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Reflected and transmitted powers of electromagnetic wave through a double-negative slab

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Reflected and transmitted powers due to the interaction of electromagnetic waves with a double-negative slab are investigated in detail. The formulations for the transverse electric wave case are provided. Transfer matrix method is used in the analysis to find the reflection and the transmission coefficients at each interface. Numerical results are presented for both transverse electric and transverse magnetic wave cases to show the effect of the structure parameters, the incidence angle and the frequency on the reflected and the transmitted powers.

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1. Introduction

A medium in which both the permittivity ε and the permeability μ are simultaneously negative in a frequency band is called the double-negative medium whose electromagnetic features are not found in nature and constructed artificially. There are several terminologies used in literature to refer to the double-negative medium such as metamaterial, left-handed medium, backward wave medium, negative refractive index material, etc. Veselago [1] first introduced the double-negative medium concept in 1967 and investigated various electromagnetic properties of the medium. He showed that the electric field vector $\boldsymbol{E}$, the magnetic field vector $\boldsymbol{H}$ and the wave vector $\boldsymbol{k}$ form a left-handed triplet of vectors, and the Poynting vector $\boldsymbol{S}$ and $\boldsymbol{k}$ are in opposite directions. Pendry [2] investigated the slab of double-negative medium to construct an unconventional lens. In [2], it is shown that "perfect lenses" can be realized by using the double-negative media in the microwave band. Also, a new composite medium which forms a left-handed medium was demonstrated and realized experimentally by Smith *et. al.* in 2000 [3]. Ziolkowski and Heyman extensively studied the wave propagation in double-negative medium both analytically and numerically [4]. Ideas for potential application of double-negative media with negative permittivity and permeability are suggested by Engheta to provide physical remarks and intuitive comments on these ideas [5]. Kong provided a general formulation for the electromagnetic wave interaction with stratified negative isotropic media [6]. In his investigation, the characteristic of the wave in the double negative media, the reflection and the transmission of TE and TM waves by these media are studied. The power analysis of plane waves through a double-negative slab has been studied by Sabah, Ögücü and Uckun to observe the characteristics of it between two different dielectric media [7].

In this paper, a theoretical study of the electromagnetic waves in double-negative slab (DNS) sandwiched between two semi-infinite dielectric media is outlined. In particular, the problem of reflected and transmitted powers of electromagnetic wave is examined. Although the electromagnetic wave interaction with stratified double-negative media is studied in literature, the reflected and transmitted powers and their variations with the structure parameters, the incident angle and the frequency have not been investigated yet. In this work, the incident electric field is assumed to be an electromagnetic plane wave with transverse electric (TE) polarization. The fields in each medium are determined, and the relationship among the incident, the reflected and the transmitted electric and the magnetic fields are derived by imposing the boundary conditions at the interfaces. Then, the reflected and the transmitted powers are obtained using transfer matrix method [8]. Finally, numerical results are presented to illustrate the effect of the structure parameters, the incident angle and the frequency on the reflected and transmitted powers.

2. Reflection and transmission by a DNS

A double-negative medium has interesting properties when it is inserted as a slab between two dielectric media. We consider an electromagnetic plane wave that is propagating within the semi-infinite dielectric medium toward the DNS and then transmitted to the other semi-infinite dielectric medium. The configuration of the structure is shown in Fig. 1. In the analysis, $\exp(j\omega t)$ time dependence is assumed and suppressed.

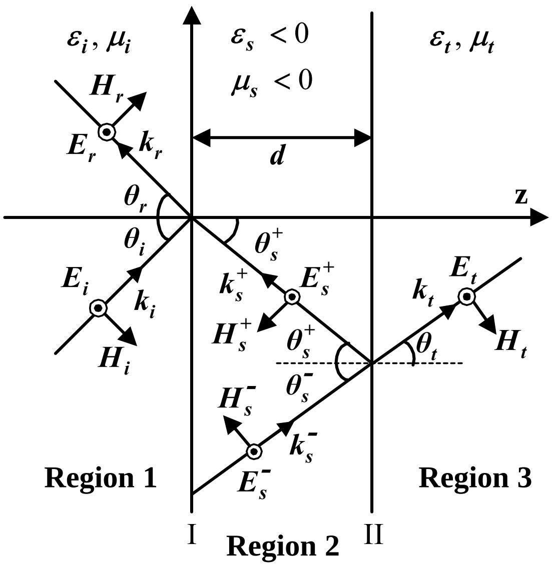

Fig. 1. DNS between two dielectric media.

