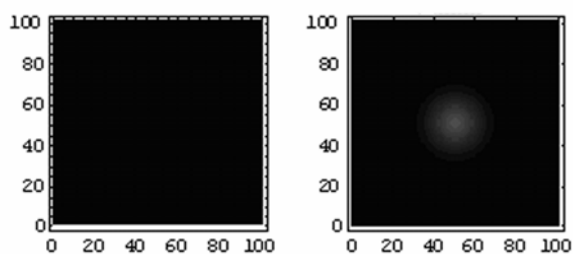


Fig. 1b (Thermal breather)

Fig. 1. The solution of the system (3a,b) for $\alpha_1=\alpha_2=1/3$, $\beta_1=-0.2$, $\beta_2=2$, $\tau=1.25$; 1.5 and the initial and boundary Gaussian conditions.

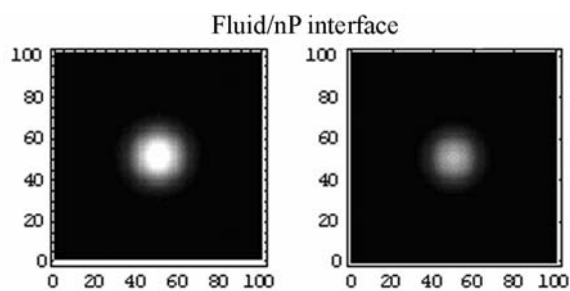


Fig. 2a (Thermal breather pair)

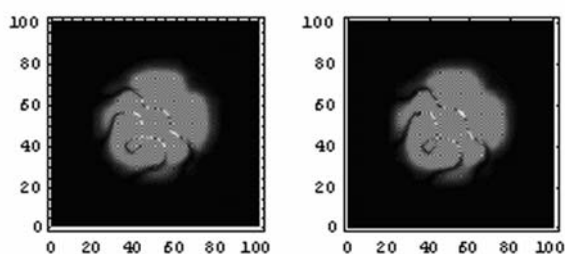


Fig. 2b (Thermal cluster)

Fig. 2. The solution of the system (3a,b) for $\alpha_1=\alpha_2=1/3$, $\beta_1=-1.8$, $\beta_2=1.9$, $\tau=0.75$; 1.25 and the initial and boundary Gaussian conditions.

The self-organizing of nP/L interface is a fractal process since the equal thermal “diffusion” curves are of

Koch type (the fractal dimension is $D \approx 1.26$ - see the similitude in Figs. 3a-e). For other details see [12,13].

It results that $\beta = \beta_1 = \beta_2 \leq 2$ is the optimal value of the fractaling effects, i.e of the self-organizing of the nP/L interface (for details see - [13]). Since from [1]

$$\tau = \frac{d^2 c_p \rho}{6k}$$

(τ is the time required for heat to move in the liquid by the distance equal to the particle size, d is the nano-particle diameter, k is the thermal conductivity of the “composite material” (mixture nano-particle/liquid), ρ is the density of the “composite material”, c_p is the specific heat of the “composite material”), and (3a,b) the parameter β defines the ratio between the time required for heat to move in the liquid along the d distance in the absence of the fractaling effects (τ) and in their presence (τ_f), one gets the restriction (the stability criterion of the thermal transfer in nano-fluid) $\delta = \tau/\tau_f \leq 2.0$. It results that, through the fractaling effects (self-organizing of the nP/L interface) one gets the increasing of the heat transfer into the nP/L interface. This restriction is verified by means of the experimental data [1,2]. For example: using Al_2O_3 particles ≈ 13 nm in diameter and water under stationary conditions, the ratio is $\delta \approx 1.3$; using Cu particles ≈ 10 nm in diameter and water under stationary conditions, the ratio is $\delta \approx 1.4$.

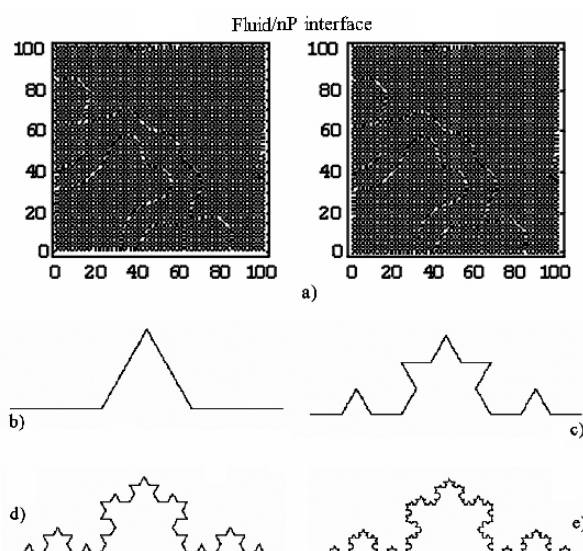


Fig 3. (a) The solution of the system (3a,b) for $\alpha_1=\alpha_2=1/3$, $\beta_1=-1.8$, $\beta_2=1.9$, $\tau=2$ and the initial and boundary Gaussian conditions; (b-e) Koch thermal curves at 1 to 4 iteration levels.

3. Conclusions

The main conclusions of this paper are as follow:

- (i) Using a reaction-diffusion type equations for the thermal transfer, the dynamics of the nP/L interface is analyzed;
- (ii) Thermal breather, thermal breather pair and thermal cluster type solutions are obtained by means of a finite differences method;
- (iii) The self-organizing of the nP/L interface is a fractal process since the equal thermal “diffusion” curves are of Koch type;
- (iv) Through self-organizing, one gets the increasing of heat transfer in the nP/L interface by means of the heat time transfer variation. In such a context, a stability criterion for thermal transfer in nP/L interface is given;
- (v) The model was verified by means of experimental data too;
- (vi) By an extension of this self-organizing process to the fluid-nanoparticles system, it is possible a global increase of the thermal transfer in nanofluids.

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*Corresponding author : sergium@phys-iasi.ro