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On the phase coexistence of band ferromagnetism and singlet superconductivity

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In this work we evaluate the conditions of the superconductivity -band ferromagnetism coexistence using a Green Zubarev function method for a singlet Cooper pairing system, where the ferromagnetic order appears as a consequence of spontaneously broken spin-rotation symmetry. In this way, we get the solutions for the Green functions and the elementary excitations spectrum of the interacting system. Using a spectral representation for the correlation functions, the self-consistent equations for the order parameters are derived and their solutions at $T = 0$ K are discussed. For this case, the relations between ferromagnetic and singlet pairing constants are emphasized. Finally the energy of the system in this state is calculated and we show that the singlet superconductivity-band ferromagnetism phase coexistence is more favorable energetically than the normal phase.

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1. Introduction

The possibility of coexistence of ferromagnetism and superconductivity (SF) was pointed out on earlier time by Ginzburg [1] in the case of that magnetization is less than termodynamic critical field and it was experimentally investigated [2]. The standard way to investigate SF state is to introduce two kinds of fermions: local *f*-electrons which could cause FM and itinerant ones susceptible to produce SC. This point of view was embraced in [3] and in the leading works [4,5] where superconducting materials with magnetic impurities were studied. The case of itinerant electrons developing both ferromagnetism and superconductivity was considered an intricate one until the occurrence of such coexistence was revealed by the recently discovered material as UGe_2 [6,], $ZrZn_2$ [7] and $URhGe$ [8]. Based on these experimental results, the theoretical studies initiated by Kharchev *et. al.* [9] produced further useful works and discussions [10, 11, 12] which lead to establish the main aspects of the phenomenon and further targets to be studied.

In our work we try to evaluate the conditions of the superconductivity-band ferromagnetism coexistence using a Green Zubarev function method for a singlet Cooper pairing system. The aim of the article is to obtain the self-consistency equations for the order parameters and their solutions for $T = 0$ case. Finally the energy of the system is calculated and the phase stability is discussed.

2. Model Hamiltonian and the spectrum of the elementary excitations

We consider a single spin $\frac{1}{2}$ fermions model where the long-range ferromagnetic order appears because of spontaneously broken spin-rotation symmetry.

In order to start up, let us write down the Hamiltonian of our problem:

$$H = \int d^3 r \sum_{\sigma} \psi_{\sigma}^{+}(\vec{r}) \left[-\frac{\nabla^2}{2m} \right] \psi_{\sigma}(\vec{r}) - g \int d^3 r \psi_{\uparrow}^{+}(\vec{r}) \psi_{\downarrow}^{+}(\vec{r}) \psi_{\downarrow}(\vec{r}) \psi_{\uparrow}(\vec{r}) - \\ - \frac{J}{2} \int d^3 r \vec{S}(\vec{r}) \vec{S}(\vec{r}) \tag{1}$$

where ψ_{σ} and ψ_{σ}^{+} are fermionic annihilation and creation field operators with spin σ. In this formula *g (>0)* is pairing coupling constant and *J (>0)* is Heisenberg ferromagnetic exchange integral and μ - the chemical potential. Expressing spin density operators in terms of annihilation and creation field operators $\vec{S}(\vec{r}) = \frac{1}{2} \sum_{\sigma,\sigma'} \psi_{\sigma}^{+}(\vec{r}) \vec{\tau}_{\sigma\sigma'} \psi_{\sigma'}^{+}(\vec{r})$ where $\vec{\tau}_{\sigma\sigma'}$ are Pauli matrices, and retaining only the $\vec{S}_z$ term, we get the model Hamiltonian of our problem in the momentum space:

$$H = \sum_{\vec{k},\sigma} \left[\varepsilon_{\vec{k}} - \mu \right] a_{\vec{k}\sigma}^{+} a_{\vec{k}\sigma} - g \sum_{\vec{k},\vec{k}'} a_{\vec{k}\uparrow}^{+} a_{-\vec{k}\downarrow}^{+} a_{-\vec{k}'\downarrow} a_{\vec{k}'\uparrow} - \frac{J}{4} \sum_{\vec{k},\vec{k}',\vec{q}} a_{\vec{k}+\vec{q}\uparrow}^{+} a_{\vec{k}'-\vec{q}\downarrow}^{+} a_{\vec{k}'\downarrow} a_{\vec{k}\uparrow} \tag{2}$$

Note that, in the first sum of (2) the chemical potential was normalized due to inclusion of the terms $-\frac{J}{8}$ resulted from the expression of the magnetic terms through annihilation and creation operators and, in the second sum of (2), only the BCS terms was considered.