As shown in Fig. 1, the TE polarized incident electric field in Region 1 can be expressed as

$$\boldsymbol{E}_i = \boldsymbol{a}_y\, E_i \cdot \exp[-jk_i(x\sin\theta_i + z\cos\theta_i)] \quad (1)$$

where k_i is the wave number and E_i is the magnitude of the electric field. It impinges onto the dielectric-DNS interface I at an angle of θ_i. Then the wave is both reflected back and transmitted to the slab, all governed by the continuity of the electric and magnetic fields at the interface. The reflected electric field can be written as

$$\boldsymbol{E}_r = \boldsymbol{a}_y\, E_r \cdot \exp[-jk_r(x\sin\theta_r - z\cos\theta_r)] \quad (2)$$

where k_r is the wave number, E_r is the magnitude, and θ_r is the reflection angle.

The wave in the slab reflects back and transmitted to the second dielectric medium upon reaching the interface II. The electric field in the slab is, thus, the sum of the two field components, namely, the field transmitted to the slab at interface I and the field reflected from the interface II. Hence we can write it as

$$\boldsymbol{E}_s = \boldsymbol{a}_y E_s^+ \cdot \exp[-jk_s(x\sin\theta_s^+ - z\cos\theta_s^+)] + \boldsymbol{a}_y E_s^- \cdot \exp[-jk_s(x\sin\theta_s^- + z\cos\theta_s^-)] \quad (3)$$

where the superscripts + and − signs refer to the backward and the forward waves, respectively, within the slab. Note that the wave number $|k_s| = |\boldsymbol{k}_s^+| = |\boldsymbol{k}_s^-|$, $\boldsymbol{k}_s^+$ being the propagation vector of backward wave and $\boldsymbol{k}_s^-$ the propagation vector of the forward wave as shown in Fig. 1. θ_s^+ and θ_s^- are the transmission and the reflection angles at interface I and interface II, respectively.

The electric field component of the transmitted wave in Region 3 is

$$\boldsymbol{E}_t = \boldsymbol{a}_y\, E_t \cdot \exp[-jk_t(x\sin\theta_t + z\cos\theta_t)] \quad (4)$$

where E_t represent transmitted electric field magnitude, θ_t is the transmission angle and k_t is the wave number of Region 3, respectively. The magnetic fields in all regions can easily be obtained by using Maxwell's equations.

By imposing the continuity of the tangential components of the electric and magnetic fields at the interfaces, both in phase and in magnitude, the transmission and the reflection coefficients at each interface can be found. The continuity of these fields in phase requires

$$k_i \sin\theta_i = k_r \sin\theta_r = k_s \sin\theta_s^+ = k_s \sin\theta_s^- = k_t \sin\theta_t \quad (5)$$

Eq. (5) is known as Snell's law which gives the relation among the incidence, reflection and transmission angles. From Eq. (5) $\theta_s = \theta_s^+ = \theta_s^-$. After applying the boundary conditions at the interfaces with the use of Eq. (5), the relationships among the fields in all regions can be expressed by the transfer matrices which are given in Eq. (6).

$$\begin{bmatrix} E_i \\ E_r \end{bmatrix} = [A] \begin{bmatrix} E_s^+ \\ E_s^- \end{bmatrix}$$

$$\begin{bmatrix} E_s^+ \\ E_s^- \end{bmatrix} = [B][E_t] \quad (6)$$

where $[A]$ is the square matrix of order 2 and $[B]$ is a 2×1 vector. The transfer matrix $[F]$ is defined as the multiplication of $[A]$ and $[B]$ whose elements are expressed as functions of the structure parameters, the incidence angle, the slab thickness and the frequency. From Eq. (6), we can get the reflected and transmitted electric fields in terms of the incident electric field as:

$$E_r = \frac{f_2}{f_1} E_i$$

$$E_t = \frac{1}{f_1} E_i \quad (7)$$

where f_1 and f_2 are the elements of the 2×1 vector $[F] = [A]\cdot[B]$.

