

# Reversible magnetization processes in scalar Preisach-type models of hysteresis

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The reversible part of magnetization is one of the purely phenomenological defined components of the Preisach-type models. The article presents a new type of reversible distribution based on physical observations and capable to describe the variation of the reversible component with the angle of the applied magnetic field. It also presents a new phenomenological approach for the description of Stoner-Wohlfarth type magnetization loops using Preisach-type models.

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## 1. Introduction

Any magnetisation process implies both irreversible and reversible processes. The irreversible processes are usually associated with the dissipation of energy and with the switching of the magnetic moment from one equilibrium position to another while the reversible magnetic processes are essentially quasistatic processes.

Both of these processes are involved, for example, in the magnetic recording process.

During recording, in the writing process, the magnetic head brings the medium into the desired magnetized state. The aim of this process is to induce irreversible changes in the magnetic state of the medium using a magnetic field strong enough to switch the magnetic moments from a certain region of the medium in order to bring them in the desired magnetic state. When the head leaves a certain area, its magnetic field acts no more on the medium and a remanence magnetization is obtained. While the reading process is taking place in the absence of an external magnetic field, the remanent magnetization is more relevant for the recording performance than the magnetisation during the application of the field. The difference between the values of the two magnetizations – in-field and remanent – is mainly due to reversible processes. In many cases the weight of the reversible part of the magnetization is considerable and, as a consequence, it is very important to describe it correctly in the hysteresis models.

Taking into consideration that the physical root of the reversible magnetization processes is the reversible rotation of the magnetic moment, one may observe that this processes fit well in the frame of the 3D vector magnetization models while the scalar models are usually trying to describe them in a phenomenological manner.

In this paper we are proposing a new approach for the description of the reversible magnetization processes in phenomenological scalar Preisach-type models, an approach which is based on the analysis of the physical

background of the magnetization processes of a single domain ferromagnetic entity.

## 2. Models for magnetic reversibility

The Classical Preisach model (CPM) [1] is a scalar model which was initially developed to describe systems of perfectly aligned single-domain ferromagnetic particles. The external magnetic field is considered to act on the easy axis of the particles. In this case, the hysteresis loop of each particle is rectangular and can be completely characterized mathematically by the value of the switching fields and the saturation magnetic moment. In a system, one considers that the switching fields are distributed which leads to the idea of a Preisach distribution which is, in this case, dealing only with the irreversible magnetization processes. The Preisach distribution also includes a distribution of the interaction fields in the system.

A simple solution for taking into account the reversible magnetization processes was the appending of a new, fully independent distribution of completely reversible hysterons or step operators in the Generalised Preisach Model (GPM) [2], [3] (Fig. 1). The most common shape of the reversible distribution is double-exponential:

$$f_{\text{rev}}(H) = M_S \frac{\gamma}{2H_{c0}} \exp\left(-\gamma \frac{|H|}{H_{c0}}\right) \quad (1)$$

$H_{c0}$  is the most probable value of the coercive field,  $M_S$  is the saturation magnetization of the system,  $M_r$  is the value of the remanent magnetization,  $S$  is the rectangularity of

the major loop  $\left(S = \frac{M_r}{M_S}\right)$  and  $\gamma$  is a fit parameter. The

algorithm for the reversible part identification is simple and involves fitting the segment between saturation and remanence on major hysteresis branch with

$$\int f_{\text{rev}}(H) dH.$$

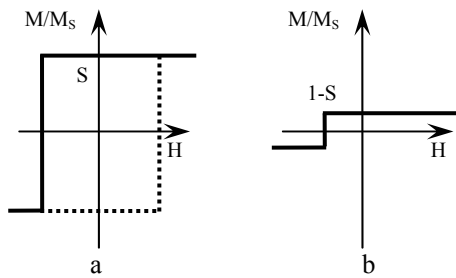


Fig. 1. Typical irreversible (a) and reversible (b) hysterons in GPM.

A more accurate technique of introducing reversible magnetization in Preisach-type models was presented by Della Torre in the series of models – DOK [4], VD-2 [5] and CMH [6]. In these models the reversible component is not independent of the irreversible distribution and instead to each point of the Preisach plane is associated a non-rectangular hysteresis loop (Fig. 2(a)) resulting a reversible component that is distributed all over the plane (while in GPM is a one-dimensional distribution).