Supposing than, the interaction described by the model Hamiltonian (2) leads to SF coexistence, the corresponding averages can be expressed in terms of double time Green Zubarev functions:

$$
\begin{aligned}
G_{11}\left(\vec{k},t-t'\right)&=-i\theta\left(t-t'\right)\left\langle\left\{a_{\vec{k}\uparrow}\left(t\right),a^{+}_{\vec{k}\uparrow}\left(t'\right)\right\}\right\rangle,\\
G_{21}\left(\vec{k},t-t'\right)&=-i\theta\left(t-t'\right)\left\langle\left\{a^{+}_{-\vec{k}\downarrow}\left(t\right),a^{+}_{\vec{k}\uparrow}\left(t'\right)\right\}\right\rangle,\\
G_{12}\left(\vec{k},t-t'\right)&=-i\theta\left(t-t'\right)\left\langle\left\{a_{\vec{k}\uparrow}\left(t\right),a_{-\vec{k}\downarrow}\left(t'\right)\right\}\right\rangle,\\
G_{22}\left(\vec{k},t-t'\right)&=-i\theta\left(t-t'\right)\left\langle\left\{a^{+}_{-\vec{k}\downarrow}\left(t\right),a_{-\vec{k}\downarrow}\left(t'\right)\right\}\right\rangle,
\end{aligned}
\tag{3}
$$

where the operators appears in Heisenberg picture, $\{.,.\}$ denotes the anticommutator and $\langle\ldots\rangle$ means the statistical averages on the ground state of interacting system. Following the Green-Zubarev procedure we will write down the time evolution equations for the Green functions (3). Using the generic form

$$
i\frac{dG_{AB}\left(t-t'\right)}{dt}=\delta\left(t-t'\right)\left\langle\left\{A(0),B(0)\right\}\right\rangle-i\theta\left(t-t'\right)\left\langle\left\{\left[A(t),H\right],B(t')\right\}\right\rangle
$$

where $[.,.]$ denotes the commutator and, supposing that only non zero averages are that from (3), the time evolution equations are

$$
\begin{aligned}
i\frac{dG_{11}\left(t-t'\right)}{dt}&=\delta\left(t-t'\right)+\bar{\varepsilon}_{\vec{k}}G_{11}\left(\vec{k},t-t'\right)-\Delta G_{21}\left(\vec{k},t-t'\right)+\frac{J}{4}n_{\downarrow}G_{11}\left(\vec{k},t-t'\right),\\
i\frac{dG_{21}\left(t-t'\right)}{dt}&=-\bar{\varepsilon}_{\vec{k}}G_{21}\left(\vec{k},t-t'\right)-\Delta^{*}G_{11}\left(\vec{k},t-t'\right)-\frac{J}{4}n_{\uparrow}G_{21}\left(\vec{k},t-t'\right),\\
i\frac{dG_{12}\left(t-t'\right)}{dt}&=\bar{\varepsilon}_{\vec{k}}G_{12}\left(\vec{k},t-t'\right)-\Delta G_{22}\left(\vec{k},t-t'\right)-\frac{J}{4}n_{\downarrow}G_{12}\left(\vec{k},t-t'\right),\\
i\frac{dG_{22}\left(t-t'\right)}{dt}&=\delta\left(t-t'\right)-\bar{\varepsilon}_{\vec{k}}G_{22}\left(\vec{k},t-t'\right)-\Delta^{*}G_{12}\left(\vec{k},t-t'\right)+\frac{J}{4}n_{\uparrow}G_{22}\left(\vec{k},t-t'\right),
\end{aligned}
\tag{4}
$$

where we put $\Delta=g\sum_{\vec{k}}\left\langle a_{-\vec{k}\downarrow}a_{\vec{k}\uparrow}\right\rangle$ for the energy gap, $n_{\downarrow(\uparrow)}=\sum_{\vec{k}}\left\langle a^{+}_{\vec{k}\downarrow(\uparrow)}a_{\vec{k}\downarrow(\uparrow)}\right\rangle$ for spin up and down occupation numbers and $\bar{\varepsilon}_{\vec{k}}=\varepsilon_{\vec{k}}-\mu$.