The reflected and the transmitted powers carried by the waves can be found by using the reflected and the transmitted electric field expressions in Eq. (7). Note that the incident power is normalized to unity. Then the reflected and transmitted powers can be expressed as

$$P_r = \left| \frac{f_2}{f_1} \right|^2$$
$$P_t = \left| \frac{k_t \mu_i \cos\theta_t}{k_i \mu_t \cos\theta_i} \cdot \frac{1}{f_1} \right|^2 \quad (8)$$

where μ_i and μ_t are the permeabilities of Region 1 and Region 3, respectively, as shown in Fig. 1. Similar formulations can be also derived for the TM polarization case.

3. Numerical results

Since the reflected and the transmitted powers are functions of the structure parameters (the permittivities and the permeabilities of the media), the incidence angle, the slab thickness and the frequency, the reflected and transmitted powers have been calculated as functions of the frequency and the incident angle for different structure parameters. Operation frequency is assumed to be $f_o = 10$ GHz and the slab thickness is $d = \lambda_o / 2$ where λ_o is the wavelength in free space at the operation frequency f_o. The thickness of the slab is assumed to be constant in all calculations. Magnitudes of the permeabilities of three media are select to be equal ($\mu_i = -\mu_s = \mu_t = \mu_o$), and six different cases are considered by changing the permittivities of the media.

Case (a): $|\varepsilon_t| < |\varepsilon_s| < |\varepsilon_i|$ ($\varepsilon_i = 9\varepsilon_o,\ \varepsilon_s = -4\varepsilon_o,\ \varepsilon_t = \varepsilon_o$); the first dielectric medium is denser than the other media and the last dielectric medium is less dense than the DNS. The critical angle at interface I is $\theta_{c1} = 41.8^o$ and at interface II it is $\theta_{c2} = 30^o$. Fig. 2(a) shows the reflected and transmitted powers, P_r and P_t, versus incidence angle. It is seen that total internal reflection occurs for TE and TM waves at the incidence angle greater than or equal to 19.5^{o}, because θ_s becomes greater than θ_{c2} for $\theta_i \geq 19.5^o$.

Case (b): $|\varepsilon_s| < |\varepsilon_t| < |\varepsilon_i|$ ($\varepsilon_i = 9\varepsilon_o,\ \varepsilon_s = -\varepsilon_o,\ \varepsilon_t = 4\varepsilon_o$); the double-negative medium is less dense than the other media. The reflected and transmitted powers versus incidence angle are presented in Fig. 2(b) for this case. Here total internal reflection occurs at the critical angle of 19.5^{o}. But in this case, P_r (P_t) for the TE wave does not go sharply to the value of one (zero) after the critical angle.

Case (c): $|\varepsilon_t| < |\varepsilon_i| < |\varepsilon_s|$ ($\varepsilon_i = 4\varepsilon_o,\ \varepsilon_s = -9\varepsilon_o,\ \varepsilon_t = \varepsilon_o$); The results obtained for this case are shown in Fig. 2(c). The critical angle is 19.5^{o} at interface II and there is no critical angle at interface I. However, as it can be observed form Fig. 2(c) that the total reflection occurs $\theta_i \geq 31^{\text{o}}$ since θ_s becomes greater than 19.5^{o}.

Case (d): $|\varepsilon_i| < |\varepsilon_t| < |\varepsilon_s|$ ($\varepsilon_i = \varepsilon_o,\ \varepsilon_s = -9\varepsilon_o,\ \varepsilon_t = 4\varepsilon_o$); Fig. 2(d) presents P_r and P_t versus incidence angle for this case. Total reflection occurs at 90^{o} for TE and TM waves. P_t becomes nearly unity around 67^{o} for TM wave. It means that Quasi-Brewster angle occurs at this angle of incidence.

Case (e): $|\varepsilon_s| < |\varepsilon_i| < |\varepsilon_t|$ ($\varepsilon_i = 4\varepsilon_o,\ \varepsilon_s = -\varepsilon_o,\ \varepsilon_t = 9\varepsilon_o$); The reflected and transmitted powers as a function of incidence angle are shown in Fig. 2(e) for the case where the DNS is less dense than the others. Characteristics of the reflected and transmitted powers of TE and TM waves are similar to the case (b) except for the total reflection angle. Here the total reflection occurs at incidence angle 30^{o}.