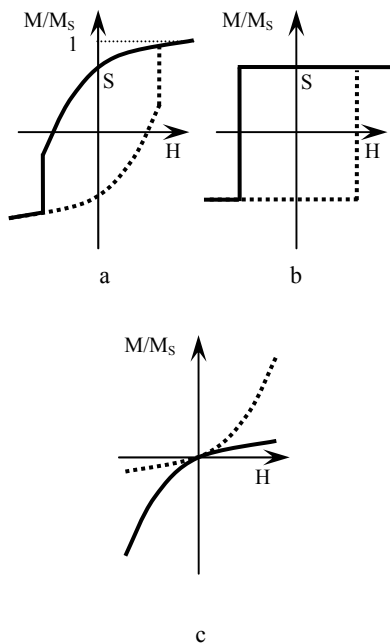


Fig. 2. Typical total (a), irreversible (b) and reversible (c) hysterons in Preisach-type models with distributed reversible part.

These Preisach-type models have tried to give a more correct equivalence between the real reversible magnetization processes and the characteristic Preisach description of the total magnetic moment of the sample. Nevertheless, it was accepted that the reversible processes are not completely given by (1) and that the effects are more complex. In this paper we are analyzing the possibility to completely and accurately include the reversible magnetization processes of the component

particles in the CPM. As a test, we are using the magnetization processes of the well known Stoner-Wohlfarth single domain ferromagnetic particles [7]. The first problem is to find the Preisach representation of the Stoner-Wohlfarth (SW) hysteresis loop.

As can be observed in Fig. 3 the SW loop contains irreversible switches and reversible moment variations. This allows a first decomposition of the SW loop into a rectangular one with the height given by the variation of the moment during the switch (Fig. 3(b)) and another hysteresis loop which contains the reversible variation of the total moment (Fig. 3(c)) computed by simply extracting the irreversible component from the total moment. It is worthy to say that the reversible part is not represented by a mathematical (single valued) reversible function, as expected, but by a hysteretic function as the irreversible component.

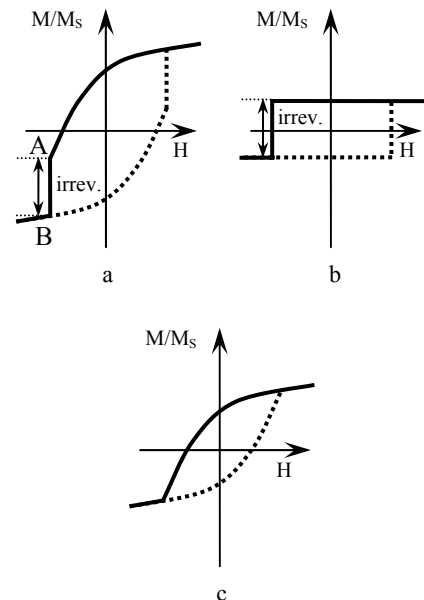


Fig. 3. A single domain particle's hysteresis loop (a) and the proposed irreversible (b) and reversible (c) components.

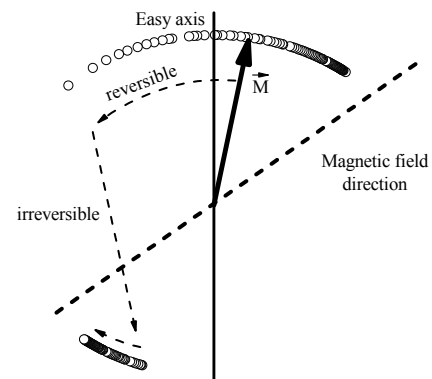


Fig. 4. Trajectory of the magnetic moment vector for a single domain particle.

This approach is validated by the analysis of the trajectory of the magnetic moment when passing from

positive to negative saturation. While the irreversible contribution appears only during the switch, one can associate to this component a value given by the difference between the projections of the magnetic moment in A and B positions (Fig. 4).

### 3. Preisach representation of the SW Loop

In order to represent the magnetization loops of a Stoner-Wohlfarth single domain ferromagnetic particle using a Preisach-type model one must identify the irreversible and irreversible distribution of the phenomenological model.

For the irreversible part it is straightforward to associate a singular distribution in the Preisach plane for the rectangular hysteresis loop (loop (b) in Fig. 3). However, it is obviously impossible to find a stable distribution for the reversible component. As a solution to this problem we propose to define a new type of reversible component, also distributed on the first bisector of the Preisach plane, as it is usually done for the reversible processes (Fig. 5). This distribution can be basically represented with a combination of exponential functions and it switches between the “up” and “down” shapes (represented with solid and dashed line respectively) as a function of the state of the irreversible component (loop (b) in Fig. 3).

By including the angle of the external applied field as a parameter in the algebraic expression of the reversible function one can simulate the magnetization curves for any angle by changing only this parameter.