Switching to the time Fourier transform of Green functions in (4), we get the algebraic system:

$$
\begin{cases}
\left(\omega-\bar{\varepsilon}_{\vec{k}}-\dfrac{J}{4}n_{\downarrow}\right)G_{11}\left(\vec{k},\omega\right)+\Delta G_{21}\left(\vec{k},\omega\right)=1\\
\Delta^{*}G_{11}\left(\vec{k},\omega\right)+\left(\omega+\bar{\varepsilon}_{\vec{k}}+\dfrac{J}{4}n_{\uparrow}\right)G_{21}\left(\vec{k},\omega\right)=0\\
\left(\omega+\bar{\varepsilon}_{\vec{k}}-\dfrac{J}{4}n_{\uparrow}\right)G_{22}\left(\vec{k},\omega\right)+\Delta^{*}G_{12}\left(\vec{k},\omega\right)=1\\
\Delta G_{22}\left(\vec{k},\omega\right)+\left(\omega-\bar{\varepsilon}_{\vec{k}}-\dfrac{J}{4}n_{\downarrow}\right)G_{12}\left(\vec{k},\omega\right)=0
\end{cases}
\tag{5}
$$

Now, one can observe that G_{11} and G_{21} can be determined by solving the first two equations from (5) while G_{22} and G_{12} can be obtained by solving the last two. With respect to this observation, the elementary excitations spectrum can be obtained from the existence conditions of the solutions. Therefore, this condition for the first two equations leads to

$$
\omega_{1,2}\left(\vec{k}\right)=-\frac{JM}{4}\pm\sqrt{\tilde{\varepsilon}^{2}_{\vec{k}}+\left|\Delta\right|^{2}}\,,
\tag{6}
$$

where magnetization M is defined as $M=\frac{1}{2}\left\langle S_{z}\right\rangle=\frac{1}{2}\left(n_{\uparrow}-n_{\downarrow}\right)$ and $\tilde{\varepsilon}_{\vec{k}}=\bar{\varepsilon}_{\vec{k}}-\bar{\mu}$ with $\bar{\mu}=\mu-\frac{JM}{8}$. The condition for the last two equations leads to the same formula (6).

The solution of system (7) can be easily written in a matrix form:

$$
\hat{G}\left(\vec{k},\omega\right)=\begin{pmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\end{pmatrix}=\frac{1}{\left(\omega-\omega_{1}\right)\left(\omega-\omega_{2}\right)}\begin{pmatrix}\omega+\tilde{\varepsilon}_{\vec{k}}+\dfrac{J}{4}n_{\uparrow} & -\Delta\\ -\Delta^{*} & \omega-\tilde{\varepsilon}_{\vec{k}}-\dfrac{J}{4}n_{\downarrow}\end{pmatrix}
\tag{7}
$$

3. Self-consistency equations for the order parameters

In order to get the equations for the order parameters Δ and M, we will express the averages from (3) in terms of spectral intensities $J_{\alpha\beta}$. Using the generic formula

$$
J_{\alpha\beta}=\frac{i}{e^{\beta\omega}+1}\left[G_{\alpha\beta}\left(\omega+i\varepsilon\right)-G_{\alpha\beta}\left(\omega-i\varepsilon\right)\right]\quad,\quad\alpha,\beta=1,2
$$

we first compute the spectral intensities then, making use of the spectral representation

$$
\left\langle\hat{A}\left(\vec{k},0\right)\hat{B}\left(\vec{k},0\right)\right\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}J_{AB}\left(\omega\right)d\omega
$$