Case (f): $|\varepsilon_i| < |\varepsilon_s| < |\varepsilon_t|$ ($\varepsilon_i = \varepsilon_o,\ \varepsilon_s = -4\varepsilon_o,\ \varepsilon_t = 9\varepsilon_o$); Fig. 2(f) illustrates the reflected and transmitted powers versus incidence angle when the first dielectric medium is less dense than the other media. At 90^{o}, total reflection occurs for both TE and TM waves. Quasi-Brewster angle for TM wave occurs around $\theta_i = 65^{\text{o}}$.

Another important parameter that influences the reflected and transmitted powers is the frequency of operation. In this paper, the reflected and the transmitted powers of the TE and the TM waves are calculated at various frequencies for different values of $\varepsilon_i, \varepsilon_s, \varepsilon_t$. Note that the permeabilities of the media and the thickness of the slab are the same as in the previous examples. It is also worth mentioning that although the permittivity and the permeability of the DNS are functions of the frequency, it is assumed that both parameters are constant over a frequency band in this study. However, this approximation also allows commenting on the effect of the frequency.

P_r and P_t versus frequency are presented in Fig. 3(a) for the case $|\varepsilon_s| < |\varepsilon_t| < |\varepsilon_i|$ ($\varepsilon_i = 9\varepsilon_o,\ \varepsilon_s = -\varepsilon_o,\ \varepsilon_t = 4\varepsilon_o$) at an incidence angle of $\theta_i = 19^{\text{o}}$ that is very close to the angle where total reflection occurs. It is seen that, at low frequency values, both the TE and the TM waves are totally transmitted. However, as the frequency increases, the incident power is almost totally reflected in the TE polarization case. The reflected and the transmitted powers remain almost constant over the frequency range studied for the TM polarization case. This result suggests the possibility of separating the TE and TM components of a general wave in a frequency band for a given incidence angle, at which the transmitted wave is almost the TM polarized wave and the reflected wave is almost the TE polarized wave.

Fig. 3(b) shows the reflected and transmitted powers as a function of frequency for the case $|\varepsilon_i| < |\varepsilon_s| < |\varepsilon_t|$ ($\varepsilon_i = \varepsilon_o$, $\varepsilon_s = -4\varepsilon_o$, $\varepsilon_t = 9\varepsilon_o$). For this case, the incident angle is chosen to be 80°, and it can be deduced that P_r and P_t make oscillations at all frequencies considered.

As can be observed from Fig. 2 and Fig. 3, the conservation of the power is satisfied for both polarizations.