For the simulations we considered two expressions for the “up” and “down” states given by (2) and (3):

$$f_{\text{rev}}^{\text{up}}(H, \theta, H_k) = S(\theta) \frac{\exp\left(-\frac{H}{H_{\sigma r}(H_k)}\right)}{H_{\sigma r}(H_k)} \quad (2)$$

$$f_{\text{rev}}^{\text{down}}(H, \theta, H_k) = S(\theta) \frac{\exp\left(\frac{H}{H_{\sigma r}(H_k)}\right)}{H_{\sigma r}(H_k)} \quad (3)$$

where  $S$  is a function of  $\theta$ , the angle between the easy axis and the direction of the applied field and  $H_{\sigma r}$  is a function of the anisotropy field  $H_k$ .

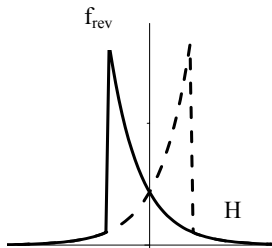


Fig. 5. The modified reversible function used to simulate Stoner-Wohlfarth-type behavior in Preisach-type models.

In order to compute the magnetic response of the system the reversible functions (2) and (3) must be

integrated like in standard Preisach-type model so the magnetization on the upper and lower branch will be:

$$M^{\text{up}}(H_0, \theta, H_k) = 1 - \int_{H_0}^{\infty} f_{\text{rev}}^{\text{up}}(H, \theta, H_k) dH \quad (4)$$

$$M^{\text{down}}(H_0, \theta, H_k) = -1 + \int_{-\infty}^{H_0} f_{\text{rev}}^{\text{down}}(H, \theta, H_k) dH \quad (5)$$

where  $H_0$  is the value of the external applied magnetic field for which the magnetization is calculated.

One can observe that the response of the system is a function of several parameters, which are the same parameters as for the Stoner-Wohlfarth model: the value of the external field, the angle between the field and the easy axis, the value of the anisotropy field. Due to the presence of the singular irreversible distribution the concept of the magnetic history of the system – which is often associated with the Preisach plane – is reduced to only two distinct states: “up” and “down”. This last parameter decides which one of the two expressions – (4) or (5) – must be used.

Fig. 6 presents the result of the Preisach simulation of the descending major hysteresis branches for several angles of the external magnetic field applied to a Stoner-Wohlfarth-type particle for a certain value of the anisotropy field  $H_{k0}$ .

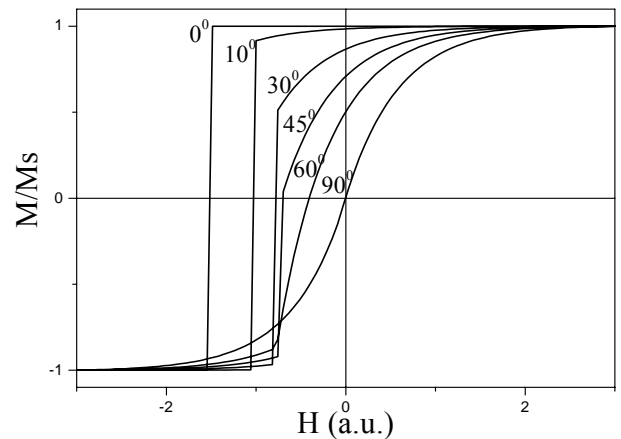


Fig. 6. The major hysteresis branches for different angles of the external applied field.

### 4. Preisach representation of a system made of many SW particles

There are several configurations worth taking into account:

- systems made of aligned particles with a switching fields distribution,
- systems made of partially aligned particles with a switching field distribution,
- systems made of random oriented particles with a switching field distribution.

For each type of systems, the reversible distribution will be obtained by adding the reversible functions for all particles. For continuous distributions, the reversible distribution is a sum of the reversible contributions of all of the particles, computed as integrals:

$$F_{\text{rev}}^{\text{up}}(H) = \int_{\theta} \int_{H_k} f_{\theta}(\theta) f_{H_k}(H_k) f_{\text{rev}}^{\text{up}}(H, \theta, H_k) dH_k d\theta$$

$$F_{\text{rev}}^{\text{down}}(H) = \int_{\theta} \int_{H_k} f_{\theta}(\theta) f_{H_k}(H_k) f_{\text{rev}}^{\text{down}}(H, \theta, H_k) dH_k d\theta$$