we will get the averages involved in (3). Therefore

$$
\begin{aligned}
\sum_{\vec{k}}\left\langle a^{+}_{\vec{k}\uparrow}a_{\vec{k}\uparrow}\right\rangle&=n_{\uparrow}=\sum_{\vec{k}}\frac{1}{2E_{\vec{k}}}\left(\frac{\omega_{1}+\bar{\varepsilon}_{\vec{k}}+\dfrac{J}{4}n_{\uparrow}}{e^{\beta\omega_{1}}+1}-\frac{\omega_{2}+\bar{\varepsilon}_{\vec{k}}+\dfrac{J}{4}n_{\uparrow}}{e^{\beta\omega_{2}}+1}\right),\\
\sum_{\vec{k}}\left\langle a^{+}_{\vec{k}\downarrow}a_{\vec{k}\downarrow}\right\rangle&=1-n_{\downarrow}=\sum_{\vec{k}}\frac{1}{2E_{\vec{k}}}\left(\frac{\omega_{1}-\bar{\varepsilon}_{\vec{k}}-\dfrac{J}{4}n_{\downarrow}}{e^{\beta\omega_{1}}+1}-\frac{\omega_{2}-\bar{\varepsilon}_{\vec{k}}-\dfrac{J}{4}n_{\downarrow}}{e^{\beta\omega_{2}}+1}\right),\\
\sum_{\vec{k}}\left\langle a_{-\vec{k}\downarrow}a_{\vec{k}\uparrow}\right\rangle&=\sum_{\vec{k}}\left\langle a^{+}_{\vec{k}\uparrow}a^{+}_{-\vec{k}\downarrow}\right\rangle=\frac{1}{g}=-\sum_{\vec{k}}\frac{1}{2E_{\vec{k}}}\left(\frac{1}{e^{\beta\omega_{1}}+1}-\frac{1}{e^{\beta\omega_{2}}+1}\right).
\end{aligned}
\tag{8}
$$

Here we used the notation $E_{k}=\sqrt{\tilde{\varepsilon}_{\vec{k}}+\left|\Delta\right|^{2}}$. Now if we put for the particles number $n=n_{\uparrow}+n_{\downarrow}$ we have the following form of the equations (8)

$$M=\sum_i \frac{\sinh\left(\beta\frac{JM}{4}\right)}{\cosh\left(\beta\frac{JM}{4}\right)+\cosh\left(\beta E_i\right)},\quad \frac{1}{g}=\sum_i \frac{1}{2E_i}\cdot\frac{\sinh\left(\beta E_i\right)}{\cosh\left(\beta\frac{JM}{4}\right)+\cosh\left(\beta E_i\right)},$$

$$n=\sum_i\left[1+\frac{\tilde{\varepsilon}_i}{E_i}\cdot\frac{\sinh\left(\beta E_i\right)}{\cosh\left(\beta\frac{JM}{4}\right)+\cosh\left(\beta E_i\right)}\right]. \tag{9}$$

It can be verified that the above equations leads to the correct solutions in the particular cases $\left(M=0,\Delta\neq 0\right)$ and $\left(M\neq 0,\Delta=0\right)$ both for $T=0$ and $T\neq 0$.

4. Results and discussion

For the $T=0$ case the form of the eqns. (9) is:

$$\frac{1}{g}=\sum_{\vec{k}}\frac{1}{2E_k}\theta\left(E_k-\frac{JM}{4}\right), \tag{10}$$

$$M=\sum_{\vec{k}}\theta\left(\frac{JM}{4}-E_k\right), \tag{11}$$

$$n=\sum_{\vec{k}}\left[1-\frac{\tilde{\varepsilon}_{\vec{k}}}{E_k}\theta\left(E_k-\frac{JM}{4}\right)\right]. \tag{12}$$

where $\theta\left(x\right)$ is the step function.