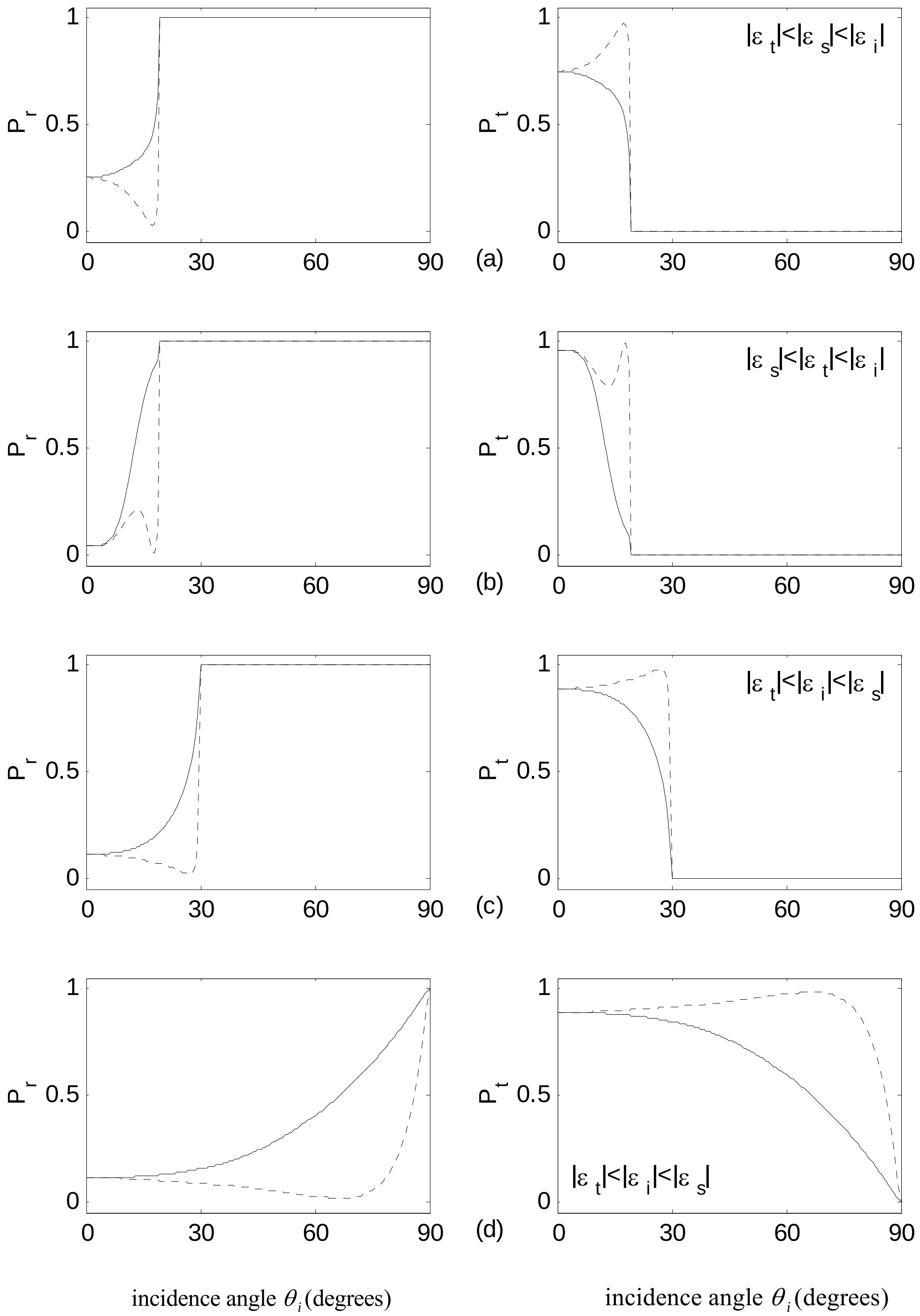

Fig. 2. Reflected and transmitted powers of TE and TM waves versus incidence angle. Figures on the left hand side denote the reflected powers and on the right hand side denote the transmitted powers. The solid lines correspond to the TE wave and the dotted lines correspond to the TM wave.