(6)

$$F_{\text{rev}}(H) = F_{\text{rev}}^{\text{up}}(H) + F_{\text{rev}}^{\text{down}}(H)$$

where  $f_{\theta}(\theta)$  is the easy axis distribution and  $f_{H_k}(H_k)$  is the anisotropy field distribution. In (6),  $F_{\text{rev}}^{\text{up}}(H)$  refers to the reversible contribution of the positive saturated zones of the Preisach plane and  $F_{\text{rev}}^{\text{down}}(H)$  refers to the reversible contribution of the negative saturated zones of the Preisach plane. This means that the profile of the reversible distribution  $F_{\text{rev}}(H)$  is state-dependent and difficult to identify for complicate states of the Preisach plane. Fig. 7 presents the evolution of the profile of the reversible distribution  $F_{\text{rev}}(H)$  computed using (6) for the case of the descending branch of the major hysteresis loop for a system made of partially aligned particles ( $\theta_0=20^\circ$ ) with a switching field distribution. We have also computed the reversible distribution for the other types of particulate systems mentioned above and we haven't found significant differences between the three cases.

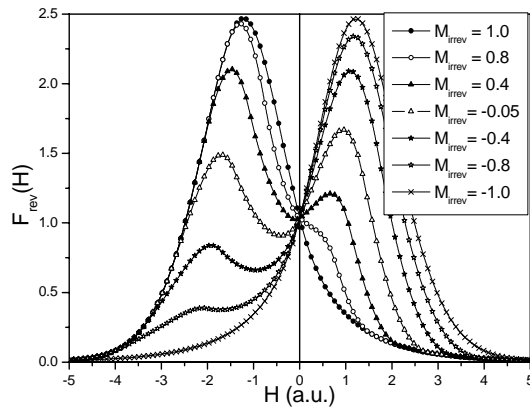


Fig. 7. The reversible distribution for different points on the descending branch of MHL for a system with  $\theta$  and  $H_k$  distributions.

The trend of reversible distribution when  $M$  changes looks similar with the dynamics of the interaction field distribution for the  $PM^2$  model presented in [8] and they have the same physical background – the switching of the magnetic state. This allows us to propose as a good

approximation a simpler algorithm for the identification of the reversible distribution.

If one notes with  $F_{\text{rev}}^+(H)$  and  $F_{\text{rev}}^-(H)$  the profile of the reversible distribution for positive and negative saturation of the irreversible part, then, for an intermediate state – characterized by a value  $M_{\text{irrev}}$  of the irreversible magnetization – the reversible distribution is:

$$F_{\text{rev}}(H) = \frac{M_{\text{irrev}}}{M_{\text{irrev}}^{\text{sat}}} F_{\text{rev}}^+(H) + \frac{(M_{\text{irrev}}^{\text{sat}} - M_{\text{irrev}})}{M_{\text{irrev}}^{\text{sat}}} F_{\text{rev}}^-(H) \quad (7)$$

where  $M_{\text{irrev}}^{\text{sat}}$  is the saturation value of the irreversible component of magnetization. One must mention that  $M_{\text{irrev}}^{\text{sat}}$  value is different from the total saturation magnetization. For example, for a one-quadrant medium – a medium for which the entire Preisach distribution is located in the third quadrant of the Preisach plane – the total saturation and remanence magnetization values are:

$$M_{\text{sat}} = M_{\text{irrev}}^{\text{sat}} + M_{\text{rev}}^{\text{sat}}, \quad (8)$$

$$M|_{H=0} = M_{\text{irrev}}^{\text{sat}} + M_{\text{rev}}|_{H=0}$$

where  $M_{\text{sat}}$  and  $M_{\text{rev}}^{\text{sat}}$  signify the saturation values of total and reversible magnetization, while  $M|_{H=0}$  and  $M_{\text{rev}}|_{H=0}$  denote the corresponding remanent values.

In an even more simpler approach, the  $F_{\text{rev}}^+(H)$  and  $F_{\text{rev}}^-(H)$  functions can be approximated with gauss or log-normal functions making the evaluation of (7) very straightforward.

## 5. Conclusions

Starting from physical considerations we designed a new type of reversible distribution for Preisach-type model capable to correctly describe the shape of the magnetic hysteresis loop of a Stoner-Wohlfarth single domain particle for any angle of the applied magnetic field.

Using as a starting point the results for one particle we calculated the reversible distribution of a system made of many particles and, following the observed dynamics of the distribution, we proposed an easy way to mimic it avoiding heavy calculations.

This study provides a method to avoid more complex representation of the reversible component as a distribution in the Preisach plane, like in the CMH model.

While this method involves only changes of the reversible part it can be fitted to any Preisach-type model in order to describe more complicated systems, like the  $PM^2$  model for strongly interacting particles [8].

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