The eqn. (10) can be integrated using the standard BCS procedure and we get:

$$e^{\frac{1}{\rho_F g}}\left[\frac{Jn}{8\varepsilon_F}m+\sqrt{\left(\frac{Jn}{8\varepsilon_F}\right)^2 m^2-\left(\frac{|\Delta|}{\varepsilon_F}\right)^2}\right]=\frac{\omega_D}{\varepsilon_F}\left[1+\sqrt{1+\left(\frac{\varepsilon_F}{\omega_D}\right)^2\left(\frac{|\Delta|}{\varepsilon_F}\right)^2}\right], \tag{13}$$

where ω_D -Debye frequency, $\varepsilon_F=\frac{\left(3\pi^2 n\right)^{2/3}}{2m^*}$ -Fermi energy and $m=\frac{2M}{n}$ -magnetization per particle.

The eqns. (11) and (12) can be integrated without any restrictions regarding density of states and the limits of integration and, after we eliminate the chemical potential, μ the result is:

$$\left(\frac{Jn}{8\varepsilon_F}\right)^2 m^2-\left(\frac{|\Delta|}{\varepsilon_F}\right)^2=\frac{1}{4}\left[\left(1+m\right)^{2/3}-\left(1-m\right)^{2/3}\right]^2 \tag{14}$$

The above relation leads to the restriction $\frac{Jnm}{8|\Delta|}>1$, which consistently expresses the physics of the problem.

Now, we will seek for the SF coexistence by searching the common solution of eqns. (13) and (14): $|\Delta|\left(J,g\right)$ and $m\left(J,g\right)$. Fig.1 shows $\frac{|\Delta|}{\varepsilon_F}$ dependence from m for a few values of parameter $\frac{Jn}{8\varepsilon_F}$ as it results from eqn. (14). It can be observed that for $0.8\div 1.3$ range of values for $\frac{Jn}{8\varepsilon_F}$ there are nonzero solutions for the order parameters. We emphasise the linear dependence $|\Delta|=|\Delta|\left(m\right)$ for small values of m [9,13]. The linear approximation becomes better as $\frac{Jn}{8\varepsilon_F}$ values are increasing. In this case, the small magnetic field cannot break the singlet Cooper paired fermions so that superconductivity and ferromagnetism can coexist. For $m\to 1$ the magnetization goes to saturation and, as it can be expected, superconductivity disappears ($|\Delta|\to 0$). This can be interpreted that strong magnetization field breaks the singlet Cooper pairs and all fermions are magnetically aligned. Another feature of the interdependence of the order parameters emphasised by Fig. 1 is that at a fixed value of m, $\frac{|\Delta|}{\varepsilon_F}$ is increasing with the increasing values of $\frac{Jn}{8\varepsilon_F}$. This observation may sustain the statement that the mechanism of the Cooper pairing in SF coexistence is a magnetically mediated interaction between fermions.

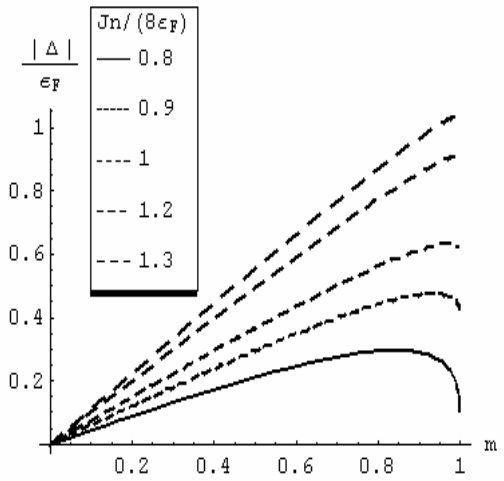

Fig. 1. *The dependence of* $\frac{|\Delta|}{\varepsilon_F}$ *on* m *resulted from eqn. (14).*

Table 1. The domain of values for $\frac{|\Delta|}{\varepsilon_F}$ *and* m *at a fixed value of* $\frac{Jn}{8\varepsilon_F}$ *(a) and at a fixed value of* $e^{\frac{1}{\rho_F g}}$ *(b).*

Tabels (1) and (2) depict the domain of values for $\frac{|\Delta|}{\varepsilon_F}$ *and* m *which eqn. (13) is verified.*