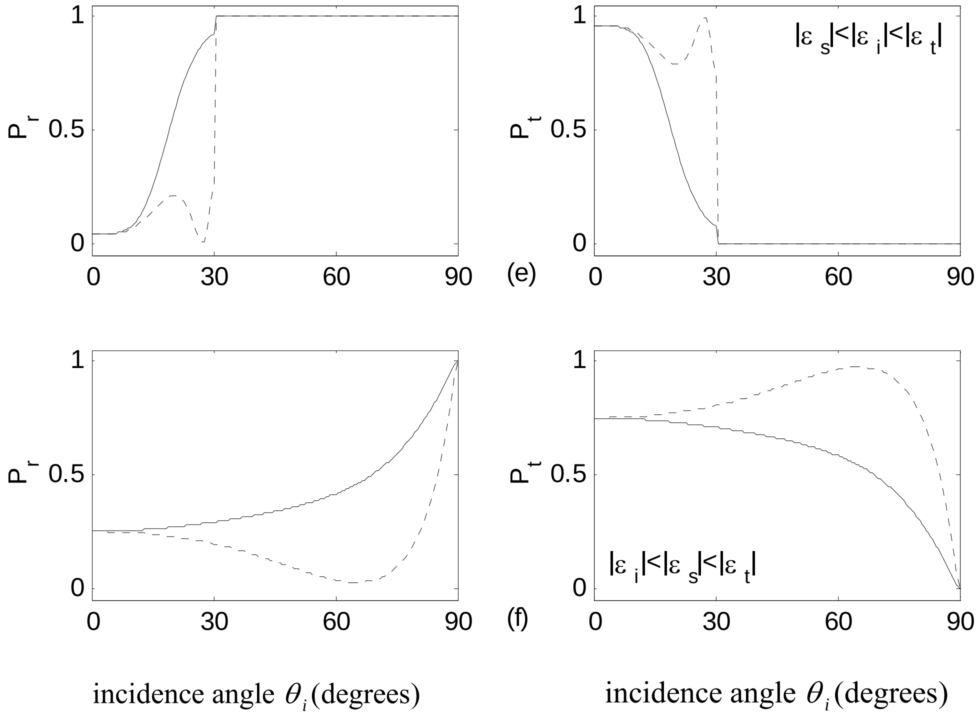

Fig. 2. (Continue) Reflected and transmitted powers of TE and TM waves versus incidence angle. Figures on the left hand side denote the reflected powers and on the right hand side denote the transmitted powers. The solid lines correspond to the TE wave and the dotted lines correspond to the TM wave.

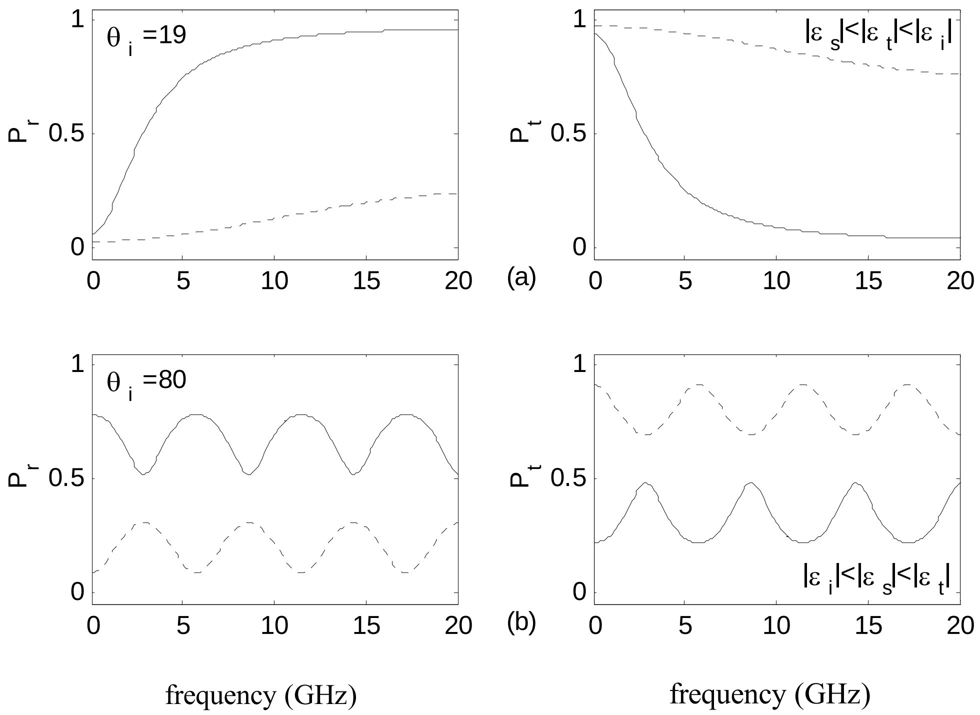

Fig. 3. Reflected and transmitted powers of TE and TM waves as a function of frequency. Figures on the left hand side denote the reflected powers and on the right hand side denote the transmitted powers. The solid lines correspond to the TE wave and the dotted lines correspond to the TM wave.

4. Conclusions

In this paper, the reflected and transmitted powers due to the interaction of electromagnetic waves with a DNS are studied. The electromagnetic field components are written in all regions and by employing the boundary conditions for the tangential electric and magnetic fields, the reflected and the transmitted powers are found by the use of the transfer matrix method. The formulations for the TE wave are given. It is shown that the reflected and the transmitted powers are functions of the structure parameters, incident angle and the frequency. Hence the effect of these parameters on the reflected and the transmitted powers for both TE and TM polarized cases are presented by the numerical results. At a specific frequency of operation, it is deduced that there occurs total internal reflection or total transmission for a given permittivity and permeability set of the structure under investigation. The frequency also changes the behavior of the reflected and the transmitted powers. A special case, where the incidence angle is close to the total reflection angle, is also studied and it is seen that the TE wave is almost totally reflected as the frequency increases as opposed to the TM wave which is almost totally transmitted throughout the frequency range studied. This observation suggests the possibility of separating the TE and TM components of a general wave for a given incidence angle in a frequency band.

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