$\frac{\omega_D}{\varepsilon_F}=10^{-1}$, $\frac{Jn}{8\varepsilon_F}=1$		
$e^{\frac{1}{\rho_F g}}$	$\frac{\lvert\Delta\rvert}{\varepsilon_F}\left(\times 10^{-3}\right)$	$m\left(\times 10^{-3}\right)$
20	$8.900 \div 9.020$	$8.900 \div 9.040$
30	$5.960 \div 5.970$	$5.995 \div 6.010$
100	$1.410 \div 1.414$	$5.980 \div 6.010$
$\frac{\omega_D}{\varepsilon_F}=2\cdot 10^{-2}$, $\frac{Jn}{8\varepsilon_F}=1$		
$e^{\frac{1}{\rho_F g}}$	$\frac{\lvert\Delta\rvert}{\varepsilon_F}\left(\times 10^{-3}\right)$	$m\left(\times 10^{-3}\right)$
20	$0.630 \div 0.638$	$1.099 \div 1.102$
30	$0.590 \div 0.598$	$0.797 \div 0.801$
100	$0.280 \div 0.289$	$0.296 \div 0.304$
$\frac{\omega_D}{\varepsilon_F}=10^{-2}$, $\frac{Jn}{8\varepsilon_F}=1$		
$e^{\frac{1}{\rho_F g}}$	$\frac{\lvert\Delta\rvert}{\varepsilon_F}\left(\times 10^{-3}\right)$	$m\left(\times 10^{-3}\right)$
20	$0.770 \div 0.780$	$0.797 \div 0.804$
30	$0.470 \div 0.477$	$0.498 \div 0.504$
100	$0.121 \div 0.130$	$0.137 \div 0.142$

a

$\frac{\omega_D}{\varepsilon_F}=10^{-1}$, $e^{\frac{1}{\rho_F g}}=30$		
$\frac{Jn}{8\varepsilon_F}$	$\frac{\lvert\Delta\rvert}{\varepsilon_F}\left(\times 10^{-3}\right)$	$m\left(\times 10^{-3}\right)$
0.9	$5.240 \div 5.260$	$5.991 \div 6.010$
1.0	$5.960 \div 5.970$	$5.995 \div 6.010$
1.2	$5.960 \div 5.969$	$4.998 \div 5.004$
$\frac{\omega_D}{\varepsilon_F}=2\cdot 10^{-2}$, $e^{\frac{1}{\rho_F g}}=30$		
$\frac{Jn}{8\varepsilon_F}$	$\frac{\lvert\Delta\rvert}{\varepsilon_F}\left(\times 10^{-3}\right)$	$m\left(\times 10^{-3}\right)$
0.9	$0.370 \div 0.380$	$0.798 \div 0.802$
1.0	$0.590 \div 0.598$	$0.797 \div 0.801$
1.2	$0.880 \div 0.888$	$0.797 \div 0.801$
$\frac{\omega_D}{\varepsilon_F}=10^{-2}$, $e^{\frac{1}{\rho_F g}}=30$		
$\frac{Jn}{8\varepsilon_F}$	$\frac{\lvert\Delta\rvert}{\varepsilon_F}\left(\times 10^{-3}\right)$	$m\left(\times 10^{-3}\right)$
0.9	$0.390 \div 0.400$	$0.497 \div 0.504$
1.0	$0.470 \div 0.477$	$0.498 \div 0.504$
1.2	$0.550 \div 0.560$	$0.467 \div 0.474$

b

It can be observed that, for a fixed value of $\frac{Jn}{8\varepsilon_F}$, the values of $\frac{|\Delta|}{\varepsilon_F}$ and m are decreasing with decreasing values of pairing coupling constant g for all fixed values of $\frac{\omega_D}{\varepsilon_F}$. (table (1)). For a fixed value of g, (Table 2) shows a not very strong dependence of $\frac{|\Delta|}{\varepsilon_F}$ and m with increasing values of J. We note that there are another one suit of a range values for $\frac{|\Delta|}{\varepsilon_F}$ and m which satisfy eq (13). For instance at $e^{\frac{1}{\rho_F g}}=30$ and $\frac{Jn}{8\varepsilon_F}=1$, $\frac{|\Delta|}{\varepsilon_F}\in[1.73,\,1.78]\times 10^{-3}$ and $m\in[1.09,1.10]\times 10^{-3}$ is also a solution of (13). It can be easily verify in (13) that, for the range values of the order parameters which lead to the real solution of (13), there is a linear dependence of $\frac{|\Delta|}{\varepsilon_F}$ on m.

The Ground State Energy. The energy of the ground state is usually obtained by averaging the canonical form of Hamiltonian (2). Therefore

$$\langle H\rangle = U = \sum_{\vec{k}}\varepsilon_{\vec{k}}\left\{\left\langle a^{+}_{\vec{k}\uparrow}a_{\vec{k}\uparrow}\right\rangle+\left\langle a^{+}_{\vec{k}\downarrow}a_{\vec{k}\downarrow}\right\rangle\right\}-g\sum_{\vec{k},\vec{k}'}\left\langle a^{+}_{\vec{k}\uparrow}a^{+}_{-\vec{k}\downarrow}\right\rangle\left\langle a_{-\vec{k}'\downarrow}a_{\vec{k}'\uparrow}\right\rangle+$$
$$+\frac{J}{4}\sum_{\vec{k},\vec{k}',\vec{q}}\left\langle a^{+}_{\vec{k}+\vec{q}\uparrow}a_{\vec{k}\uparrow}\right\rangle\left\langle a^{+}_{\vec{k}'-\vec{q}\downarrow}a_{\vec{k}\downarrow}\right\rangle \qquad (15)$$

In addition, making use of the identity $n_\uparrow n_\downarrow=\frac{1}{4}\left[\left(n_\uparrow+n_\downarrow\right)^2-\left(n_\uparrow-n_\downarrow\right)^2\right]=\frac{1}{4}n^2-4M^2$ and of the notation $m=\frac{2M}{n}$ we get from (15):

$$U=\sum_{\vec{k}}\varepsilon_{\vec{k}}\left\{\left\langle a^{+}_{\vec{k}\uparrow}a_{\vec{k}\uparrow}\right\rangle+\left\langle a^{+}_{\vec{k}\downarrow}a_{\vec{k}\downarrow}\right\rangle\right\}-\frac{|\Delta|^2}{g}-\frac{Jm^2n^2}{4}+\frac{Jn^2}{16}, \qquad (16)$$

which express the ground state energy of the interacting system in SF phase.

For $|\Delta|=0$ and $m=0$, (16) gives the ground state energy for the normal phase:

$$U_0=\sum_{\vec{k}}\varepsilon_{\vec{k}}\left\{\left\langle a^{+}_{\vec{k}\uparrow}a_{\vec{k}\uparrow}\right\rangle+\left\langle a^{+}_{\vec{k}\downarrow}a_{\vec{k}\downarrow}\right\rangle\right\}+\frac{Jn^2}{16}. \qquad (17)$$

Now it can be easily observed that

$$U - U_0 = -\frac{|\Delta|^2}{g} - \frac{Jm^2n^2}{4} < 0 \qquad (18)$$

therefore, in the ground state, SF phase is more favourable than the normal phase. The SF phase preference in the ground state of interacting system was also emphasised by a mean field approach in [13]

5. Conclusions

In this paper a Green functions solution for band ferromagnetism and singlet superconductivity coexistence was investigated and shown that for the ground state, there are nontrivial results. The range of values for the order parameters around 10^{-3} emphasises that the coexistence is possible for weak magnetization. This result may sustain the natural interpretation that the weak internal magnetic field does not destroy the superconductivity. Evaluating the energy of the ground state we show that SF coexistence state is more favourable that the normal state. Although the nature of superconducting order parameter is an open one, we show that an s-wave Cooper pairing is qualitatively viable.

As a result of the general part of the theory, the elementary excitations spectrum and self-consistency equations for finite temperatures were obtained. Note that, for $T = 0$ case, the self-consistency equations for the order parameters was established by eliminating the chemical potential using the equation for the particle number. We appreciate this procedure is correct because there is no approximation involved, as in the case of putting ε_F instead μ.

Finally we must say that we choose the Green Zubarev functions theoretical model because of it's possibility to develops calculus in higher order of perturbation theory, opposite with a mean field approach which does not allow further corrections of the results. A second order decoupling of the time evolution Green functions for our system will be the object of a further work.

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